A crack opening stress equation for in-phase and out-of-phase thermomechanical fatigue loading

Carl Fischer\textsuperscript{a}, Christoph Schweizer\textsuperscript{a}, Thomas Seifert\textsuperscript{b}

\textsuperscript{a}Fraunhofer Institute for Mechanics of Materials IWM, Wöhlerstrasse 11, 79102 Freiburg, Germany
\textsuperscript{b}Offenburg University of Applied Science, Badstrasse 24, 77652 Offenburg, Germany

Abstract

In this paper, a crack opening stress equation for in-phase and out-of-phase thermomechanical fatigue (TMF) loading is proposed. The equation is derived from systematic calculations of the crack opening stress with a temperature dependent strip yield model for both plane stress and plane strain, different load ratios and different ratios of the temperature dependent yield stress in compression and tension. Using a load ratio scaled by the ratio of the yield stress in compression and tension, the equation accounts for the effect of the temperature dependent yield stress and the constraint on the crack opening stress. Based on the scaling relation established in this paper, Newman’s crack opening stress equation for isothermal loading is enabled to predict the crack opening stress under TMF loading.

\textit{Keywords:} Fatigue crack closure, thermomechanical fatigue, strip yield model, crack opening stress equation, constraint
1. Introduction

High temperature components made of nickel-based superalloys such as gas turbine blades or combustion chamber components are exposed to thermomechanical loadings, which can lead to crack initiation and crack growth. The phase angle between temperature and the mechanical loading varies depending on the location. From uniaxial strain controlled thermomechanical fatigue tests it is known that the phase angle has a strong influence on fatigue lives of polycrystalline nickel-based superalloys [1, 2, 3, 4]. At high mechanical loadings, in-phase (IP, phase angle of 0°) TMF loading usually results in shorter fatigue lives than out-of-phase (OP, phase angle of 180°) TMF loading. This tendency is found to be reversed at low mechanical loadings. For phase-shift loading and especially a phase angle of ±90° fatigue lives are generally higher than for both IP and OP TMF-loading.

Due to the temperature dependence of the mechanical properties such as Young’s modulus or the yield stress, strain controlled TMF tests show load ratios with either positive (OP loading) or negative mean stresses (IP loading). This is especially true for high mechanical strain amplitudes and large temperature ranges, where the stresses in tension and compression are bounded by the temperature dependent yield stress. In contrast to isothermal loadings increasing mean stresses do not generally lead to shorter fatigue lives. Hence, it was concluded in [5] that the interaction between the temperature dependent mechanical properties and the history dependent mean stress evolution under thermomechanical fatigue loading has not been fully understood yet.

From a mechanism-based point of view, the influence of the mean stress
on fatigue crack growth is explained with plasticity-induced fatigue crack closure. In fatigue crack growth models, typically an effective (reduced) stress intensity factor

\[ \Delta K_{\text{eff}} = \Delta K \cdot U. \]  

(1)
is used to account for crack closure effects (see e.g. [6, 7]), where \( \Delta K \) is the range of the stress intensity factor and the function \( U \) describes the ratio

\[ U = \frac{\sigma_{\infty,\text{max}} - \sigma_{\text{op}}}{\sigma_{\infty,\text{max}} - \sigma_{\infty,\text{min}}} = \frac{1 - \frac{\sigma_{\text{op}}}{\sigma_{\infty,\text{max}}}}{1 - R_{\sigma}}. \]  

(2)

\( \sigma_{\infty,\text{max}} \) and \( \sigma_{\infty,\text{min}} \) denote the maximum and minimum stresses in a loading cycle and \( \sigma_{\text{op}} \) is the crack opening stress. \( R_{\sigma} \) is the load ratio \( \sigma_{\infty,\text{min}}/\sigma_{\infty,\text{max}} \). By correlating measured fatigue crack growth rates with \( \Delta K_{\text{eff}} \) instead of \( \Delta K \), the influence of the load ratio on the fatigue crack growth rates can be compensated. Various empirical formulas have been proposed to calculate \( U \) as a function of the load ratio \( R_{\sigma} \), see e.g. [7, 8].

