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Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe werden sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Work Effort, Consumption, and Portfolio Selection: When the Occupational Choice Matters

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July 5, 2010

Abstract

We consider a highly-qualified individual with respect to her choice between two distinct career paths. She can choose between a mid-level management position in a large company and an executive position within a smaller listed company with the possibility to directly affect the company's share price. She invests in the financial market including the share of the smaller listed company. The utility maximizing strategy from consumption, investment, and work effort is derived in closed form for logarithmic utility. The power utility case is discussed as well. Conditions for the individual to pursue her career with the smaller listed company are obtained. The participation constraint is formulated in terms of the salary differential between the two positions. The smaller listed company can offer less salary. The salary shortfall is offset by the possibility to benefit from her work effort by acquiring own-company shares. This gives insight into aspects of optimal contract design. Our framework is applicable to the pharmaceutical and financial industry, and the IT sector.

2000 MSC Subject Classification: 49L20, 91B28, 93E20
Key Words: portfolio choice, work effort, consumption, occupational choice

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1 Introduction

It is widely supported that the remuneration of managers should be linked to performance, see, e.g., Ross (1973), Jensen and Meckling (1976), Holmstrom (1979) and others, for the fundamentals of agency theory, and the summaries of Murphy (1999) and Core, Guay and Larcker (2003). In contrast to past research, we investigate the motivation for an individual to voluntarily performance-link her private wealth by acquiring shares in the own-company. We consider a highly-qualified individual with respect to her choice between two distinct career paths. She can choose between a mid-level management position in a large company and an executive position in a smaller listed company with the possibility to directly affect the company's share price. The individual is assumed to be utility maximizing, deriving utility from terminal wealth and intertemporal consumption, and negative utility (disutility or cost) from work effort. The investment opportunities include the share of the smaller listed company and thus the individual can capitalize on her work effort by investing in own-company shares. Taking up the mid-level management position with the large company is the outside option in our setting. The outside option rules out the possibility to affect the share price of the smaller company. The individual is characterized by two time preference parameters (ρ, discount rate for utility from consumption, and $\tilde{\rho}$, discount rate for the disutility from work effort), and two work effectiveness parameters (κ, representing inverse work productivity, and α, representing disutility stress).

First, we analyze the individuals optimal control problem under the assumption that she takes up the offer from the smaller listed company. The optimal investment strategy ($\pi^\star$), consumption ($k^\star$), and work effort ($\lambda^\star$), respectively, are derived in closed form in the log-utility setting using stochastic control theory and the corresponding Hamilton-Jacobi-Bellman equations. We demonstrate that an executive with higher work effectiveness (quality) undertakes more work effort. Additionally, the broader constant relative risk aversion setting is explored. By imposing a sensible parameter restriction we are able to reduce the problem to a Riccati equation which we can solve in closed form. As second step, we identify conditions for the individual to work for the smaller listed company. The participation constraint is given in terms of the salary differential of the two job alternatives. In particular, we derive the minimal required salary $\delta^\star$ that needs to be offered by the smaller company to attract the individual and thereby characterize the

participation constraint. In general, we find that a more talented individual requires a lower salary to be attracted to the smaller listed company. The salary shortfall is offset by the possibility to benefit from her work effort by acquiring shares of the company. This salary pattern can be observed in practice, e.g., in the pharmaceutical industry, the IT sector, and the financial industry. Other technical papers similarly concerned with dynamic principal-agent models include Cadenillas, Cvitanic and Zapatero (2004), Desmettre, Gould and Szimayer (2010), Korn and Kraft (2008) and Ou-Yang (2003), for example.

The paper is organized as follows. Section 2 introduces the notation and terminology. In Section 3 the Hamilton-Jacobi-Bellman equations characterizing the utility and consumption maximization problem are derived, and closed-form solutions for the log-utility case are established. The power utility case is discussed as well. The results are illustrated in Section 4. Section 5 concludes and gives an outlook for future research. Technical proofs are in the Appendix.

2 Notation and Setup

We consider an individual endowed with given initial wealth. She manages here financial objectives by investing in the financial market and choosing her instantaneous consumption. The individual can also choose the level of work effort she applies.

2.1 Financial Market

First we specify the financial market. We are given a filtered probability space $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t\geq 0})$ satisfying the usual hypothesis and large enough to support two independent standard Brownian motions, $W^P = (W^P_t)_{t\geq 0}$ and $W = (W_t)_{t\geq 0}$. The investment opportunities available are a money market account, a diversified market portfolio, and shares of a small listed company making a job offer to the individual.

The risk-free money market account has the price process $B = (B_t)_{t\geq 0}$, with dynamics

$$\mathrm{d}B_t = r\, B_t\, \mathrm{d}t\,, \quad B_0 = 1\,, \tag{1}$$

where r is the instantaneous risk-free rate of return, hence $B_t = e^{r\,t}$.

The price process of the market portfolio, $P = (P_t)_{t\geq 0}$, follows the stochastic differential equation (SDE)

$$\mathrm{d}P_t = P_t\left(\mu_P\,\mathrm{d}t + \sigma_P\,\mathrm{d}W_t^P\right), \quad P_0 \in \mathbb{R}^+, \tag{2}$$

where $\mu_P \in \mathbb{R}$ and $\sigma_P > 0$ are respectively the expected return rate and volatility of the market portfolio. The corresponding Sharpe ratio is then $\lambda_P = (\mu_P - r)/\sigma_P$.

The company's stock price process, $S^u = (S_t^u)_{t\geq 0}$, is a controlled diffusion with SDE

$$\mathrm{d}S_t^u = S_t^u\left([r + \lambda_t\,\sigma]\,\mathrm{d}t + \beta\left[\frac{\mathrm{d}P_t}{P_t} - r\mathrm{d}t\right] + \sigma\,\mathrm{d}W_t\right), \quad S_0^u \in \mathbb{R}^+, \tag{3}$$

where $\mu = r + \lambda\,\sigma$ is the company's expected return rate in excess of the beta-adjusted market portfolio's expected excess return rate (i.e. the expected return compensation for non-systematic risk), σ is the company's non-systematic volatility, and $\lambda = (\lambda_t)_{t\geq 0}$ is a control process collected in the control vector process u that will be specified below.

2.2 Controls and Wealth Process

The individual is endowed with the initial wealth $V_0 > 0$. She receives an instantaneous salary proportional to her current wealth at a relative rate δ. For an exogenously given time horizon, $T > 0$, the individual seeks to maximize her total utility by controlling the portfolio holdings, consumption, and work effort.

The portfolio is determined by a self-financing trading strategy given by the bivariate control process $\pi = (\pi^P, \pi^S)$, where $\pi^P = (\pi_t^P)_{t\geq 0}$ is the fraction of wealth invested in the market portfolio and $\pi^S = (\pi_t^S)_{t\geq 0}$ is the fraction of wealth invested in the company's stock. The remainder in the risk-free account, that is, the strategy is self-financing. The individual consumes instantaneously at the relative rate $k = (k_t)_{t\geq 0}$ proportional to the wealth V_t^π at time t, where $k_t \geq 0$, leading to a total consumption rate $k_t\,V_t^\pi$. Further, she influences the small company's stock price dynamics by choice of the control strategy $\lambda = (\lambda_t)_{t\geq 0}$, which is specified to be associated with work effort. The control strategy can be conceptualized as deriving from the individual's corporate investment. For example, identifying and initiating positive net present value projects. Value is added if $\mu = r + \lambda\,\sigma$ is greater than r, indicating excess return compensation for non-systematic risk. To ensure sensible

solutions we require $\lambda \geq 0$, which effectively bars her from destroying company value ($\lambda < 0$) and potentially profiting by shorting the company's stock. All controls are collected in the vector process $u = (\pi^P, \pi^S, k, \lambda)$.

For a fixed salary rate δ, initial wealth $V_0 > 0$, and a control strategy u, the wealth process, $V^u = (V_t^u)_{t \geq 0}$, with starting value $V_0^u = V_0$ is given by

$$\mathrm{d}V_t^u = V_t^u \left(\left[1 - \pi_t^P - \pi_t^S\right] \frac{\mathrm{d}B_t}{B_t} + \pi_t^P \frac{\mathrm{d}P_t}{P_t} + \pi_t^S \frac{\mathrm{d}S_t^u}{S_t^u} + \delta \,\mathrm{d}t - k_t \,\mathrm{d}t \right), t \geq 0\,. \tag{4}$$

The above equation can be rewritten as follows

$$\begin{aligned} \mathrm{d}V_t^u = V_t^u \Big(& \left[r + \delta - k_t + (\pi_t^P + \beta\, \pi_t^S)\, \lambda_P\, \sigma_P + \pi_t^S\, \lambda_t\, \sigma\right] \mathrm{d}t \\ & + \left[\pi_t^P + \beta\, \pi_t^S\right] \sigma_P \,\mathrm{d}W_t^P + \pi_t^S\, \sigma \,\mathrm{d}W_t \Big), t \geq 0\,. \end{aligned} \tag{5}$$

2.3 Stochastic Control Problem

The individual is assumed to maximize the expected value of the terminal utility of her wealth for time horizon T, subject to some utility function U_1 and her consumption rate over the time period $[t, T]$, subject to some utility function U_2. The disutility for work effort is quantified by the cost function C. Both utility functions and the cost function will be specified when deriving closed-form solutions.

Assuming control of the company's stock price behavior λ is determined exogenously and comes at zero cost, the individual's *optimal investment and consumption decision* is then described by

$$\widehat{\Phi}(t, v) = \sup_{(\pi,k) \in \Pi(t,v)} \mathbb{E}^{t,v} \left[U_1(V_T^{(\pi,k)}) + \int_t^T U_2(s, V_s^{(\pi,k)}, k_s)\, \mathrm{d}s \right], \tag{6}$$

for $(t, v) \in [0, T] \times \mathbb{R}^+$, where $\Pi(t, v)$ denotes the set of all admissible portfolio processes (π, k) at time t corresponding to portfolio value (i.e. wealth) $v = V_t > 0$ (see for example Korn and Korn (2001)), U_1 and U_2 are utility functions, and $\mathbb{E}^{t,v}$ denotes the expectation conditional on t and v.

The *optimal investment and consumption control decision including work effort* is then the solution of

$$\Phi(t, v) = \sup_{u \in A(t,v)} \mathbb{E}^{t,v} \left[U_1(V_T^u) + \int_t^T U_2(s, V_s^u, k_s)\, \mathrm{d}s - \int_t^T C(s, V_s^u, \lambda_s)\, \mathrm{d}s \right], \tag{7}$$

for $(t,v) \in [0,T] \times \mathbb{R}^+$. The set of admissible strategies for the maximization $A(t,v)$ problem is made precise in the following definition.

