Projection-Based Handling of Contact Constraints

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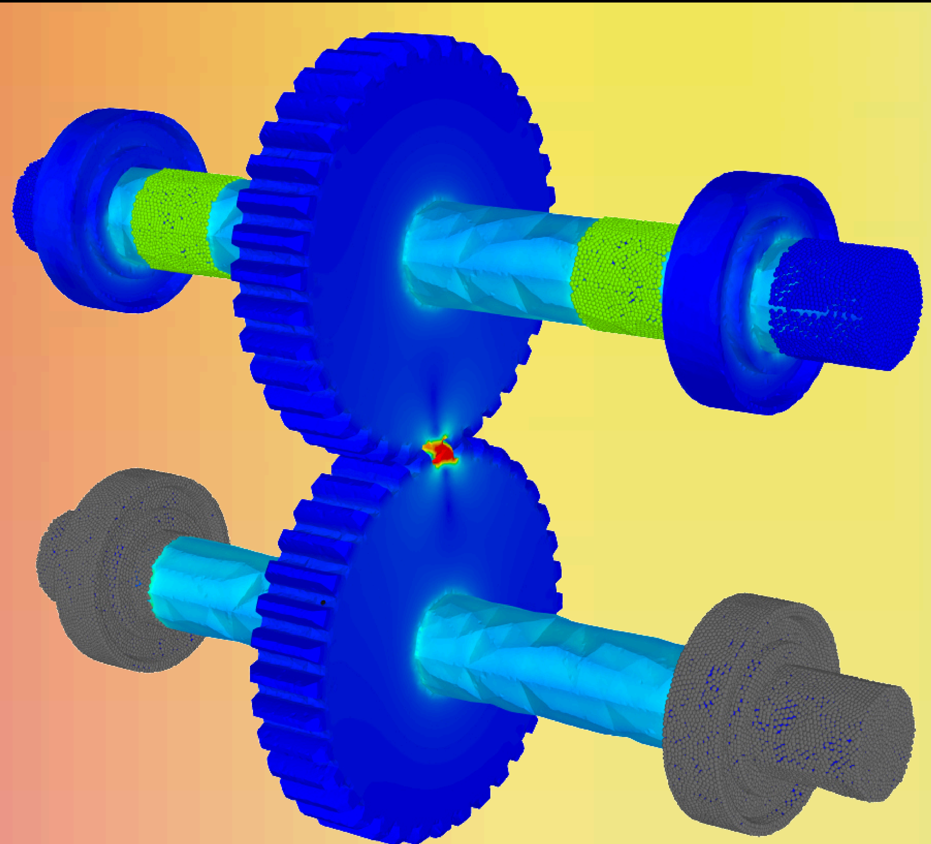
PROBLEM

Simulating **contact between deformable** objects requires the solution of **indefinite systems**, which generally are solved using **direct methods**. For high numbers of degrees of freedom this can be very **inefficient**, instead using **iterative solvers** like the preconditioned conjugate gradient (PCG) method is favorable, which can only be used for **symmetric positive definite systems**.



RELATED WORK

Li et al. [1] simulate contact by solving a non-linear optimization problem using the barrier-method. This requires a direct solver to solve the resulting indefinite systems.

Cheshmi et al. [2] introduce a solver for large-scale quadratic programs based on the active-set method. In each active-set iteration the matrix factorisation is updated to solve the current indefinite system.

CONTRIBUTION

We extend

- MPCG by Baraff and Witkin [3]
- Prefiltered approach by Tamstorf et al. [4]

to handle contact constraints between deformables.

▶ Solve QPs using (GPU-parallelized) iterative solvers

REFERENCES

[1] Minchen Li et al.. 2020. Incremental potential contact: intersection-and inversion-free, large-deformation dynamics. ACM Transactions on Graphics 39, 4

[2] Kazem Cheshmi et al. 2020. NASOQ: Numerically Accurate Sparsity-Oriented QP Solver. ACM Transactions on Graphics 39, 4

[3] David Baraff and Andrew Witkin. 1998. Large steps in cloth simulation. In Proceedings of the 25th annual conference on Computer graphics and interactive techniques -SIGGRAPH '98. ACM Press.