An alternative, analytical approach was proposed by Newman [9] on the basis of results obtained with a modified Dugdale model. The so called strip yield model (SYM) accounts for a growing fatigue crack, which leaves plastically deformed material behind the moving crack-tip. In the SYM, the crack opening stress is calculated from the contact stress profile at minimum load. In contrast to the empirical approaches, the SYM accounts for the complete loading history and is often used for the assessment of variable amplitude loading [10, 11]. Based on the results of the SYM, Newman [12] developed a crack opening stress equation, which is capable of describing the influence of the load ratio \( R_{\sigma} \), the ratio \( \sigma_{\text{max}}/\sigma_{Y} \) (\( \sigma_{Y} \) is the yield stress) and the constraint, i.e. plane stress or plane strain.
While crack opening stress equations as e.g. the Newman model are well established for isothermal loading conditions, an analytical crack closure model for nonisothermal conditions does not exist. Thus, the effect of crack closure cannot be reasonably taken into account in TMF applications, resulting in uncertain fatigue life predictions and misinterpretation of experimental results.

Experimental measurements of the crack closure effect under TMF loading red can rarely be found in literature. In [13] the crack closure strain was measured using a potential drop method for Inconel 718. Analogously to Eq. (2) a function $U$ was defined and evaluated for IP and OP TMF tests performed at different strain ratios. It was found, that $U$ varies with the phase angle and the strain ratio. In [14] the effect of dwell times on the thermo-mechanical fatigue crack growth under IP TMF loading between $T = 50 - 550^\circ$C was studied for Inconel 718. The crack closure stress was evaluated for the single-edge notched specimen by monitoring the change of stiffness attributed to the point when the crack faces get into contact during unloading. With increasing dwell time the crack closure stress decreased significantly, but always stayed in the positive stress regime. The effect of IP and OP TMF loading on fatigue crack growth in IN792 was studied by [15] in the temperature range between $T = 100 - 750^\circ$C using a single-edge notched specimen. The IP TMF tests were conducted with a strain ratio of $R_\epsilon = 0$ while the OP TMF tests were performed with $R_\epsilon = -\infty$. The crack closure force detected from the change of stiffness similarly to [14] was always lower for OP TMF loading than for IP TMF loading.

In a recent paper [5] published by the authors, the SYM was modified to
account for temperature dependent elastic and plastic material properties. For two different load ratios under plane stress conditions it was demonstrated, that plasticity-induced crack closure differs strongly for IP and OP TMF loading, as long as the temperature fluctuations go along with a significant change of the yield stress during a TMF cycle. With a mechanism-based lifetime model and the results of the modified SYM it was shown, that IP TMF tests have shorter lifetimes than OP TMF tests at high mechanical loadings and that the lifetime curves overlap at lower mechanical loadings. These effect can partly explained by plasticity-induced fatigue crack closure.

It is the aim of this paper to develop a crack opening stress equation for TMF loading, which is valid for both plane stress and plane strain. To this end, the data base of IP and OP crack opening stresses in [5] is extended by calculations with the SYM for two more load ratios under plane stress and also under plane strain. The extension of the temperature dependent SYM is presented in section 2, while the results for all loading conditions are shown in section 3. Based on the simulation results, a scaling relation is developed in section 4. The scaling relation is able to describe all results obtained with the SYM independent of the phase angle, the constraint and the temperature dependent yield stress. The scaling relation is then used to modify crack opening stress equation from Newman [12] to TMF loading. The temperature dependent SYM and the scaling relation with their limitations are discussed in section 5 and concluded in section 6.
2. Temperature dependent strip yield model for plane stress and plane strain

In the previous work [5] the analytical strip yield model (SYM) from [9] for a center-cracked (infinite) plate with crack length \(2a\) under mode I loading was extended to thermo-cyclic loading by accounting for the temperature dependent elastic and plastic material properties. However, solely plane stress conditions were investigated. Thus, some modifications for plane strain conditions are introduced next, before the plane strain SYM is validated. For the full details on the equations and implementation of the temperature dependent SYM, the (interested) reader is referred to [5].