Definition 2.1 *Fix $(t,v) \in [0,T] \times \mathbb{R}^+$, then $u = \left(\pi^P, \pi^S, k, \lambda\right)$ is in the set of admissible strategies $A(t,v)$, if and only if u is an $\{\mathcal{F}_s\,;\, t \le s \le T\}$-predictable processes, such that*

(i) the stock price equation

$$\mathrm{d}S_s^u = S_s^u \left([r + \lambda_s\, \sigma]\, \mathrm{d}s + \beta \left[\frac{\mathrm{d}P_s}{P_s} - r \mathrm{d}s \right] + \sigma\, \mathrm{d}W_s \right) ,$$

with initial condition $S_t^u \in \mathbb{R}^+$ admits a non-negative solution and

$$\int_t^T \left(S_s^u\right)^2 \left(\sigma^2 + \beta^2\, \sigma_P^2\right)\, \mathrm{d}s \;<\; \infty \quad P - a.s.\,;$$

(ii) the wealth equation

$$\mathrm{d}V_s^u = V_s^u \left(\left[1 - \pi_s^P - \pi_s^S\right] \frac{\mathrm{d}B_s}{B_s} + \pi_s^P \frac{\mathrm{d}P_s}{P_s} + \pi_s^S \frac{\mathrm{d}S_s^u}{S_s^u} + \delta\, \mathrm{d}s - k_s\, \mathrm{d}s \right),$$

with initial condition $V_t^u = v$ has a unique non-negative solution and

$$\int_t^T (V_s^u)^2 \left(([\pi_s^P + \beta\, \pi_s^S]\, \sigma_P)^2 + (\pi_s^S \sigma)^2 \right)\, \mathrm{d}s \;<\; \infty \quad P - a.s.\,;$$

(iii) and the utility of wealth and consumption, and the disutility of control satisfy

$$\mathbb{E}\left[U_1(V_T^u)^- + \int_t^T U_2(s, V_s^u, k_s)^-\, \mathrm{d}s + \int_t^T C(s, V_s^u, \lambda_s)\, \mathrm{d}s \right] \;<\; \infty\,.$$

2.4 Outside Option

The individual can choose between two job offers at $t = 0$. As an alternative to taking on the executive position with the company with share price S^u, she can pursue her outside option and decide to work for a large company in a mid-management position paying a salary at rate $\widehat{\delta}$. In the latter case she cannot affect the stock price process any longer and hence $\widehat{\lambda} = 0$. The classical optimal investment and consumption decision applies.

Assume that portfolio process follows Eq. (5) where we set $\delta = \widehat{\delta}$ and $\lambda = \widehat{\lambda} = 0$. Then the optimal investment decision problem in Equation (6) determines the value of the outside option $\widehat{\Phi}(0, V_0)$ at time $t = 0$ for initial wealth $V_0 > 0$.

3 Optimal Strategies

In this section we use stochastic control techniques to derive closed-form solutions to the control problem in (7). Our main focus is placed on the log utility specification for utility from terminal wealth and consumption and disutility that is a power function of work effort applied. In addition, we also discuss the general constant relative risk aversion specification.

3.1 Hamilton-Jacobi-Bellman Equation

Having formulated the optimal investment and control decision problem including consumption with respect to the parameter set $u = (\pi, k, \lambda)$ as given by (7), we can write down the corresponding Hamilton-Jacobi-Bellman equation. Note that we formulate this equation with respect to a general utility functions U_1 and U_2 and a general cost function C. For $(t, v) \in [0, T) \times \mathbb{R}^+$ we have

$$\frac{\partial \Phi}{\partial t}(t,v) + \sup_{u \in \mathcal{U}} \left[(L^u \, \Phi)(t,v) + U_2(t,v,k) - C(t,v,\lambda) \right] \, = \, 0 \, , \tag{8}$$

with terminal condition $\Phi(T, v) = U_1(v)$, for $v \in \mathbb{R}^+$, where $\mathcal{U} = \mathbb{R}^2 \times [0, \infty)^2$ and the differential operator L^u is defined by

$$\begin{aligned} &(L^u g)(t,v) = \\ &\frac{\partial g}{\partial v}(t,v)\, v \left(r + \delta + \pi^S(t,v)\, \lambda(t,v)\, \sigma + \left[\pi^P(t,v) + \beta\, \pi^S(t,v) \right] \lambda_P \, \sigma_P - k(t,v) \right) \\ &+ \frac{1}{2} \frac{\partial^2 g}{\partial v^2}(t,v)\, v^2 \left([\pi^S(t,v)\, \sigma]^2 + [\pi^P(t,v)\, \sigma_P + \beta\, \pi^S(t,v)\, \sigma_P]^2 \right) \, . \end{aligned} \tag{9}$$

Potential maximizers $\pi^{P\star}$, $\pi^{S\star}$, $k^\star$ and $\lambda^\star$ of the HJB (8) can be calculated by establishing the first order conditions:

$$\begin{aligned} \pi^{P\star}(t,v) &= -\frac{\lambda_P}{v\, \sigma^P} \, \frac{\Phi_v(t,v)}{\Phi_{vv}(t,v)} - \beta\, \pi^{S\star}(t,v) \, , \\ \pi^{S\star}(t,v) &= -\frac{\lambda^\star(t,v)}{v\, \sigma} \, \frac{\Phi_v(t,v)}{\Phi_{vv}(t,v)} \, , \end{aligned} \tag{10}$$

and $\lambda^\star$ is the solution of the implicit equation

$$\lambda \, \frac{\Phi_v^2(t,v)}{\Phi_{vv}(t,v)} + \frac{\partial C}{\partial \lambda}(t,v,\lambda) = 0 \, , \qquad \text{for all } (t,v) \in [0,T] \times \mathbb{R}^+ \, , \tag{11}$$

where we have already used (10) to simplify the equation, and $k^\star$ is the solution of the equation

$$\frac{\partial U_2}{\partial k}(t,v,k) - v\,\Phi_v(t,v) = 0. \tag{12}$$

Substituting the maximizers (10) in the HJB (8) yields:

$$\begin{aligned}\Phi_t(t,v)+\Phi_v(t,v)\,v\,(r+\delta-k^\star(t,v)) - \frac{1}{2}(\lambda^\star(t,v))^2\,\frac{\Phi_v^2(t,v)}{\Phi_{vv}(t,v)} \\ - \frac{1}{2}\lambda_P^2\,\frac{\Phi_v^2(t,v)}{\Phi_{vv}(t,v)} + U_2(t,k^\star(t,v)) - C(t,v,\lambda^\star(t,v)) = 0\,.\end{aligned} \tag{13}$$

In the following we solve (13) with particular choices for the utility and disutility functions.

3.2 Closed-Form Solution for the Log-Utility Case

We specify the utility functions to be of log-utility type, belonging to the constant relative risk aversion class. The utility function of the final wealth U_1 is

$$U_1(v) = K\,\log(v)\,, \quad \text{for } v \in \mathbb{R}^+\,, \tag{14}$$

for a constant $K > 0$, the utility function of consumption U_2 is

$$U_2(t,k,v) = e^{-\rho\,t}\,\log(v\,k)\,, \quad \text{for } (t,v,k) \in [0,T]\times\mathbb{R}^+\times\mathbb{R}_0^+\,, \tag{15}$$

where $\rho \in \mathbb{R}$ parametrizes the time preference, and the cost function of work effort C is

$$C(t,v,\lambda) = e^{-\tilde{\rho}\,t}\,\kappa\,\frac{\lambda^\alpha}{\alpha}\,, \quad \text{for } (t,v,\lambda) \in [0,T]\times\mathbb{R}^+\times\mathbb{R}_0^+\,, \tag{16}$$

where $\kappa > 0$ and $\alpha > 2$ are the individual's work effectiveness parameters, respectively termed 'inverse work productivity' and 'disutility stress', and $\tilde{\rho} \in \mathbb{R}$ is a time preference parameter. The constant κ directly relates the her work effort disutility to the quality of his control decision as indicated by the non-systematic Sharpe ratio λ, and α indicates how rapidly her work effort disutility will rise for the sake of an improved λ. The requirement $\alpha > 2$ is a consequence of our set-up that ensures the executive's disutility grows with work effort, i.e. λ, at a rate that offsets (at some level of λ) the rate

of her utility gain due to the flow-on from her work effort to the value of his own-company stockholding; this becomes evident with derivation of the solution to (7). A higher quality individual is able to achieve a given λ with lower disutility, and is able to improve λ with lower incremental disutility. That is, higher individual quality (i.e. higher work effectiveness) is implied by lower values of κ and α.

For the remainder of the paper we assume that the optimal investment and control problem (7) admits a value function $\Phi \in C^{1,2}$. To guarantee that the candidates we will derive for the executive's optimal investment and control strategy (i.e. the choices for own-company stockholding, market portfolio holding and non-systematic Sharpe ratio) and value function are indeed optimal, we have to consider a more restrictive class of admissible strategies as follows.

Definition 3.1 *Fix $(t, v) \in [0, T] \times \mathbb{R}^+$. Then by $A'(t, v)$ we denote the set of admissible strategies $u \in A'(t, v)$, such that $u \in A(t, v)$ and*

$$\mathbb{E}\left[\int_t^T \left(\pi_s^P + \beta\, \pi_s^S\right)^2 (\sigma_P)^2 + \left(\pi_s^S \sigma\right)^2 \,\mathrm{d}s\right] < \infty\,, \tag{17}$$

Restating the optimal investment and control problem:

$$\Phi(t, v) = \sup_{u \in A'(t,v)} \mathbb{E}^{t,v}\left[U_1(V_T^u) + \int_t^T U_2(s, V_s^u, k_s)\,\mathrm{d}s - \int_t^T C(s, V_s^u, \lambda_s)\,\mathrm{d}s\right], \tag{18}$$

for $(t, v) \in [0, T] \times \mathbb{R}^+$.

A closed-form solution is obtained for the optimal investment and control problem in (18) using the utility and disutility functions (14), (15) and (16).

Theorem 3.1 *The full solution of the maximization problem (18) can be summarized by the strategy*

$$\begin{aligned} \pi^{P\star}(t, v) &= \frac{\lambda_P}{\sigma_P} - \beta\, \pi^{S\star}(t, v)\,, \quad \pi^{S\star}(t, v) = \frac{\lambda^\star(t, v)}{\sigma}\,, \\ \lambda^\star(t, v) &= \left(\frac{e^{\tilde{\rho}\, t}}{\kappa}\, f(t)\right)^{\frac{1}{\alpha-2}} \quad, \quad k^\star(t, v) \;=\; \frac{e^{-\rho\, t}}{f(t)}\,, \end{aligned} \tag{19}$$

and value function

$$\Phi(t, v) = f(t)\, \log(v) + g(t)\,, \tag{20}$$

with

$$f(t) = \begin{cases} K + \frac{e^{-\rho\, t} - e^{-\rho\, T}}{\rho}\,, & \textit{for } \rho \neq 0\,, \\ K + T - t\,, & \textit{for } \rho = 0\,, \end{cases}$$

and

$$\begin{aligned} g(t) = \left(r + \delta + \frac{1}{2}\,\lambda_P^2\right) \int_t^T f(s)\,\mathrm{d}s + \frac{\alpha - 2}{2\,\alpha} \int_t^T \left(\frac{e^{\tilde{\rho}\, s}}{\kappa}\right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}}\,\mathrm{d}s \\ - \int_t^T (1 + \rho\, s)\, e^{-\rho\, s}\,\mathrm{d}s - \int_t^T e^{-\rho\, s}\, \log(f(s))\,\mathrm{d}s\,. \end{aligned}$$