[4] Rasmus Tamstorf et al.. 2015. Smoothed aggregation multigrid for cloth simulation. ACM Transactions on Graphics 34, 6

software campus

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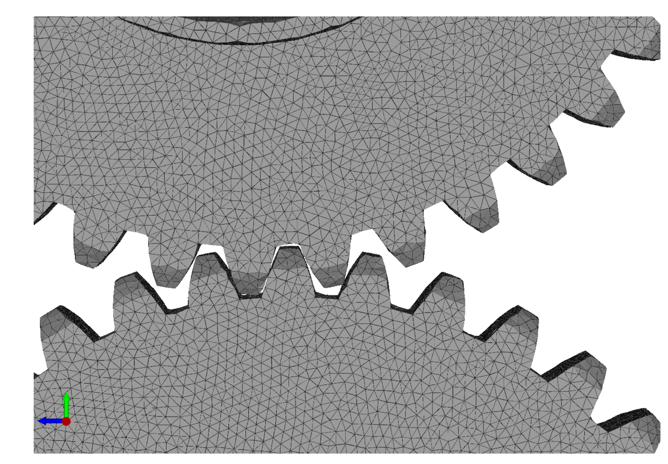
BACKGROUND

- Compute deformation using FEM – results in large sparse systems
- Contact modeled as distance constraints between node-triangle and edge-edge pairs



▶ Quadratic Program:

$$\min \mathbf{u}^T\mathbf{K}\mathbf{u} - \mathbf{u}^T\mathbf{f} \quad \text{s.t.} \quad \mathbf{C}_{\text{ineq}}\mathbf{u} \geq \mathbf{g}$$

APPROACH

ACTIVE-SET METHOD

Compute step direction $\mathbf{d}$

$$\min \mathbf{d}^T\mathbf{K}\mathbf{d} - \mathbf{d}^T\mathbf{b}_k \quad \text{s.t.} \quad \mathbf{C}_k\mathbf{d} = 0$$

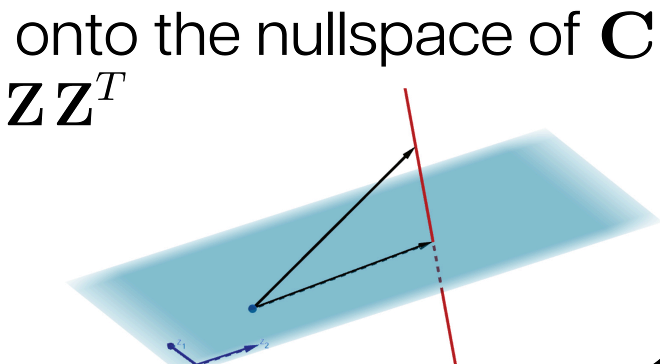
take step $\mathbf{u}_k + \alpha\mathbf{d}_k$

update $\mathbf{C}_k$ and $\mathbf{b}_k$

PROJECTION-BASED METHODS

$\mathbf{P}$ orthogonal projection onto the nullspace of $\mathbf{C}$

$\mathbf{P} = \mathbf{Z}\mathbf{Z}^T$



MPCG FOR CONTACT CONSTRAINTS

Solve $\mathbf{K}\mathbf{d} = \mathbf{b}_k$ while enforcing $\mathbf{C}_k\mathbf{d} = 0$

In each MPCG iteration: Project search direction $\mathbf{p}_i$ onto feasible space

$$\mathbf{p}_i \leftarrow \mathbf{P}\mathbf{p}_i$$

PREFILTERING FOR CONTACT CONSTRAINTS

Better preconditioning if prefiltered system is used

$$(\mathbf{PKP} + \mathbf{I} - \mathbf{P})\mathbf{d} = \mathbf{P}\mathbf{b}_k$$

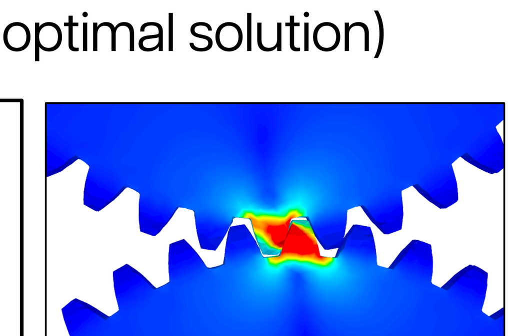
solve using PCG method

Post-Filtering: $\mathbf{d} \leftarrow \mathbf{P}\mathbf{d}$
Required to fulfill requested accuracy

RESULTS

Gears simulation scenario:
2.2 M degrees of freedom, ~1T potential contacts (3 active at optimal solution)

	MPCG	Prefiltering and PCG
Solve Time	6.8 s	11.2 s
Residual	5.1 e-4	2.8 e-4
Max. constraint violation	1.9 e-19	8.5 e-20



- GPU-parallelized iterative methods to solve QPs
- Constraints are enforced with very high accuracy
- Minimization error independant of constraint violation error

LIMITATIONS

- Requires feasible starting point – currently only usable in combination with a primal active-set method
- Recompuation of projection matrix in each active-set iteration necessary
- Prefiltering: Recomputation of preconditioner in each active-set iteration