For assessing plasticity-induced crack closure under plane strain conditions the constraint factors \(\alpha\) and \(\eta\) according to [9] are set to \(\alpha = 3\) and \(\eta = \nu\), where \(\nu\) denotes the Poisson’s ratio. As described in [5], \(\eta\) enters into the calculation of the crack-surface displacements. The length of the plastic zone \(\omega\) in front of the physical crack tip under thermomechanical fatigue loading is computed by:

\[
\omega = \left[ a \left( \cos \left( \frac{\pi \sigma_\infty}{2 \alpha \sigma_Y(T)} \right) \right)^{-1} - 1 \right],
\]

where \(a\) is the physical crack length, \(\sigma_\infty\) is the outer applied stress and \(\sigma_Y(T)\) is the temperature dependent yield stress. For \(\alpha = 1\), i.e. plane stress conditions, the equal equation for \(\omega\) given in [5] is obtained. In order to guarantee that Eq. (3) yields the maximum plastic zone size during a loading cycle, the maximum value of \(\sigma_\infty/\sigma_Y\) within a loading cycle is used \((\sigma_\infty < \sigma_Y)\).

The yield stress ratio \(R_Y = -\sigma_{Y,c}/\sigma_{Y,t}\) defined in [5] \((\sigma_{Y,t}\) is the yield
stress at the maximum applied load in tension, \( \sigma_{Y,c} \) is the yield stress at the minimum applied load in compression), which will be used for the scaling relation derived in section 4, now takes the form:

\[
R_Y = \frac{\sigma_{Y,c}}{\alpha \sigma_{Y,t}}
\]

(4)

Thus, for isothermal loading, where \( \sigma_Y = \sigma_{Y,c} = \sigma_{Y,t} \) is assumed, \( R_Y = 1/\alpha \).

In order to validate the implementation of the SYM under plane strain (constraint factor \( \alpha = 3 \)) conditions, the calculated crack opening stresses are compared in terms of \( U \) from Eq. (2) to the results of the crack opening stress equation of Newman [12]. To this end, temperature independent (i.e. constant) material properties are chosen.

The results are shown in Fig. 1 as a function of the normalized applied maximum stress \( \frac{\sigma_{\infty,\text{max}}}{\alpha \sigma_{Y,t}} \). For all considered load ratios \( R_\sigma \) the results are in good agreement with the crack opening stress equation of Newman, although the validity range of the crack opening equation was originally limited to \( R_\sigma \geq -1 \) [12].

3. Results of the SYM for IP and OP TMF loading under plane stress and plane strain for different load ratios

3.1. Material properties and loading

In this paper and the previous paper [5] the temperature dependent elastic and plastic material properties are taken from the polycrystalline nickel-based superalloy MAR-M247 CC [16]. The material properties are summarized in Table 1 and stem from stabilized LCF hysteresis loops. The yield stress \( \sigma_Y \) is defined as the half of the cyclic yield stress \( \sigma_{CY} \) from Fig. 2: \( \sigma_Y = \sigma_{CY}/2 \).
Figure 1: Calculated crack opening stresses in comparison to the crack opening stress equation of Newman [12] for different load ratios and plane strain.

Table 1: Material properties for MAR-M247 CC from [16].

<table>
<thead>
<tr>
<th>Temperature in °C</th>
<th>$E$ in GPa</th>
<th>$\sigma_Y$ in MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>199</td>
<td>830.5</td>
</tr>
<tr>
<td>300</td>
<td>199</td>
<td>999.5</td>
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<tr>
<td>750</td>
<td>174</td>
<td>909.5</td>
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<tr>
<td>850</td>
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<td>654.5</td>
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<tr>
<td>950</td>
<td>159</td>
<td>421</td>
</tr>
<tr>
<td>1050</td>
<td>142</td>
<td>235.75</td>
</tr>
</tbody>
</table>
Figure 2: Temperature dependent static $R_{p0.2}$ and cyclic 0.2%-offset yield stress $\sigma_{CY}$ for the nickel-base superalloy MAR-M247.