Proof. First observe that a function F of the form $F(\lambda) = a\,\lambda^2 - b\,\lambda^\alpha$, $\lambda \geq 0$, for given constants $a, b > 0$ and $\alpha > 2$, has a unique maximizer $\lambda^\star$ and maximized value $F(\lambda^\star)$ given by

$$\lambda^\star = \left(\frac{2\,a}{\alpha\, b}\right)^{\frac{1}{\alpha-2}}\,, \quad \text{and} \quad F(\lambda^\star) = (\alpha - 2)\,\alpha^{-\frac{\alpha}{\alpha-2}}\, 2^{\frac{2}{\alpha-2}}\, a^{\frac{\alpha}{\alpha-2}}\, b^{-\frac{2}{\alpha-2}}\,. \tag{21}$$

Using this insight, the first order condition for $\lambda^\star$ in (11) is now solved. Set

$$a = \frac{1}{2}\,\frac{\Phi_v^2}{-\Phi_{vv}}\,, \quad \text{and} \quad b = e^{-\tilde{\rho}\, t}\,\frac{\kappa}{\alpha}\,,$$

then (21) gives

$$\lambda^\star = \left(\frac{e^{\tilde{\rho}\, t}}{\kappa}\,\frac{\Phi_v^2}{-\Phi_{vv}}\right)^{\frac{1}{\alpha-2}}\,, \text{ and } F(\lambda^\star) = \frac{\alpha - 2}{2\,\alpha}\,\left(\frac{e^{\tilde{\rho}\, t}}{\kappa}\right)^{\frac{2}{\alpha-2}} \left(\frac{\Phi_v^2}{-\Phi_{vv}}\right)^{\frac{\alpha}{\alpha-2}}\,. \tag{22}$$

Having specified the utility function U_2 of the consumption rate as $U_2(t, v, k) = e^{-\rho\, t}\, \log(v\, k)$, we can also solve the first order condition for the optimal consumption rate. Equation (12) then gives:

$$k^\star = \frac{e^{-\rho\, t}}{v\,\Phi_v}\,. \tag{23}$$

Substituting $\lambda^\star$ and $k^\star$ in equation (13) we get:

$$\begin{aligned} 0 = \Phi_t + \Phi_v\, v\,(r + \delta) + \frac{1}{2}\,\lambda_P^2\,\frac{\Phi_v^2}{-\Phi_{vv}} + \frac{\alpha - 2}{2\,\alpha}\,\left(\frac{e^{\tilde{\rho}\, t}}{\kappa}\right)^{\frac{2}{\alpha-2}} \left(\frac{\Phi_v^2}{-\Phi_{vv}}\right)^{\frac{\alpha}{\alpha-2}} \\ - e^{-\rho\, t} - \rho\, t\, e^{-\rho\, t} - e^{-\rho\, t}\, \log(\Phi_v)\,. \end{aligned} \tag{24}$$

Using the ansatz $\Phi(t,v) = \log(v)\, f(t) + g(t)$ with $f(T) = K$ and $g(T) = 0$ results in

$$\Phi_t = \log(v)\dot{f}(t) + \dot{g}(t)\,, \quad \Phi_v = \frac{1}{v}\, f(t)\,, \quad \Phi_{vv} = -\frac{1}{v^2}\, f(t)\,, \text{ and}$$
$$\Phi(T,v) = K\, \log(v) = U_1(v)\,.$$

Then (24) reduces to

$$\begin{aligned} 0 = \log(v)\dot{f}(t) + \dot{g}(t) + f(t)\left(r + \delta + \frac{1}{2}\,\lambda_P^2\right) + \frac{\alpha - 2}{2\,\alpha}\left(\frac{e^{\tilde{\rho}\,t}}{\kappa}\right)^{\frac{2}{\alpha-2}} f(t)^{\frac{\alpha}{\alpha-2}} \\ - e^{-\rho\,t} - \rho\,t\,e^{-\rho\,t} + e^{-\rho\,t}\,\log(v) - e^{-\rho\,t}\,\log(f(t))\,. \end{aligned} \tag{25}$$

Taking the derivative of this equation w.r.t. v gives:

$$\frac{1}{v}\,\dot{f}(t) + \frac{1}{v}\,e^{-\rho\,t} = 0 \quad \Longleftrightarrow \quad \dot{f}(t) = -e^{-\rho\,t}\,.$$

Using the condition $f(T) = K$ we then get by integration

$$f(t) = \begin{cases} K + \frac{e^{-\rho\,t} - e^{-\rho\,T}}{\rho}\,, & \text{for } \rho \neq 0\,, \\ K + T - t\,, & \text{for } \rho = 0\,. \end{cases}$$

Following the derivation of f we can eliminate the $\log(v)$ in (25)

$$\begin{aligned} -\dot{g}(t) = f(t)\left(r + \delta + \frac{1}{2}\,\lambda_P^2\right) + \frac{\alpha - 2}{2\,\alpha}\left(\frac{e^{\tilde{\rho}\,t}}{\kappa}\right)^{\frac{2}{\alpha-2}} f(t)^{\frac{\alpha}{\alpha-2}} \\ - e^{-\rho\,t} - \rho\,t\,e^{-\rho\,t} - e^{-\rho\,t}\,\log(f(t))\,, \quad \text{and} \;\; g(T) = 0\,. \end{aligned} \tag{26}$$

Equation (26) can now be solved by simple integration:

$$\begin{aligned} g(t) = \left(r + \delta + \frac{1}{2}\,\lambda_P^2\right)\int_t^T f(s)\,\mathrm{d}s + \frac{\alpha - 2}{2\,\alpha}\int_t^T \left(\frac{e^{\tilde{\rho}\,s}}{\kappa}\right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}}\,\mathrm{d}s \\ - \int_t^T (1 + \rho\,s)\,e^{-\rho\,s}\,\mathrm{d}s - \int_t^T e^{-\rho\,s}\,\log f(s)\,\mathrm{d}s\,, \end{aligned}$$

where $f(t)$ is given as above.
Combining the results for the functions f and g we then get the claimed result

for the value function. Noting that $\Phi_v/\Phi_{vv} = -v$ and using the first order conditions in (10) establishes the claimed optimal strategies $\pi^{P\star}$ and $\pi^{S\star}$. Finally noting that $\Phi_v^2/\Phi_{vv} = -f(t)$ and using the solved first order condition (22), we get the desired result for the optimal sharpe ratio $\lambda^\star$ and plugging in $v\,\Phi_v = f$ in (23) we get the claimed result for the optimal consumption rate $k^\star$. The claimed optimal investment and control choices are deterministic and the optimal consumption rate are continuous on a compact support, so they are uniformly bounded implying $u^\star = (\pi^{S\star}, \pi^{P\star}, \lambda^\star, k^\star) \in A'(t,v)$. □

Remark 3.1 *The expression for g in Theorem 3.1 can be partially calculated fairly explicitly. For $\rho \neq 0$ we obtain*

$$\begin{aligned} g(t) = &\left(r + \delta + \frac{1}{2}\,\lambda_P^2\right)\left(K\,[T-t] + \frac{1}{\rho^2}\,\left[e^{-\rho\,t} - e^{-\rho\,T}(1+\rho[T-t])\right]\right) \\ &- \frac{1}{\rho}\,\left(e^{-\rho\,t} - e^{-\rho\,T}\right) - t\,e^{-\rho\,t} + T\,e^{-\rho\,T} + K\,\log(K) \\ &- \log\left(K + \frac{1}{\rho}\left[e^{-\rho\,t} - e^{-\rho\,T}\right]\right)\left(K + \frac{1}{\rho}\left[e^{-\rho\,t} - e^{-\rho\,T}\right]\right) \\ &+ \frac{\alpha-2}{2\,\alpha}\int_t^T \left(\frac{e^{\tilde{\rho}\,s}}{\kappa}\right)^{\frac{2}{\alpha-2}}\,f(s)^{\frac{\alpha}{\alpha-2}}\,\mathrm{d}s\,. \end{aligned}$$

The integral in the last line can in general not be computed in closed form. However, it can be expressed as a hypergeometric function. For $\rho = 0$, the function g can be obtained by continuity in ρ, i.e. fix t and then compute the limit for $\rho \to 0$.

The solutions of the maximization problems given in Theorem 3.1 are candidates for the optimal investment and control choices as well as for the optimal consumption rate for the problem in (18). In the following theorem we verify that under sufficient assumptions these solutions are indeed optimal.

Theorem 3.2 (Verification) *Let $\kappa > 0$ and $\alpha > 2$. Assume the executive's utility function of wealth, the utility function of the consumption rate as well as the cost function are given by (14), (15) and (16). Then the candidates given in (19) and (20) are the optimal investment and control strategy (i.e. own-company stockholding, market portfolio holding and non-systematic Sharpe ratio strategy), the optimal consumption rate and value function of the optimal control problem (18).*

Proof. Define the performance functional of our optimal investment, consumption and control decision by

$$J'(t,v;\pi,k,\lambda) := \mathbb{E}^{t,v}\left[U_1\left(V_T^u\right) + \int_t^T U_2(s,V_s^u,k_s)\,\mathrm{d}s - \int_t^T C(s,V_s^u,\lambda_s)\,\mathrm{d}s\right]. \tag{27}$$

Our candidates are optimal if we have

$$\begin{aligned} &J'(t,v;\pi^\star,k^\star,\lambda^\star) = \Phi(t,v) \text{ and} \\ &J'(t,v;\pi,k,\lambda) \leq \Phi(t,v)\,, \text{ for all } (\pi,k,\lambda) \in A_1'(t,v)\,. \end{aligned} \tag{28}$$

Let $u \in A_1'(t,v)$. Since $\Phi \in C^{1,2}$, we obtain by Ito's formula:

$$\begin{aligned} &\Phi(T,V_T^u) + \int_t^T e^{-\rho\, s}\,\log(V_s^u k_s)\,\mathrm{d}s - \int_t^T e^{-\tilde{\rho}\, s}\kappa\frac{\lambda_s^\alpha}{\alpha}\,\mathrm{d}s \\ &= \Phi(t,v) + \int_t^T \left(\Phi_t(s,V_s^u) + e^{-\rho\, s}\log(V_s^u k_s) - e^{-\tilde{\rho}\, s}\kappa\frac{\lambda_s^\alpha}{\alpha}\right)\mathrm{d}s \\ &\quad + \int_t^T \Phi_v(s,V_s^u)V_s^u\left(r+\delta-k_s+\left[\pi_s^P+\beta\pi_s^S\right]\lambda_P\sigma_P+\pi_s^S\lambda_s\sigma\right)\mathrm{d}s \\ &\quad + \frac{1}{2}\int_t^T \Phi_{vv}(s,V_s^u)\left(V_s^u\right)^2\left(\left[(\pi_s^P+\beta\,\pi_s^S)\sigma_P\right]^2+\left[\pi_s^S\sigma\right]^2\right)\mathrm{d}s \\ &\quad + \int_t^T \Phi_v(s,V_s^u)V_s^u\left(\pi_s^P+\beta\pi_s^S\right)\sigma_P\,\mathrm{d}W_s^P + \int_t^T \Phi_v(s,V_s^u)V_s^u\pi_s^S\sigma\,\mathrm{d}W_s. \end{aligned} \tag{29}$$

First, we investigate the optimal control $u^\star = (\pi^{P\star},\pi^{S\star},\lambda^\star,k^\star)$ given in (19). To show that the local martingale component in (29) vanishes in expectation we check the sufficient integrability condition

$$\mathbb{E}\left[\int_t^T \left(\Phi_v(s,V_s^{u^\star})V_s^{u^\star}\right)^2\left(\left[\pi_s^{P\star}+\beta\,\pi_s^{S\star}\right]^2\sigma_P^2+\left[\pi_s^{S\star}\right]^2\sigma^2\right)\mathrm{d}s\right] < \infty\,. \tag{30}$$

From (19) and (20) we obtain

$$\left(\Phi_v(s,V_s^{u^\star})V_s^{u^\star}\right)^2\left(\left[\pi_s^{P\star}+\beta\pi_s^{S\star}\right]^2\sigma_P^2+\left[\pi_s^{S\star}\right]^2\sigma^2\right) = \left(f(s)\right)^2\left(\lambda_P^2+\left[\lambda^\star(s)\right]^2\right).$$

Now, f and $\lambda^\star$ are deterministic continuous functions on the compact $[0,T]$, and thus the above expression is uniformly bounded. Accordingly the expectation in (30) is finite, and the Wiener integrals in (29) vanish in expectation.