The considered stress-controlled loading cycles for IP and OP TMF loading are shown in Fig. 3. In all simulations the minimum temperature is

Figure 3: Loading history for IP and OP TMF loading.
300 °C, while the maximum temperature is varied between 750 °C and 1050 °C. Depending on the plastic zone size in front of the crack-tip, the number of strip elements in the plastic zone was varied between 5 and 1000.

3.2. Results for plane stress

In Fig. 4 the simulation results for plane stress are shown for $R_\sigma = -R_Y$, $-1$, $-0.5$ and 0. The results for $R_\sigma = -R_Y$ and $R_\sigma = -1$ correspond to those presented in [5]. For all considered load ratios, crack closure is more pronounced under IP TMF loading than under OP TMF loading as long as the minimum and maximum temperatures in the cycle lead to a yield stress ratio $R_Y \neq 1$. The difference increases with increasing maximum temperature. For $R_\sigma \geq -1$ the slope of the IP curves at high $R_Y$ values, i.e. high temperatures, increases. This effect was already discussed in [5] and is associated with a partial loss of the crack face contact in compression. In case of OP TMF loading the dependency of $U$ can only be studied for a certain fraction of $\sigma_{\infty,\max}/(\alpha \sigma_{Y,t})$, because the minimum applied load $\sigma_{\infty,\min}$ reaches the yield stress in compression.

3.3. Results for plane strain

The simulation results for IP and OP TMF loading for $R_\sigma = -R_Y$, $-1$, $-0.5$ and 0 are presented in Fig. 5 for different maximum temperatures and plane strain. Under IP TMF loading plasticity-induced crack closure is generally more pronounced than under OP TMF loading, leading to smaller values in $U$. With decreasing maximum temperature there is only a small difference between IP and OP TMF, because $R_Y$ converges against $\sigma_{\infty,\max}/(\alpha \sigma_{Y,t}) = -0.33$ with $\alpha = 3$. 

10
Figure 4: $U$ for IP- and OP-TMF loading under plane stress conditions with different maximum temperatures for a) $R_\sigma = -R_Y$, b) $R_\sigma = -1$, c) $R_\sigma = -0.5$ and d) $R_\sigma = 0$.

4. A crack opening stress equation for IP and OP TMF loading

4.1. A scaling relation for the crack opening stress under TMF loading

The results of the temperature dependent strip yield model show, that plasticity-induced crack closure and consequently the function $U$ is influenced by the stress ratio, the yield stress ratio (through the maximum temperature), the constraint and the phase angle between mechanical strain and temperature (see Figs. 4 and 5). In order to develop a crack opening stress equation for TMF loading, the primary aim is therefore to find an appropriate representation to consistently describe all IP, OP TMF and isothermal
Figure 5: $U$ for IP- and OP-TMF loading under plane strain conditions with different maximum temperatures for a) $R_\sigma = -R_Y$, b) $R_\sigma = -1$, c) $R_\sigma = -0.5$ and d) $R_\sigma = 0$.

simulation results within a single diagram.

Such a representation is found by correlating $(\sigma_{op} - \sigma_{\infty, min})/\sigma_{Y,c}$ with $\sigma_{\infty, max}/(\alpha \sigma_{Y, t})$ and introducing the yield stress corrected load ratio $R_\sigma^*$ as

$$R_\sigma^* = \frac{R_\sigma}{R_Y}.$$ (5)

For $\sigma_{Y,c} = \sigma_{Y,t}$ it holds e.g. $R_\sigma^* = \alpha R_\sigma$. In Fig. 6 selected isothermal ($\sigma_{Y,c} = \sigma_{Y,t}$) simulation results for plane stress and plane strain are shown. All results can be described in common bands with constant $R_\sigma^*$. Selected TMF and isothermal simulation results are shown in Figs. 7a and 7b for plane stress and plane strain, respectively. Additionally, the isothermal simulation
Figure 6: Scaled crack opening stresses for $\sigma_{Y,c} = \sigma_{Y,t}$ as a function of the constraint factor $\alpha$ and the ratio $R^*_\sigma$ in comparison to the crack opening stress equation from Newman [12].

Results from Fig. 6 are included. As for the isothermal results from Fig. 6, common bands with constant $R^*_\sigma$ ratios are formed. For constant values of $R^*_\sigma$, results from IP TMF, OP TMF and isothermal simulations fall together.

The scaling relation according to $(\sigma_{op} - \sigma_{\infty, min})/\sigma_{Y,c}$ and the usage of
$R_\sigma^*$ lead to a unified correlation of isothermal and nonisothermal simulation results independent of the stress state and the yield stress ratio $R_Y$. Thus, isothermal simulation results (index: $iso$) with temperature independent yield stress can be used to predict results under TMF loading (index: $TMF$). If $\sigma_{\infty,max}^{TMF} |_{TMF} = \sigma_{\infty,max}^{iso} |_{iso}$ and $R_{\sigma,TMF}^* = R_{\sigma,iso}^*$ are fulfilled, then

$$\frac{\sigma_{\infty,max}^{TMF} |_{TMF} - R_{\sigma,TMF}^* \sigma_{Y,c}^{TMF}}{\sigma_{Y,c}^{TMF}} = \frac{\sigma_{\infty,max}^{iso} |_{iso} - R_{\sigma,iso}^* \sigma_{Y,c}^{iso}}{\sigma_{Y,c}^{iso}}. \quad (6)$$

Insertion of the definitions for $R_{\sigma}$ and $R_Y$ leads to:

$$\frac{\sigma_{\infty,max}^{TMF} |_{TMF}}{\sigma_{\infty,max}^{iso} |_{iso}} = \frac{\sigma_{\infty,max}^{TMF} |_{TMF}}{R_{Y,TMF}^* \sigma_{\infty,max}^{iso} |_{iso}} - \frac{R_{\sigma,TMF}^*}{R_{\sigma,iso}^*}. \quad (7)$$

By using $R_{Y,iso}^* = 1/\alpha_{iso}$, Eq. (7) can be solved for $\frac{\sigma_{op} |_{TMF}}{\sigma_{\infty,max}^{TMF}}$:

$$\frac{\sigma_{op} |_{TMF}}{\sigma_{\infty,max}^{TMF}} = R_{Y,TMF}^* \alpha_{TMF} \frac{\sigma_{op} |_{iso}}{\sigma_{\infty,max}^{iso}} - R_{Y,TMF}^* R_{\sigma,iso}^* \alpha_{iso} + R_{\sigma,TMF}^*. \quad (8)$$

With $R_{\sigma,iso}^* \alpha_{iso} = R_{\sigma,iso}^* = R_{\sigma,TMF}^*$ this further simplifies to

$$\frac{\sigma_{op} |_{TMF}}{\sigma_{\infty,max}^{TMF}} = R_{Y,TMF}^* \alpha_{iso} \frac{\sigma_{op} |_{iso}}{\sigma_{\infty,max}^{iso}}. \quad (9)$$

and provides a relationship between the normalized crack opening stress for TMF and isothermal loading as a function of the isothermal constraint factor $\alpha_{iso}$ and the yield stress ratio $R_{Y,TMF}$ (note that $R_{Y,TMF}$ contains $\alpha_{TMF}$).