Furthermore, Φ satisfies the HJB equation (8) implying

$$\begin{aligned}0 =& \Phi_v(s, V_s^{u^\star}) V_s^{u^\star} \left(r + \delta - k_s + \left[\pi_s^{P^\star} + \beta \pi_s^{S^\star}\right] \lambda_P \sigma_P + \pi_s^{S^\star} \lambda_s^\star \sigma\right) \\ &+ \frac{1}{2} \Phi_{vv}(s, V_s^{u^\star}) \left(V_s^{u^\star}\right)^2 \left([(\pi_s^{P^\star} + \beta\, \pi_s^{S^\star})\sigma_P]^2 + [\pi_s^{S^\star} \sigma]^2\right) \\ &+ \Phi_t(s, V_s^{u^\star}) + e^{-\rho s} \log(V_s^{u^\star} k_s^\star) - e^{-\tilde{\rho} s} \kappa \frac{(\lambda_s^\star)^\alpha}{\alpha}, \quad \text{for } t \le s \le T\,.\end{aligned}$$

Then, using that $\Phi(T, v) = U_1(v)$ the expectation of (29) is:

$$\begin{aligned}\Phi(t, v) &= \mathbb{E}^{t,v}\left[\Phi(T, V_T^{u^\star}) + \int_t^T e^{-\rho s} \log(V_s^{u^\star} k_s^\star)\, \mathrm{d}s - \int_t^T e^{-\tilde{\rho} s} \kappa \frac{(\lambda_s^\star)^\alpha}{\alpha}\, \mathrm{d}s\right] \\ &= \mathbb{E}^{t,v}\left[U_1(V_T^{u^\star})) + \int_t^T U_2(s, V_s^{u^\star}, k_s^\star)\, \mathrm{d}s - \int_t^T C(s, V_s^{u^\star}, \lambda_s^\star)\, \mathrm{d}s\right] \\ &= J'(t, v; \pi^\star, \lambda^\star, k^\star)\,.\end{aligned}$$

Thus we have verified the first part of (28).

Next, fix $u \in A'(t, v)$. By the HJB equation (8), we have

$$\begin{aligned}0 \ge& \Phi_t(s, V_s^u) + \Phi_v(s, V_s^u) V_s^u \left(r + \delta - k_s + \left[\pi_s^P + \beta \pi_s^S\right] \lambda_P \sigma_P + \pi_s^S \lambda_s \sigma\right) \\ &+ \frac{1}{2} \Phi_{vv}(s, V_s^u) \left(V_s^u\right)^2 \left([(\pi_s^P + \beta\, \pi_s^S)\sigma_P]^2 + [\pi_s^S \sigma]^2\right) \\ &+ e^{-\rho s} \log(V_s^u k_s) - e^{-\tilde{\rho} s} \kappa \frac{\lambda_s^\alpha}{\alpha}, \quad \text{for } t \le s \le T\,.\end{aligned}$$

Substituting this in (29) and recalling that $\Phi_v(t, v) = \frac{1}{v} f(t)$ we get:

$$\begin{aligned}&\Phi(T, V_T^\pi) + \int_t^T e^{-\rho s} \log(V_s^u k_s) \mathrm{d}s - \int_t^T e^{-\tilde{\rho} s} \kappa \frac{\lambda_s^\alpha}{\alpha} \mathrm{d}s \\ &\quad \le \Phi(t, v) + \int_t^T f(s)\, \left)(\pi_s^P + \beta\, \pi_s^S\right) \sigma_P\, \mathrm{d}W_s^P + \int_t^T f(s)\, \pi_s^S \sigma\, \mathrm{d}W_s\,.\end{aligned}$$

Taking the expectation on both sides and keeping in mind that $\Phi(T, v) =$

$U_1(v)$ then yields

$$
\begin{aligned}
&J'(t,v;\pi,\lambda,k) \\
&\quad = \mathbb{E}^{t,v}\left[U_1(V_T^u)) + \int_t^T U_2(s,V_s^u,k_s)\,\mathrm{d}s - \int_t^T C(s,V_s^u,\lambda_s)\,\mathrm{d}s\right] \\
&\quad = \mathbb{E}^{t,v}\left[\Phi(T,V_T^u) + \int_t^T e^{-\rho s}\log(V_s^u k_s)\mathrm{d}s - \int_t^T e^{-\tilde{\rho}s}\kappa\frac{\lambda_s^\alpha}{\alpha}\,\mathrm{d}s\right] \\
&\quad \leq \Phi(t,v) + \underbrace{\mathbb{E}^{t,v}\left[\int_t^T f(s)\,\left(\pi_s^P + \beta\,\pi_s^S\right)\sigma_P\,\mathrm{d}W_s^P + \int_t^T f(s)\,\pi_s^S\sigma\,\mathrm{d}W_s\right]}_{=0,\text{ by (17)}}.
\end{aligned}
$$

The Wiener integral vanishes in expectation since the corresponding integrand is square integrable, since f is uniformly bounded and (17). □

3.3 Participation Constraint for the Log-Utility Case

The optimal strategies in Theorem 3.1 and Theorem 3.2 above apply in case the individual decides to work for the smaller listed company. However, she has the opportunity to take up an outside option, that is, working for a larger company in a mid-level management position. The outside option offers a contract that differs in the salary rate and foregoes the possibility of controlling the stock price of the smaller listed company. Next, we calculate the value of the outside option and derive the participation constraint.

The outside option pays a salary rate $\widehat{\delta}$. Taking on the position results in the loss of the ability to influence the stock price of the smaller listed company and therefore $\widehat{\lambda} = 0$. She can invest in the financial market. The classical optimal investment and consumption decision applies. For the remainder of this subsection we assume that the portfolio process follows Eq. (5) where we set $\delta = \widehat{\delta}$ and $\lambda = \widehat{\lambda} = 0$. Then the optimal investment decision problem in Equation (6) determines the value of the outside option $\widehat{\Phi}(0,V_0)$ at time $t = 0$ for initial wealth $V_0 > 0$. The solution $\widehat{\Phi}$ can be obtained as a simplification of the results in Theorem 3.1 and Theorem 3.2, i.e. $\widehat{\Phi}(t,v) = \widehat{f}(t)\,\log(v) + \widehat{g}(t)$ with

$$
\widehat{f}(t) = \begin{cases} K + \frac{e^{-\rho\,t} - e^{-\rho\,T}}{\rho}\,, & \text{for } \rho \neq 0\,, \\ K + T - t\,, & \text{for } \rho = 0\,, \end{cases}
$$

and

$$\widehat{g}(t) = \left(r + \widehat{\delta} + \frac{1}{2}\,\lambda_P^2\right) \int_t^T \widehat{f}(s)\,\mathrm{d}s \\ - \int_t^T (1+\rho\,s)\,e^{-\rho\,s}\,\mathrm{d}s - \int_t^T e^{-\rho\,s}\,\log(\widehat{f}(s))\,\mathrm{d}s\,.$$

Observe that $\widehat{f} = f$ and

$$g(t) - \widehat{g}(t) = (\delta - \widehat{\delta}) \int_t^T f(s)\,\mathrm{d}s + \frac{\alpha - 2}{2\,\alpha} \int_t^T \left(\frac{e^{\tilde{\rho}\,s}}{\kappa}\right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}}\,\mathrm{d}s\,. \quad (31)$$

Based on the discussion above we can state the participation constraint.

Theorem 3.3 *Let $\widehat{\delta}$ be the salary rate of the outside option. Then the value of the outside option is the solution $\widehat{\Phi}$ to optimal investment and consumption problem in* (6) *with dynamics* (5) *where we set $\delta = \widehat{\delta}$ and $\lambda = \widehat{\lambda} = 0$. The participation constraint for the individual is*

$$\delta \;\geq\; \widehat{\delta} - \frac{(\alpha-2)}{2\,\alpha}\,\frac{\int_0^T \left(\frac{e^{\tilde{\rho}\,t}}{\kappa}\right)^{\frac{2}{\alpha-2}} f(t)^{\frac{\alpha}{\alpha-2}}\,\mathrm{d}t}{\int_0^T f(t)\,\mathrm{d}t}\,, \quad (32)$$

where f is given in Theorem 3.1.

Proof. The value function is of the form $\widehat{\Phi} = \widehat{f}(t)\,\log(v) + \widehat{g}(t)$ with $\widehat{f} = f$ and $\widehat{g} - g$ given in (31). Then we have of course $\Phi(t,v) - \widehat{\Phi}(t,v) = g(t) - \widehat{g}(t)$ and the participation constraint $\Phi(0,V_0) \geq \widehat{\Phi}(0,V_0)$ follows as stated in (32). □

3.4 Discussion of the Power-Utility Case

In this subsection, we derive a closed-form solution for the case of power utility. In particular, we specify a constant relative risk aversion utility-disutility set-up. For the relative risk aversion parameter $\gamma > 1$, the utility function of the final wealth U_1 is

$$U_1(v) = \frac{v^{1-\gamma}}{1-\gamma}\,, \quad \text{for } \gamma > 1\,, \quad (33)$$

the utility function of the consumption U_2 is

$$U_2(k, v) = \frac{(v\,k)^{1-\gamma}}{1-\gamma}\,, \quad \text{for } \gamma > 1\,, \tag{34}$$

and the disutility of control (i.e. work effort) C is

$$C(v, \lambda) = \kappa\, v^{1-\gamma}\, \frac{\lambda^\alpha}{\alpha}\,, \quad \text{for } \gamma > 1\,, \tag{35}$$

where $\kappa > 0$ and $\alpha > 2$ are as in the log-utility part.

Compared to the log-utility setup we have made the simplifying assumption that utility from consumption in (34) and the cost from work effort in (35) are not depending on time, see (15) and (16) for time preferences in the log-utility setup. These assumption enable us to obtain a tractable formulation of the problem. However, we require a further structural assumption linking the cost function parameter α to the relative risk aversion γ. The following condition is assumed to hold:

$$\alpha = 2\gamma + 2\,. \tag{36}$$

Condition (36) enables us to reduce an ODE of inhomogeneous Bernoulli type that appears in the HJB equation to an ODE of Riccati type, which we are able to solve in closed-form. This restriction is however not counterintuitive. A more risk averse individual is implicitly assumed to be more sensitive towards work. When focusing on the optimal work effort $\lambda^\star$ as a main result we can rely on Desmettre et al. (2010) discussing a related framework although without consumption and salary. Their results indicate that $\lambda^\star$ decreases with increasing risk aversion as well as with increasing disutility stress. So by relating those two parameters via (36) we do not change the qualitative behavior of the optimal work effort.