4.2. Newman’s crack opening stress equation for TMF loading

Eq. (9) states that an analytical expression for the crack opening stress under isothermal loading can be used to predict the crack opening stress under TMF loading. Such an analytical description is e.g. given by Newman [12]. In order to demonstrate its applicability, the results obtained with the
Newman formula from [12] are shown in Fig. 6 together with the isothermal simulation results from the SYM for both plane stress (solid line) and plane strain (dashed line). The application of the scaling relation in Fig. 6 collapses the isothermal plane stress and plane strain curves into narrow bands. Since the Newman equation for plane stress ($\alpha = 1$) gives a better description of the simulation results over the whole range of $\sigma_{\infty,\text{max}}/(\alpha \sigma_Y,t)$, plane stress is considered in the following to predict the crack opening stresses under TMF loading.

In Fig. 7 the Newman formula for plane stress (solid line) is used to predict the scaled crack opening stresses under IP and OP TMF loading for plane stress and plane strain. For $\sigma_{\infty,\text{max}}/(\alpha \sigma_Y,t) << 1$ the agreement with the simulation results is generally good for all values of $R_\sigma^*$. For $\sigma_{\infty,\text{max}}/(\alpha \sigma_Y,t) \to 1$, some visible deviations occur for $R_\sigma^* = -1$, which are also present in Fig. 6.

The full set of equations from Newman [12] (index: New) for plane stress ($\alpha_{\text{iso}} = 1$) is finally summarized in order to calculate the function $U$ according to Eq. (2). Replacing $\left.\frac{\sigma_{\text{op}}}{\sigma_{\infty,\text{max}}}\right|_{\text{ISO}}$ in Eq. (9) by $\left.\frac{\sigma_{\text{op}}}{\sigma_{\infty,\text{max}}}\right|_{\text{New}}$ yields

$$
\left.\frac{\sigma_{\text{op}}}{\sigma_{\infty,\text{max}}}\right|_{\text{TMF}} = R_{Y,\text{TMF}} \left.\frac{\sigma_{\text{op}}}{\sigma_{\infty,\text{max}}}\right|_{\text{New}}
$$

with

$$
\left.\frac{\sigma_{\text{op}}}{\sigma_{\infty,\text{max}}}\right|_{\text{New}} = A_0 + A_1 R_{\sigma,\text{TMF}}^* + A_2 R_{\sigma,\text{TMF}}^{*2} + A_3 R_{\sigma,\text{TMF}}^{*3} \quad \text{for } R_{\sigma,\text{TMF}}^* \geq 0,
$$

$$
\left.\frac{\sigma_{\text{op}}}{\sigma_{\infty,\text{max}}}\right|_{\text{New}} = A_0 + A_1 R_{\sigma,\text{TMF}}^* \quad \text{for } R_{\sigma,\text{TMF}}^* < 0.
$$
and

\[ A_0 = 0.535 \cos \left( \frac{\pi \sigma_{\infty, \max}}{2 \alpha \sigma_{Y, t}} \right)_{\text{TMF}}, \quad (13) \]

\[ A_1 = 0.344 \frac{\sigma_{\infty, \max}}{\alpha \sigma_{Y, t}}_{\text{TMF}}, \quad (14) \]

\[ A_2 = 1 - A_0 - A_1 - A_3 \quad \text{and} \]

\[ A_3 = 2A_0 + A_1 - 1. \quad (15) \]

\[ 4.3. \text{Validation of the scaling relation} \]

The scaling relation from Eq. (9) is now used to predict \( U \) for IP and OP TMF loading as a function of the maximum temperature and \( \sigma_{\infty, \max}/(\alpha \sigma_{Y, t}) \) for \( R_\sigma = -R_Y \) and \( R_\sigma = 0 \) in the case of plane stress and plane strain. According to Eq. (2) the function \( U \) can be calculated for TMF loading with:

\[ U = \frac{1 - \sigma_{op}}{\sigma_{\infty, \max}}_{\text{TMF}}, \quad (17) \]

For a given TMF loading cycle, \( R_{Y, \text{TMF}}, R_{\sigma, \text{TMF}} \) and thus \( R^{*}_{\sigma, \text{TMF}} \) are known. \( U \) is predicted by

1. scaling isothermal SYM results for plane stress and \( \alpha_{\text{iso}} = 1 \) at \( R_{\sigma, \text{iso}} = R^{*}_{\sigma, \text{iso}} = R^{*}_{\sigma, \text{TMF}} \) through Eq. (9) and alternatively

2. by using the set of equations given in Eqs. (10) to (16).