Analogously to the log-utility case, to guarantee indeed the optimality of the candidates we will derive for the executive's optimal investment and control strategy and value function, we consider again a more restrictive class of admissible strategies as follows.

Definition 3.2 *Fix $(t, v) \in [0, T] \times \mathbb{R}^+$. Then for $\gamma > 1$, we denote by $A'_\gamma(t, v)$ the set of admissible strategies $u \in A'_\gamma(t, v)$, such that $u \in A_\gamma(t, v)$*

and

$$\int_t^T (\pi_s^P + \beta\,\pi_u^S)^4 (\sigma^P)^4 + \left(\pi_u^S \sigma\right)^4 \,\mathrm{d}u \leq C_1 < \infty\,, \quad \textit{for some } C_1 \in \mathbb{R}_0^+\,, \tag{37}$$

$$\int_t^T \pi_u^S \sigma \lambda_u \,\mathrm{d}u \geq C_2 > -\infty\,, \quad \textit{for some } C_2 \in \mathbb{R}_0^+\,, \tag{38}$$

$$\int_t^T k_u \,\mathrm{d}u \leq C_3 < \infty\,, \quad \textit{for some } C_3 \in \mathbb{R}_0^+\,. \tag{39}$$

Theorem 3.4 (The power-utility case: $\gamma > 1$) *Suppose that the relative risk aversion parameter γ and the disutility stress parameter α are connected via the relation* (36)*, then the full solution of the maximization problem (18) can be summarized by the strategy*

$$\begin{aligned} \pi^{P\star}(t,v) =& \frac{\mu^P - r}{\gamma\,(\sigma^P)^2} - \beta\,\pi^{S\star}(t,v)\,, \quad \pi^{S\star}(t,v) = \frac{\lambda^\star(t,v)}{\gamma\,\sigma^\star(t,v,\lambda^\star(t,v))}\,, \\ \lambda^\star(t,v) =& \left(\frac{1}{\kappa\,\gamma}\,f(t)\right)^{\frac{1}{2\gamma}}\,, \quad k^\star(t,v) = (f(t))^{-\frac{1}{\gamma}} \quad , \end{aligned} \tag{40}$$

and value function

$$\Phi(t,v) = \frac{v^{1-\gamma}}{1-\gamma}\,f(t)\,, \tag{41}$$

where

$$f(t) = \left(\frac{2(1-g_P)\sqrt{C_0}}{2\sqrt{C_0}e^{-2\sqrt{C_0}(T-t)} + (1-g_P)\left(e^{-2\sqrt{C_0}(T-t)} - 1\right)} + g_P\right)^{-\gamma}\,, \tag{42}$$

with

$$C_0 = \frac{(\gamma-1)^2}{4\gamma^2}\left(r + \delta + \frac{1}{2}\frac{\lambda_P^2}{\gamma}\right)^2 - \frac{\kappa}{2}\frac{(1-\gamma)}{(1+\gamma)}\left(\frac{1}{\kappa\gamma}\right)^{\frac{\gamma+1}{\gamma}}\,, \tag{43}$$

and

$$g_P = -\frac{1-\gamma}{2\gamma}\left(r + \delta + \frac{1}{2}\frac{\lambda_P^2}{\gamma}\right) + \sqrt{C_0}\,. \tag{44}$$

Proof. First observe that a function F of the form $F(\lambda) = a\,\lambda^2 - b\,\lambda^\alpha$, $\lambda \geq 0$, for given constants $a, b > 0$ and $\alpha > 2$, has a unique maximizer $\lambda^\star$ and maximized value $F(\lambda^\star)$ given by

$$\lambda^\star = \left(\frac{2\,a}{\alpha\,b}\right)^{\frac{1}{\alpha-2}}, \quad \text{and} \quad F(\lambda^\star) = (\alpha-2)\,\alpha^{-\frac{\alpha}{\alpha-2}}\,2^{\frac{2}{\alpha-2}}\,a^{\frac{\alpha}{\alpha-2}}\,b^{-\frac{2}{\alpha-2}}. \tag{45}$$

Using this insight the first order condition for $\lambda^\star$ in (11) is now solved. Set

$$a = \frac{1}{2}\,\frac{\Phi_v^2}{-\Phi_{vv}}, \quad \text{and} \quad b = \frac{\kappa}{\alpha}\,v^{1-\gamma},$$

then (45) gives

$$\lambda^\star = \left(\frac{1}{\kappa\,v^{1-\gamma}}\,\frac{\Phi_v^2}{-\Phi_{vv}}\right)^{\frac{1}{\alpha-2}}, \quad F(\lambda^\star) = \frac{\alpha-2}{2\,\alpha}\,\left(\kappa\,v^{1-\gamma}\right)^{-\frac{2}{\alpha-2}}\,\left(\frac{\Phi_v^2}{-\Phi_{vv}}\right)^{\frac{\alpha}{\alpha-2}}.$$

Having specified the utility function as $U_2(t,k_t) = \frac{(v\,k)^{1-\gamma}}{1-\gamma}$, the first order condition (12) for the optimal consumption rate becomes:

$$k^\star = \frac{1}{v}\,(\Phi_v)^{-\frac{1}{\gamma}}.$$

Substituting $\lambda^\star$ and $k^\star$ in (13) then yields:

$$\begin{aligned} 0 \;&= \Phi_t + \Phi_v\,v\,(r+\delta) + \frac{1}{2}\,\frac{\Phi_v^2}{-\Phi_{vv}}\,\left(\lambda^P\right)^2 \\ &\quad + \frac{\alpha-2}{2\,\alpha}\,\left(\kappa\,v^{1-\gamma}\right)^{-\frac{2}{\alpha-2}}\,\left(\frac{\Phi_v^2}{-\Phi_{vv}}\right)^{\frac{\alpha}{\alpha-2}} + \frac{\gamma}{1-\gamma}\,(\Phi_v)^{\frac{\gamma-1}{\gamma}}. \end{aligned} \tag{46}$$

Using the separation ansatz $\Phi(t,v) = f(t)\,\frac{v^{1-\gamma}}{1-\gamma}$ results in

$$\Phi_t = \dot{f}\,\frac{v^{1-\gamma}}{1-\gamma}, \quad \Phi_v = f\,v^{-\gamma}, \quad \Phi_{vv} = -\gamma\,f\,v^{-\gamma-1}, \quad \text{and} \quad f(T) = 1. \tag{47}$$

Thus (46) becomes

$$\begin{aligned} 0 = \dot{f}\,\frac{v^{1-\gamma}}{1-\gamma} + f\,v^{1-\gamma}\,(r+\delta) + \frac{1}{2}\,\frac{f\,v^{1-\gamma}}{\gamma}\,\left(\lambda^P\right)^2 \\ + \frac{\alpha-2}{2\,\alpha}\,\left(\kappa\,v^{1-\gamma}\right)^{-\frac{2}{\alpha-2}}\,\left(\frac{f\,v^{1-\gamma}}{\gamma}\right)^{\frac{\alpha}{\alpha-2}} + \frac{\gamma}{1-\gamma}v^{1-\gamma}f^{\frac{\gamma-1}{\gamma}}. \end{aligned}$$

Dividing by $\frac{v^{1-\gamma}}{1-\gamma}$ and then defining

$$\begin{aligned} a_1 =&(1-\gamma)\left(r+\delta+\frac{1}{2}\frac{\lambda_P^2}{\gamma}\right)\ ,\ a_n=(1-\gamma)\frac{\kappa}{2}\frac{\alpha-2}{\alpha}\ \left(\frac{1}{\kappa\,\gamma}\right)^{\frac{\alpha}{\alpha-2}}\ , \\ a_m =&\gamma\,,\quad n=\frac{\alpha}{\alpha-2}\,,\quad \text{and}\quad m=\frac{\gamma-1}{\gamma}\,. \end{aligned} \tag{48}$$

results in an ordinary differential equation of the form

$$\dot{f}+a_1\,f+a_n\,f^n+a_m\,f^m=0\,. \tag{49}$$

The ansatz $g=f^{1-n}$ yields $\dot{g}=\dfrac{1-n}{f^n}\,\dot{f}$ and thus

$$\dot{g}+a_1\,(1-n)\,g+a_m\,(1-n)\,g^{\frac{m-n}{1-n}}=-a_n\,(1-n)\quad,\quad g(T)=1\,.$$

Using (36), i.e. $\alpha=2+2\,\gamma$, and plugging in the coefficients in (48) we obtain the following ODE of Riccati type

$$\dot{g}-\frac{1-\gamma}{\gamma}\left(r+\delta+\frac{1}{2}\frac{\lambda_P^2}{\gamma}\right)g-g^2=\frac{\kappa}{2}\,\frac{1-\gamma}{1+\gamma}\left(\frac{1}{\kappa\,\gamma}\right)^{\frac{\gamma+1}{\gamma}}\ . \tag{50}$$

This ODE can be solved if we know a particular solution g_P, since then we can reduce this ODE by using the standard ansatz

$$h=1/(g-g_P)$$

to the following linear form:

$$\dot{h}+\left[2g_P+\frac{\gamma-1}{\gamma}\left(r+\delta+\frac{1}{2}\frac{\lambda_P^2}{\gamma}\right)\right]h+1=0\,,\qquad h(T)=\frac{1}{1-g_P}\,.$$

This equation can now be solved by variation of constants. A nonnegative particular solution of (50) is

$$\begin{aligned} g_P=&-\frac{1-\gamma}{2\gamma}\left(r+\delta+\frac{1}{2}\frac{\lambda_P^2}{\gamma}\right) \\ &+\sqrt{\frac{(\gamma-1)^2}{4\,\gamma^2}\left(r+\delta+\frac{1}{2}\frac{\lambda_P^2}{\gamma}\right)^2-\frac{\kappa}{2}\frac{(1-\gamma)}{(1+\gamma)}\left(\frac{1}{\kappa\gamma}\right)^{\frac{\gamma+1}{\gamma}}}\,, \end{aligned}$$

which means that we have to solve the following inhomogeneous linear ODE

$$\dot{h} + \left[2\sqrt{\frac{(\gamma-1)^2}{4\gamma^2}\left(r+\delta+\frac{1}{2}\frac{\lambda_P^2}{\gamma}\right)^2 - \frac{\kappa}{2}\frac{(1-\gamma)}{(1+\gamma)}\left(\frac{1}{\kappa\gamma}\right)^{\frac{\gamma+1}{\gamma}}} \right] h + 1 = 0\,. \tag{51}$$

Now applying variation of constants and using that $h(T) = 1/(1-g_P)$, the solution of this ODE is

$$h(t) = \frac{1}{1-g_P}\, e^{2\sqrt{C_0}\,(T-t)} + \frac{1}{2\sqrt{C_0}}\left(e^{2\sqrt{C_0}\,(T-t)} - 1\right)\,, \tag{52}$$

where

$$C_0 = \frac{(\gamma-1)^2}{4\gamma^2}\left(r+\delta+\frac{1}{2}\frac{\lambda_P^2}{\gamma}\right)^2 - \frac{\kappa}{2}\frac{(1-\gamma)}{(1+\gamma)}\left(\frac{1}{\kappa\gamma}\right)^{\frac{\gamma+1}{\gamma}}\,.$$

Transforming the result back to the function f we get

$$f(t) = \left(g_P + \frac{2(1-g_P)\sqrt{C_0}}{2\sqrt{C_0}e^{2\sqrt{C_0}(T-t)} + (1-g_P)\left(e^{2\sqrt{C_0}(T-t)} - 1\right)} \right)^{-\gamma}\,. \tag{53}$$

Using the representations (47) we get

$$\lambda^\star(t,v) = \left(\frac{1}{\kappa\, v^{1-\gamma}}\,\frac{\Phi_v^2}{-\Phi_{vv}}\right)^{\frac{1}{\alpha-2}} = \left(\frac{1}{\kappa\gamma} f(t)\right)^{\frac{1}{\alpha-2}} = \left(\frac{1}{\kappa\gamma} f(t)\right)^{\frac{1}{2\gamma}}\,,$$

and

$$\begin{aligned}
\pi^{P^\star}(t,v) &= -\frac{(\mu^P - r)}{v(\sigma^P)^2}\,\frac{\Phi_v(t,v)}{\Phi_{vv}(t,v)} - \beta\,\pi^{S^\star}(t,v) = \frac{\mu^P - r}{\gamma\,(\sigma^P)^2} - \beta\,\pi^{S^\star}(t,v)\,,\\
\pi^{S^\star}(t,v) &= -\frac{\lambda^\star(t,v)}{v\sigma^\star(t,v,\lambda^\star(t,v))}\,\frac{\Phi_v(t,v)}{\Phi_{vv}(t,v)} = \frac{\lambda^\star(t,v)}{\gamma\,\sigma^\star(t,v,\lambda^\star(t,v))}\,,
\end{aligned}$$

as well as

$$k^\star(t,v) = \frac{1}{v}\left(\phi_v(t,v)\right)^{-\frac{1}{\gamma}} = \frac{1}{v}\,\left(f(t)\,v^{-\gamma}\right)^{-\frac{1}{\gamma}} = \left(f(t)\right)^{-\frac{1}{\gamma}}\,.$$

And the proof is finished. □

Remark 3.2 *Establishing the solution is based on the function f in* (49)*. The transformation $g = f^{-1/\gamma}$ is applied and requires f to be nonnegative. Accordingly, the function g satisfies the Riccati ODE in* (50) *and lives also on $\mathbb{R}^+$. As a solution strategy we identifying a particular solution g_P. This works for $\gamma > 1$, since then $g_P > 0$, i.e. the particular solution is in the region where g is specified on. However, the solution strategy brakes down for $0 < \gamma < 1$. Then we would have $g_P < 0$ and this candidate is not an admissible solution. This explains why we cannot provide a solution for the case $0 < \gamma < 1$, at least, with our methods at hand.*

Again, we need to show that the candidates derived in Theorem (3.4) are indeed optimal. This is done in the following verification theorem. The proof is provided in the Appendix.

Theorem 3.5 (Verification Result for the Case $\gamma > 1$) *Let $\kappa > 0$ and $\alpha > 2$ and $\alpha = 2\gamma + 2$. Assume the utility function of wealth, the utility function of the consumption rate and the disutility function are given by (33), (34) and (35), respectively. Then the candidates given via (40) - (44) are the optimal investment and control strategy (i.e. own-company stockholding, market portfolio holding and non-systematic Sharpe ratio strategy), the optimal consumption rate and value function of the optimal control problem (18) for the case $\gamma > 1$.*

4 Discussion and Implications of Results

The previous section established results on the optimal behavior of the individual and derived the participation constraint, i.e. conditions for the her to accept the offer by the smaller listed company. In the following we discuss the results by investigating the sensitivities of the optimal strategies and the participation constraint when varying model parameters.

4.1 Optimal Work Effort

Theorems 3.1 and 3.2 indicate the individual's maximized utility and associated optimal behavior in terms of personal portfolio selection, consumption and work effort decision, given that she accepts to job offer by the smaller listed company, all subject to the log utility set-up. We now investigate the sensitivity of the optimal work effort to variations of the risk aversion and

work effectiveness characteristics and time preference. Note that the portfolio selection and consumption are in line with standard results in the log utility setup and are here of limited interest.

The individual is characterized by the work effectiveness parameters work productivity ($1/\kappa$, with $\kappa > 0$), and disutility stress ($\alpha > 2$) and the time preferences of consumption from work effort ($\rho \in \mathbb{R}$) and disutility ($\tilde{\rho} \in \mathbb{R}$), respectively. To produce results that have relativity to a base-level of work effort, as indicated by a base-level non-systematic Sharpe ratio control decision $\lambda_0 > 0$, the disutility C given by (16) is reparameterized to

$$C(t, v, \lambda) = e^{-\tilde{\rho}\, t}\, \frac{\tilde{\kappa}}{\alpha} \left(\frac{\lambda}{\lambda_0}\right)^{\alpha}, \quad \text{for } \lambda \geq 0\,, \quad \gamma > 0,$$

and the utility of wealth U_1 and the utility of consumption U_2 remain unchanged.

The individual's optimal work effort for the new disutility parametrization is $\lambda^\star(t, v) = \lambda_0^{\frac{\alpha}{\alpha-2}} \, (\frac{e^{\tilde{\rho}\, t}}{\tilde{\kappa}} f(t))^{\frac{1}{\alpha-2}}$ (see Theorem 3.1 for the optimal choice under the original parametrization). Assuming that the inverse work productivity satisfies $1/\tilde{\kappa} > \lambda_0^{-2}\, e^{|\tilde{\rho}|\, T}/K$ we guarantee that the optimal work effort $\lambda^\star$ is not less than the base level λ_0, i.e. $\lambda^\star \geq \lambda_0 > 0$. If not stated otherwise, the default values for the parameters are $\alpha = 5$, $1/\tilde{\kappa} = 1000$, $r = 0.05$, $\lambda_P = 0.20$, $\lambda_0 = 0.10$, $\rho = 0.10$, $\tilde{\rho} = -0.10$, $K = 1$, $\widehat{\delta} = 0.20$, and $T = 10$.

The individual's optimal work effort choice is positively related to her work productivity and negatively related to her disutility stress. This result is illustrated by Figures 1 and 2, which graph the optimal work effort $\lambda^\star$ versus time t and work productivity $1/\tilde{\kappa}$ and, time t and disutility stress α, respectively. Both figures indicate that the individual's optimal work effort is negatively related to time, i.e. $\lambda^\star$ is decreasing over time. The individual spends in general more work effort at the beginning of the time horizon. Note that the monotonicity of the optimal work effort depends on the sign of ρ, see discussion of Figure 4 below.

Figure 3 shows the optimal work effort choice $\lambda^\star$ w.r.t. the time preference of consumption ρ and time t. The figure indicates that with increasing time the optimal work effort decreases as already observed above. This implies that the individual is more productive at the beginning of her career path. The optimal work effort is also decreasing for increasing time preference of consumption ρ. An individual which has a higher consumption preference will deliver a lower work effort, especially at the beginning of the time horizon.

Figure 4 graphs the optimal work effort choice $\lambda^\star$ w.r.t. the time preference of disutility $\tilde{\rho}$ and time t. The optimal work effort is positively related to the time preference of work related disutility $\tilde{\rho}$, i.e. with increasing value of $\tilde{\rho}$ the individual is becoming more productive and delivers a higher level of the optimal work effort indicating a reasonable behavior: The higher the cost for spending work effort the lower is the optimal work effort. Note that positive values of $\tilde{\rho}$ are associated with work effort becoming cheaper over time. For this parameter set, we first observe over time an increase of work effort and then a decrease at the end of the time horizon. However, typically we expect $\tilde{\rho}$ to be negative, i.e., work effort becomes more expensive with the passing of time.

4.2 Participation Constraint

The participation constraint is given in Theorem 3.3. Denote $\delta^\star$ the minimal salary rate such that the participation constraint holds, i.e. $\delta^\star = \inf\{\delta \in \mathbb{R} : \delta \text{ satisfies (32)}\}$. Taking account of the reparametrization gives

$$\delta^\star = \begin{cases} \delta_0 - \dfrac{(\alpha-2)}{2\,\alpha}\,\lambda_0^{\frac{2\alpha}{\alpha-2}}\,\dfrac{\int_0^T \left(\frac{e^{\tilde{\rho}\,s}}{\tilde{\kappa}}\right)^{\frac{2}{\alpha-2}}\, f(s)^{\frac{\alpha}{\alpha-2}}\,\mathrm{d}s}{K\,T + \frac{1}{\rho^2}\,[1-e^{-\rho T}(1+\rho T)]}\,, & \text{for } \rho \neq 0\,, \\ \delta_0 - \dfrac{(\alpha-2)}{2\,\alpha}\,\lambda_0^{\frac{2\alpha}{\alpha-2}}\,\dfrac{\int_0^T \left(\frac{e^{\tilde{\rho}\,s}}{\tilde{\kappa}}\right)^{\frac{2}{\alpha-2}}\, f(s)^{\frac{\alpha}{\alpha-2}}\,\mathrm{d}s}{K\,T + \frac{1}{2}\,T^2}\,, & \text{for } \rho = 0\,. \end{cases} \tag{54}$$

Now, $\alpha > 2$ by assumption and $f > 0$ by Theorem 3.1. And the minimal salary rate of the smaller listed company that is satisfying the participation constraint is always below the salary rate of the outside option, i.e. $\delta^\star < \delta$. The salary rate discount can be explained by the fact that the smaller company is offering in return for the reduced salary the possibility to affect the share price by work effort and thereby to increase the utility derived from the individual's investment. In the following we investigate the minimal required salary rate $\delta^\star$ depending on the individual's parameters (work productivity $1/\tilde{\kappa}$, disutility stress α, time preference of consumption ρ and time preference for work effort $\tilde{\rho}$) to characterize individuals that are attracted by an offer of the smaller listed company.

Figure 5 displays the minimal required salary rate $\delta^\star$ w.r.t. disutility stress α and work productivity $1/\tilde{\kappa}$. The minimal required salary rate is

decreasing with increasing work productivity and increasing with increasing disutility stress. This means that a more productive individual is willing to accept a lower salary rate because she can compensate the loss of utility by the ability to improve the unsystematic Sharpe ratio λ. On the other hand, an individual with a higher disutility stress requires a higher salary rate to accept the contract from the smaller listed company.

The effect of the time preferences is shown in Figure 6. The required minimal salary rate $\delta^\star$ is graphed against the time preference of consumption ρ and the time preference of disutility from work effort $\tilde{\rho}$, respectively. Increasing the time preference parameter for consumption increases the minimal required salary rate. In contrast, the required minimal salary rate decreases with increasing time preference of disutility. This is attributed to the average disutility from work effort being lower for a higher value of $\tilde{\rho}$. The individual will deliver a higher work effort, see also Figure 4.

We summarize that the offered salary rate δ can act as a selection device for the smaller listed company. Under the assumption that potential job candidates have an identical outside option, the group of individuals satisfying a more restrictive participation constraint is in general more talented, i.e. the individuals exhibit a lower disutility stress α, a higher productivity $1/\tilde{\kappa}$, a lower time preference for consumption ρ, and a higher time preference for disutility from work effort $\tilde{\rho}$. Viewing the holdings in the own-company shares ($\pi^{S\star}(t) = \lambda^\star(t)/\sigma$) as a way of voluntarily linking the pay to performance, our results reflect common practice in executive remuneration. A more talented manager is in general attracted by a lower fixed salary component and a higher performance linked salary component.