Figure 8 shows the predicted values for the function \( U \). The predicted curves received from the scaled isothermal SYM calculations are shown with open triangular symbols. The quantitative agreement with the temperature dependent SYM as a function of the maximum temperature and \( \sigma_{\infty, \max}/(\alpha \sigma_{Y, t}) \)
Figure 8: Prediction of $U$ for plane stress and plane strain under IP and OP TMF conditions with different maximum temperatures based on isothermal calculations with the SYM for plane stress, which are scaled by Eq. (9) and calculations with the Newman-formula [12] from Eqs. (10) to (16) at a), b) $R_\sigma = -1$ and c), d) $R_\sigma = 0$.

is excellent for OP TMF loading irrespective of the load ratio and the constraint. For IP TMF loading the agreement is mostly of comparable quality. For $R_\sigma = 0$ and $T_{\text{max}} = 950\, ^\circ\text{C}$ and $1050\, ^\circ\text{C}$, $U$ increases steeply with $\sigma_{\infty,\text{max}}/(\alpha\sigma_Y t) \to 1$ (see Fig. 8c). While this tendency is correctly captured, the quantitative agreement deteriorates somewhat. The Newman formula [12] from Eqs. (10) to (16) (dashed lines in Fig. 8) leads to very good predictions for $U$ in the case of plane strain (see Figs. 8b and d). For plane stress (see Figs. 8a and c) the quantitative agreement generally deteriorates
for $\sigma_{\infty,\text{max}}/(\alpha\sigma_{Y,1}) \rightarrow 1$ and especially IP TMF loading. Still the tendency, that OP TMF loading leads to less pronounced crack closure with increasing maximum temperature and IP TMF loading leads to more pronounced crack closure with increasing maximum temperature is well predicted.

5. Discussion

The scaling relation from section 4.1 leads to a unified description of isothermal and nonisothermal simulation results, which is independent of the constraint factor, the yield stress ratio and whether IP or OP TMF loading is considered (see Figs. 6 and 7). The usage of $(\sigma_{op} - \sigma_{\infty,\text{min}})/\sigma_{Y,c}$ in Figs. 6 and 7 can be motivated on the basis of the crack opening stress calculation within the SYM. In the SYM, the crack opening stress $\sigma_{op}$ is derived from the condition, that the stress intensity caused by the contact stresses within the physical crack is annihilated by the external stress intensity range with respect to the minimum load. For the considered (infinite) center cracked plate this leads to:

$$\sigma_{op} = \sigma_{\infty,\text{min}} - \sum_{i=1}^{i=CT-1} \frac{2}{\pi} \sigma_i \left[ \arcsin \frac{b_{2,i}}{a - \Delta a} - \arcsin \frac{b_{1,i}}{a - \Delta a} \right].$$

Here, $b_{1,i}$ and $b_{2,i}$ are the $x$-coordinates of the yield strips, the index $CT$ denotes the index of the physical crack-tip and $\Delta a$ corresponds to the crack growth increment of the previous loading cycle. For a homogeneous temperature distribution and thus a location independent yield stress in compression $\sigma_{Y,c}$, the element stresses at the physical crack-tip are bounded by $\sigma_{Y,c}$. Assuming, that the element stresses $\sigma_i$ at minimum applied load scale with $\sigma_{Y,c}$,
directly leads to

$$\sigma_{op} - \sigma_{\infty, \text{min}} \propto \sigma_{Y,c}.$$  \hspace{1cm} (19)$$