5 Conclusion and Outlook

We establish a model framework that gives insight into an individual's occupational decision when she can choose between two different positions. She is offered an executive position in smaller listed company where she can affect the company's share price by work effort. Alternatively, she can take up a mid-level management position with a larger company but then forgoes the possibility to affect the other company's share price. We identify conditions for the individual to work for the smaller listed company where the participation constraint is given in terms of the salary differential of the two job alternatives. In particular, we derive the minimal required salary $\delta^\star$ that

needs to be offered by the smaller company to attract the individual and thereby characterize the participation constraint. In general, we find that a more talented individual requires a lower salary to be attracted to the smaller listed company. This salary pattern can be observed in practice, e.g., in the pharmaceutical industry, the IT sector, and the financial industry.

Given that the participation constraint holds, we give explicit solutions for the individual's utility maximizing behavior in terms of the investment strategy ($\pi = (\pi^P, \pi^S)$), consumption (k), and work effort (λ). Overall, our results depend sensibly on her characteristics, work productivity $1/\kappa$, disutility stress α, time preference of consumption ρ, and time preference of work effort $\tilde{\rho}$. We demonstrate that an executive with higher work effectiveness (quality) undertakes more work effort, which is associated with a lower minimal required salary $\delta^\star$. The main analysis is performed in the log-utility setting. However, we also explore the broader setup of constant relative risk aversion.

A future development of this work is to extend the semi-static game between the individual and the smaller listed company to a stochastic differential game. The aim of the company is then to maximize share holder value. The additional control available to the company is the quantity of share-based payments granted to the individual that affect her holdings in the company's shares. The stochastic differential game can then be investigated for equilibria. This setup is likely to provide more insight into the design of optimal share-based payments.

Appendix

Proof of Theorem 3.5. Define the performance functional of our optimal investment, consumption and control decision again by (27). Our candidates are optimal if we have

$$\begin{aligned} &J'(t,v;\pi^\star,\lambda^\star,k^\star) = \Phi(t,v) \text{ and} \\ &J'(t,v;\pi,\lambda,k) \leq \Phi(t,v)\,, \text{ for all } (\pi,\lambda,k) \in A'_\gamma(t,v)\,. \end{aligned}$$

Let $u \in A'_\gamma(t,v)$. Since $\Phi \in C^{1,2}$, we obtain by Ito's formula:

$$\Phi(T,V_T^u) - \int_t^T \kappa(V_s^u)^{1-\gamma}\frac{\lambda_s^\alpha}{\alpha}\,\mathrm{d}s + \int_t^T \frac{(V_s^u\,k)^{1-\gamma}}{1-\gamma}\,\mathrm{d}s = \Phi(t,v) +$$
$$\int_t^T \Big\{\Phi_t(s,V_s^u) + \Phi_v(s,V_s^u)V_s^u\big[r + \pi_s^S\,\lambda_s\,\sigma + (\pi_s^P + \beta\,\pi_s^S)\lambda^P\sigma^P + \delta - k_s\big]$$
$$+ 1/2\Phi_{vv}(s,V_s^u)\,(V_s^u)^2\,\big[(\pi_s^P + \beta\,\pi_s^S)^2(\sigma^P)^2 + (\pi_s^S\sigma)^2\big]$$
$$- \kappa(V_s^u)^{1-\gamma}\frac{\lambda_s^\alpha}{\alpha}\mathrm{d}s + \frac{(V_s^u\,k)^{1-\gamma}}{1-\gamma}\Big\}$$
$$+ \int_t^T \Phi_v(s,V_s^u)V_s^u(\pi_s^P + \beta\,\pi_s^S)\sigma^P\,\mathrm{d}W_s^P + \int_t^T \Phi_v(s,V_s^u)V_s^u\pi_s^S\sigma\,\mathrm{d}W_s\,. \quad (55)$$

For the optimality candidates given in (19), the local martingale component in (55) disappears. A sufficient condition to verify this is the square integrability condition

$$\mathbb{E}\left[\int_t^T \left(\Phi_v(s,V_s^{u^\star})V_s^{u^\star}\right)^2\left([\pi_s^{P\star} + \beta\,\pi_s^{S^\star}]^2(\sigma^P)^2 + [\pi_s^{S^\star}\sigma]^2\right)\right]\mathrm{d}s \;<\; \infty\,. \quad (*)$$

Now substituting the candidates from (40) - (44) yields

$$\left(\Phi_v(s,V_s^{u^\star})V_s^{u^\star}\right)^2\left([\pi_s^{P\star} + \beta\,\pi_s^{S^\star}]^2(\sigma^P)^2 + [\pi_s^{S^\star}\sigma]^2\right)$$
$$= \frac{\left(V_s^{u^\star}\right)^{2(1-\gamma)}\,f(s)^2}{\gamma^2}\left[(\lambda^P)^2 + \left(\frac{1}{\kappa\gamma}f(s)\right)^{\frac{1}{\gamma}}\right]. \quad (**)$$

The RHS of $(**)$ is $\left(V_s^{u^\star}\right)^{2(1-\gamma)}$ times a deterministic and continuous function on the compact set $[0,T]$. The deterministic part is uniformly bounded. Therefore it is sufficient to focus on the stochastic component: $V_s^{u^\star}$ satisfies the wealth equation

$$\mathrm{d}V_t^{u^\star} = V_t^{u^\star}\Big[r\,\mathrm{d}t + \frac{\lambda_P^2}{\gamma}\,\mathrm{d}t + \frac{(\lambda^\star(t,V_t^{u^\star}))^2}{\gamma}\,\mathrm{d}t\; - (f(t))^{-\frac{1}{\gamma}}\,\mathrm{d}t + \delta\mathrm{d}t$$
$$+ \frac{\lambda_P}{\gamma}\,\mathrm{d}W_t^P + \frac{\lambda^\star(t,V_t^{u^\star})}{\gamma}\,\mathrm{d}W_t\Big]\,,$$

for which we have substituted the optimality candidates (40) in the original wealth equation. Recalling that $\lambda^\star(t,v)$ is a deterministic function in t and

further does not depend on v and that $f(t)$ is a deterministic function as well, we see that $V_t^{u^\star}$ follows a log-normal distribution for all $t \geq 0$ with parameters being uniformly bounded for all $t \in [0,T]$. Since all moments of a log-normally distributed random variable exist, we have proven $(*)$. Furthermore Φ satisfies the HJB equation (8), i.e. for $u = u^\star = (\pi^{P^\star}, \pi^{S^\star}, \lambda^\star, k^\star)$, the choice (16) of the disutility function and the choice (15) of the consumpion rate we have:

$$\begin{aligned}
&\Phi_t(s, V_s^{u^\star}) \,+\, \Phi_v(s, V_s^{u^\star}) V_s^{u^\star} \left[r + \pi_s^{S^\star}\, \lambda_s^\star\, \sigma + \left(\pi_s^{P^\star} + \beta\, \pi_s^{S^\star}\right) \lambda^P \sigma^P + \delta - k_s^\star \right] \\
&+ 1/2\, \Phi_{vv}(s, V_s^{u^\star}) \left(V_s^{u^\star}\right)^2 \left[(\pi_u^{P^\star} + \beta\, \pi_u^{S^\star})^2 (\sigma^P)^2 + (\pi_s^{S^\star} \sigma)^2 \right] \\
&- \kappa \left(V_s^{u^\star}\right)^{1-\gamma} \frac{(\lambda_s^\star)^\alpha}{\alpha} + \frac{\left(V_s^{u^\star} k^\star\right)^{1-\gamma}}{1-\gamma} = 0\,.
\end{aligned}$$

Then, for $u = u^\star$, the expectation of equation (55) using $\Phi(T, v) = v^{1-\gamma}/(1-\gamma)$ is:

$$\begin{aligned}
&\mathbb{E}^{t,v}\left[\frac{\left(V_T^{u^\star}\right)^{1-\gamma}}{1-\gamma} \right] - \mathbb{E}^{t,v}\left[\int_t^T \kappa \left(V_s^{u^\star}\right)^{1-\gamma} \frac{(\lambda_s^\star)^\alpha}{\alpha} \mathrm{d}s \right] + \mathbb{E}^{t,v}\left[\int_t^T \frac{\left(V_s^{u^\star} k^\star\right)^{1-\gamma}}{1-\gamma}\, \mathrm{d}s \right] \\
&= J'(t, v; \pi^\star, \lambda^\star, k^\star) = \Phi(t, v)\,.
\end{aligned}$$

The optimality of our candidates is finally shown if we have for all $(\pi, \lambda, k) \in A'_\gamma(t, v)$:

$$\begin{aligned}
&\mathbb{E}^{t,v}\left[\frac{(V_T^u)^{1-\gamma}}{1-\gamma} \right] - \mathbb{E}^{t,v}\left[\int_t^T \kappa\, (V_s^u)^{1-\gamma} \frac{(\lambda_s)^\alpha}{\alpha}\, \mathrm{d}s \right] + \mathbb{E}^{t,v}\left[\int_t^T \frac{(V_s^u k)^{1-\gamma}}{1-\gamma}\, \mathrm{d}s \right] \\
&= J'(t, v; \pi, \lambda, k) \leq \Phi(t, v)\,.
\end{aligned} \tag{56}$$

Also, since Φ satisfies the HJB equation (8), we get for all $(\pi, \lambda, k) \in A'_\gamma(t, v)$:

$$\begin{aligned}
&\Phi_t(s, V_s^u) \,+\, \Phi_v(s, V_s^u) V_s^u \left[r + \pi_s^S\, \lambda_s\, \sigma + (\pi_s^P + \beta\, \pi_s^P) \lambda^P \sigma^P + \delta - k_s \right] \\
&\quad + 1/2\, \Phi_{vv}(s, V_s^u) \left(V_s^u\right)^2 \left[(\pi_u^P + \beta\, \pi_u^S)^2 (\sigma^P)^2 + (\pi_s^{S^\star} \sigma)^2 \right] \\
&\quad - \kappa\, (V_s^u)^{1-\gamma} \frac{(\lambda_s)^\alpha}{\alpha} + \frac{(V_s^u k)^{1-\gamma}}{1-\gamma} \leq 0\,.
\end{aligned}$$

Substituting this in (55) , recalling that $\Phi_v(t,v) = f(t)\, v^{-\gamma}$, we get:

$$\Phi(T, V_T^u) - \int_t^T \kappa\, (V_s^u)^{1-\gamma}\, \frac{\lambda_s^\alpha}{\alpha}\, \mathrm{d}s + \int_t^T \frac{(V_s^u k)^{1-\gamma}}{1-\gamma}\, \mathrm{d}s \;\leq \Phi(t,v)$$
$$+ \underbrace{\int_t^T (V_s^u)^{1-\gamma} f(s) \left(\pi_s^P + \beta\, \pi_s^S\right) \sigma^P\, \mathrm{d}W_s^P + \int_t^T (V_s^u)^{1-\gamma} f(s) \pi_s^S \sigma\, \mathrm{d}W_s}_{=:M_T^t} . \tag{57}$$

To verify equation (56), we impose conditions under which the local martingale M^t is a martingale. Recall $\Phi_v(t,v) = f(t)\, v^{-\gamma}$ and calculate the quadratic variation of M^t

$$\langle M^t \rangle_T = \int_t^T (V_s^u)^{2(1-\gamma)} f^2(s) \left([\pi_s^P + \beta\, \pi_s^S]^2 (\sigma^P)^2 + [\sigma \pi_s^S]^2 \right) \mathrm{d}s$$
$$\leq \frac{1}{2} \sup_{0 \leq s \leq T} f(s)^2 \Big(\int_t^T (V_s^u)^{4(1-\gamma)}\, \mathrm{d}s + \int_t^T \left([\pi_s^P + \beta\, \pi_s^S]^2 (\sigma^P)^2 + [\sigma \pi_s^S]^2 \right)^2 \mathrm{d}s \Big) , \tag{58}$$

where the second line is a straightforward upper bound. We show that M^t is a martingale by deriving the integrability of the quadratic variation $\langle M^t \rangle_T$. First we use that f is a continuous function on the compact set $[0,T]$ and is uniformly bounded, and thus $\sup_{0 \leq s \leq T} f(s)^2$ is finite. We are left to deal with the two expressions in the brackets of (58). The second expression is bounded in expectation by assumption, see (37) in Def. 3.2. In what follows we establish that that the first expression is finite by showing that $\mathbb{E}^{t,v}[(V_s^u)^\xi] < \infty$ uniformly, where $\xi = 4(1-\gamma) < 0$ for $\gamma > 1$.

Applying variation of constants, the solution of the wealth equation (4) expressed with respect to the parameter λ is

$$V_t^u = V_0^u\, e^{(r+\delta)t + \int_0^t \left((\pi_s^P + \beta\, \pi_s^S) \lambda^P \sigma^P + \pi_s^S \lambda_s \sigma - k_s \right) \mathrm{d}s}\, e^{L_t - \frac{1}{2}\langle L \rangle_t} ,$$

where $L_t = \int_0^t (\pi_s^P + \beta\, \pi_s^S) \sigma^P \mathrm{d}W_u^P + \int_0^t \pi_s^S \sigma\, \mathrm{d}W_u$ and $\langle L \rangle_t = \int_0^t (\pi_s^P + \beta\, \pi_s^S)^2 (\sigma^P)^2 + (\pi_s^S \sigma)^2 \mathrm{d}s$.
Using this we have

$$(V_t^u)^\xi = (V_0^u)^\xi\, e^{\xi L_t - \frac{1}{2}\xi^2 \langle L \rangle_t} \times e^{\xi \left[\frac{1}{2}(\xi - 1) \langle L \rangle_t + (r+\delta)t + \int_0^t \left((\pi_s^P + \beta\, \pi_s^S) \lambda^P \sigma^P + \pi_s^S \lambda_s \sigma - k_s \right) ds \right]} .$$

The second factor is uniformly bounded by a constant, compare the conditions (37), (38) and (39) of Definition 3.2, recalling that $\xi < 0$ for $\gamma > 1$, and

keeping in mind that $k_t \geq 0$, $t \leq s \leq T$, by assumption. It remains to prove that the first factor, $Z_t := e^{\xi L_t - \frac{1}{2}\xi^2 \langle L \rangle_t} \in L^2(P)$, $t \leq u \leq T$, is integrable. However, Z^t is strictly positive local martingale since it is the stochastic exponential of the local martingale $\xi\, L^t$. The Novikov condition holds by (37), i.e.: $\mathbb{E}^{t,v}(e^{\frac{1}{2}\xi^2 \langle L^t \rangle_T}) < \infty$, and hence Z^t is a true martingale and $\mathbb{E}^{t,v}(Z^t_s) = 1$, $t \leq s \leq T$. The local martingale M^t is therefore a martingale vanishing in expectation in (57), implying (56) for $u = (\pi, \lambda, k) \in \mathcal{A}'_\gamma(t, v)$. □

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Figures

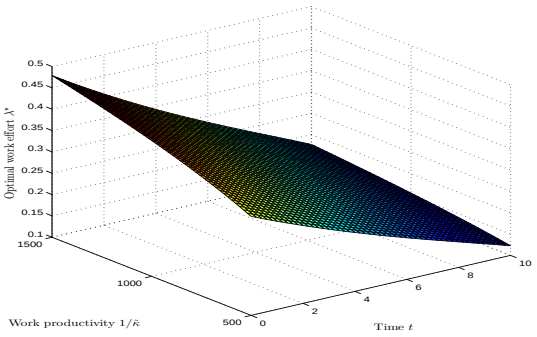

Figure 1: Optimal work effort $\lambda^\star$ w.r.t. work productivity $1/\tilde{\kappa}$ and time t for fixed disutility stress $\alpha = 5$, time preferences $\rho = 0.10$ and $\tilde{\rho} = -0.10$, $K = 1$, base-level work effort $\lambda_0 = 0.10$ and time horizon $T = 10$ years.

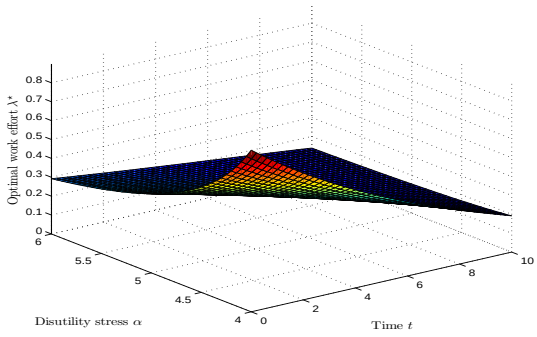

Figure 2: Optimal work effort $\lambda^\star$ w.r.t. disutility stress α and time t for fixed work productivity $1/\tilde{\kappa} = 1000$, time preferences $\rho = 0.10$ and $\tilde{\rho} = -0.10$, $K = 1$, base-level work effort $\lambda_0 = 0.10$ and time horizon $T = 10$ years.

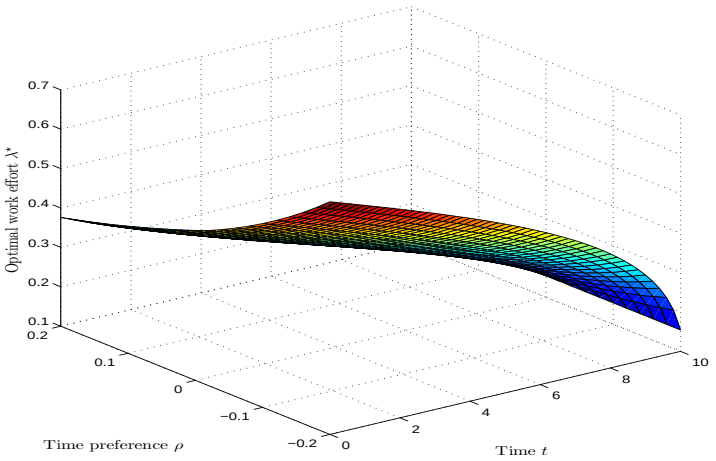

Figure 3: Optimal work effort $\lambda^\star$ w.r.t. the time preference of consumption ρ and time t for fixed work productivity $1/\tilde{\kappa} = 1000$, $\alpha = 5$, time preference $\tilde{\rho} = -0.10$, $K = 1$, base-level work effort $\lambda_0 = 0.10$ and time horizon $T = 10$ years.

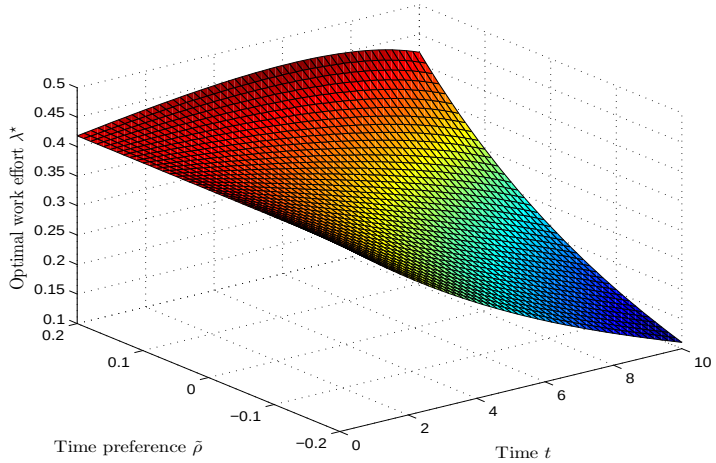

Figure 4: Optimal work effort $\lambda^\star$ w.r.t. the time preference of disutility $\tilde{\rho}$ and time t for fixed work productivity $1/\tilde{\kappa} = 1000$, disutility stress $\alpha = 5$, time preference $\rho = 0.10$, $K = 1$, base-level work effort $\lambda_0 = 0.10$ and time horizon $T = 10$ years.

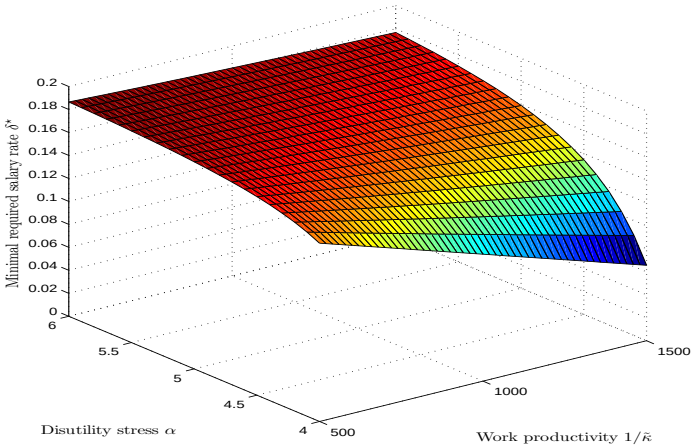

Figure 5: Minimal required salary rate $\delta^\star$ w.r.t. disutility stress α and work productivity $1/\tilde{\kappa}$ for fixed time preferences $\rho = 0.10$ and $\tilde{\rho} = -0.10$, $K = 1$, base-level work effort $\lambda_0 = 0.10$, outside salary rate $\widehat{\delta} = 0.2$, and time horizon $T = 10$ years.

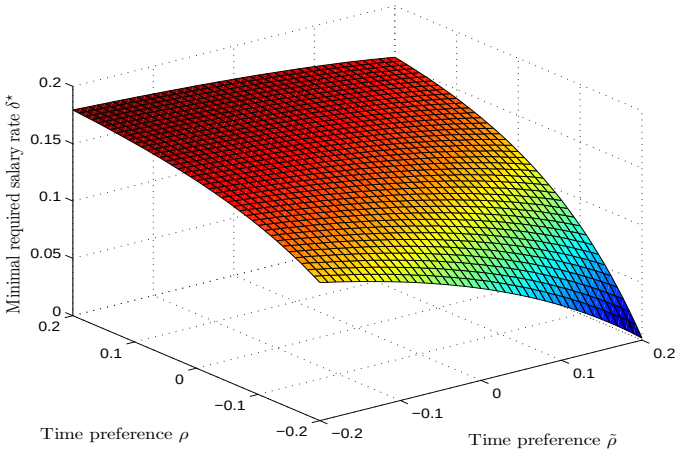

Figure 6: Minimal required salary rate $\delta^\star$ w.r.t. the time preferences ρ and $\tilde{\rho}$ for fixed work productivity $1/\tilde{\kappa} = 1000$, disutility stress $\alpha = 5$, $K = 1$, base-level work effort $\lambda_0 = 0.10$ and time horizon $T = 10$ years.

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