Equation (19) thus is a measure for the stress intensity needed to overcome the compressive contact stresses, while the usage of $$\sigma_{\infty, \text{max}}/(\alpha \sigma_{Y,t})$$ on the $$x$$-axes in Figs. 6 and 7 is a measure for the monotonic plastic zone size created in tension (compare with Eq. (3)). All isothermal and nonisothermal simulation results can now be described in common bands with a constant value of $$R_{\sigma}^*.$$ The interpretation of $$R_{\sigma}^*$$ is received by using Eq. (5) and the definition for $$R_Y$$ from Eq. (4), which yields:

$$R_{\sigma}^* = \frac{R_{\sigma}}{R_Y} = \frac{\sigma_{\infty, \text{min}}}{\sigma_{\infty, \text{max}} \alpha \sigma_{Y,t}}$$ \hspace{1cm} (20)$$

Hence, $$R_{\sigma}^*$$ can be interpreted as a yield stress corrected load ratio. Since the constraint factor $$\alpha$$ simply scales the yield stress in tension, there is no difference between increasing $$\alpha$$ and increasing the yield stress $$\sigma_{Y,t}$$ itself.

The modified Newman-formula from Eqs. (10) to (16) can be used to predict the crack opening stresses under TMF loading. However, the model shows some weaknesses at high $$\sigma_{\infty, \text{max}}/(\alpha \sigma_{Y,t})$$ values (see Figs. 6, 7 and 8), which are not attributed to the scaling relation from Section 4.1. This can be seen by comparing the predictions made with the modified Newman-formula to the scaled results of the isothermal SYM calculations in Fig. 8. The scaled isothermal SYM calculations represent the best possible accuracy which can be achieved from scaling relation and are generally in better agreement with the results of the temperature dependent SYM. Obviously, the accuracy of Eqs. (10) to (16) could in principle be improved by a readjustment of the model coefficients.
However, the current study has limitations that are related to the elastic perfectly plastic material behavior, which forms the basis for the SYM and which can only be a crude estimate for the material behavior of real technical alloys. This is especially true for thermomechanical fatigue loading with high maximum temperatures. While the SYM does not differ between stress or strain controlled conditions, the macroscopic material behavior always stays elastic, a difference is to be expected for real alloys at high temperatures due to either creep enhanced cyclic ratchetting (under stress controlled conditions) or cyclic mean stress relaxation (under strain controlled conditions). Additionally, most TMF tests go along with macroscopically inelastic strains, since the tests are typically performed in the low cycle fatigue regime. In that case, the crack opening stress equation is extrapolated to $\sigma_{\infty, \text{max}}/\sigma_y > 1$.

The SYM only considers plasticity-induced fatigue crack closure. In reality other mechanisms such as roughness- or oxide-induced fatigue crack closure might contribute as well. All results presented in this paper are only valid for a center-cracked (infinite) plate. The transfer of the Newman equation to other specimen geometries could be done as earlier proposed in [10], where the methodology of [17] was used to calculate the crack opening stresses for a compact tension specimen.

In any case, the proposed scaling relation is easy to use and shows reasonable results for a broad range of TMF loading conditions.

Note on experimental data of introduction: Due to the very limited amount of experimental data on fatigue crack closure measurements, it is not possible to draw any conclusions concerning the predicted tendencies presented in this paper. The results from [15] seem to indicate, that crack
closure stresses are higher for IP loading than for OP loading. However, the different strain ratios applied for IP and OP loading as well as the specimen geometry impede any further comparison to the results of the SYM. The effect of a dwell time on crack closure as studied in [14] cannot be evaluated because the SYM does not account for time dependent material behavior.

6. Conclusions

In this paper a scaling relation was presented which allows to predict the crack opening stress under in-phase and out-of-phase thermomechanical fatigue loading as a function of the temperature dependent yield stress and the constraint factor by introducing a yield stress corrected load ratio. The scaling relation was validated on the basis of crack opening stress calculations with a temperature dependent strip yield model. Based on the scaling relation, an existing crack opening stress equation was modified in order to be applicable to thermomechanical fatigue loading. The crack opening stress equation can predict all observed tendencies and mostly shows a good quantitative agreement with the results from the temperature dependent strip yield model.

References


