

# Fast Multitarget Tracking via Strategy Switching for Sensor-Based Sorting

Georg Maier<sup>1</sup>, Florian Pfaff<sup>2</sup>, Christoph Pieper<sup>3</sup>, Robin Gruna<sup>1</sup>, Benjamin Noack<sup>2</sup>, Harald Kruggel-Emden<sup>3</sup>, Thomas Längle<sup>1</sup>, Uwe D. Hanebeck<sup>2</sup>, Siegmart Wirtz<sup>3</sup>, Viktor Scherer<sup>3</sup> and Jürgen Beyerer<sup>1</sup>

**Abstract**—State-of-the-art sensor-based sorting systems provide solutions to sort various products according to quality aspects. Such systems face the challenge of an existing delay between perception and separation of the material. To reliably predict an object’s position when reaching the separation stage, information regarding its movement needs to be derived. Multitarget tracking offers approaches through which this can be achieved. However, processing time is typically limited since the sorting decision for each object needs to be derived sufficiently early before it reaches the separation stage. In this paper, an approach for multitarget tracking in sensor-based sorting is proposed which supports establishing an upper bound regarding processing time required for solving the *measurement to track* association problem. To demonstrate the success of the proposed method, experiments are conducted for datasets obtained via simulation of a sorting system. This way, it is possible to not only demonstrate the impact on required runtime but also on the quality of the association.

## I. INTRODUCTION

Sensor-based sorting provides solutions for sorting various products, for instance according to quality aspects [1]. Typical fields of application are found in food processing [2], [3], recycling [4], [5], and handling of industrial minerals [6], [7]. In many cases, a sorting task aims at separating low-quality objects from high-quality objects and hence results in an *accept or reject* task. Corresponding systems usually include components for transportation of the material, sensors for perception thereof, and a separation mechanism. Selection of an appropriate sensor or several thereof is typically based on the product to be handled as well as the sorting task itself. Applied sensors include RGB cameras in the visible spectrum, near-infrared and ultraviolet spectra, X-ray and hyper-spectral cameras. In many cases, information retrieved by the sensor may be represented in terms of an image. Hence, image evaluation is applied in order to classify perceived objects and derive an according sorting decision.

Typically, the material to be sorted is perceived while being transported. Scanning sensors, such as line-scan cameras, are usually utilized and provide advantages for instance regarding necessary illumination. As can be seen from Fig. 1, a delay exists between perception and separation of the

material. This delay is mainly due to required processing time of the evaluation system. Also, sensors and separation mechanisms are usually mounted at different stages of the system. Consequently, only information extracted at the time of perception can be utilized to predict an object’s position when reaching the separation stage. This does not allow for any assumptions regarding its movement.

Due to vast advances in area-scan camera technology, application of sensors of this kind becomes feasible in sensor-based sorting. An advantage lies in the chance to observe objects at multiple time points instead of only once. Performing multitarget tracking then enables evaluation of an object’s movement. In the course of sensor-based sorting, such information is crucial for reliable triggering of the separation mechanism, for example compressed air nozzles. Compared with static approaches, an enormous increase in precision of separation may be achieved.

In this work, including multitarget tracking into an evaluation system for sensor-based sorting is considered. Due to the movement of the material towards the separation mechanism, a deadline exists before which a sorting decision needs to be derived. Therefore, strict deadlines are to be met by algorithms employed. This challenge is addressed by providing a mechanism which selects an algorithm for solving the association problem, which poses a high computational burden upon the multitarget tracking system, in accordance to a present situation in terms of estimated system load. Contributions of this paper lie in demonstrating how sensor-based sorting systems can benefit from multitarget tracking and which steps can be taken to implement approaches which support that deadlines for deriving a sorting decision for each object are met.

This paper is organized as follows. Following this brief introduction, related work from the field of sensor-based sorting is revised in Sec. II. Sec. III starts by demonstrating the drawbacks of only utilizing static information to predict an object’s position at the stage of separation. Also, it is shown how multitarget tracking approaches can be integrated into evaluation systems in sensor-based sorting. An approach to support meeting the real-time requirement for multitarget tracking is presented in Sec. IV. Following that, experimental results are presented in Sec. V. Sec. VI concludes the paper and presents promising directions for future research.

## II. RELATED WORK

Sensor-based sorting systems consist of several hardware components to handle material feeding, transportation and

<sup>1</sup>G. Maier, R. Gruna, T. Längle and J. Beyerer are with the Fraunhofer Institute of Optronics, System Technologies and Image Exploitation IOSB, Karlsruhe, Germany. [name.surname@iosb.fraunhofer.de](mailto:name.surname@iosb.fraunhofer.de)

<sup>2</sup>F. Pfaff, B. Noack and U. D. Hanebeck are with the Intelligent Sensor-Actuator-Systems Laboratory, Karlsruhe Institute of Technology (KIT), Germany. [name.surname@kit.edu](mailto:name.surname@kit.edu)

<sup>3</sup>C. Pieper, H. Kruggel-Emden, S. Wirtz and V. Scherer are with the Department of Energy Plant Technology (LEAT), Ruhr-Universität Bochum (RUB), Germany. [surname@leat.ruhr-uni-bochum.de](mailto:surname@leat.ruhr-uni-bochum.de)

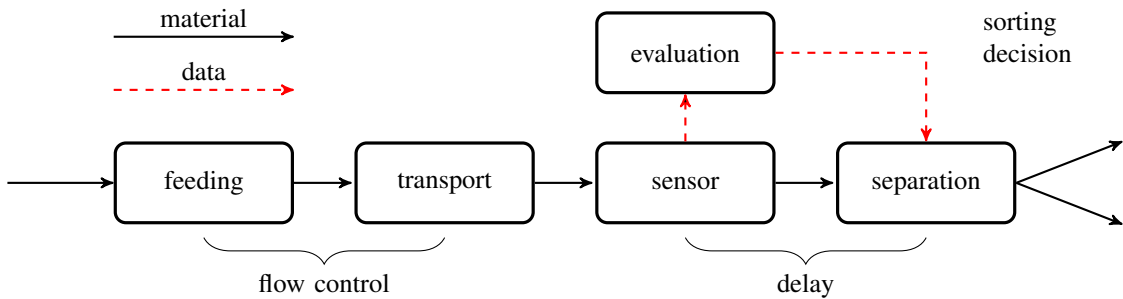


Fig. 1. Basic workflow in sensor-based sorting.

flow control, sensors, illumination, evaluation systems and separation mechanisms, see Fig. 1. Therefore, the design of high-quality systems stimulates research in various fields.

Material feeding heavily depends on the entire processing pipeline. Considering recycling applications, sensor-based sorting often is one step of many to achieve the processing task which also consists of other sorting schemes such as magnetic sorting [5]. Regarding transportation, it is desired to support flow control of the material as good as possible. Examples include conveyor belts [4] and chutes [8], whereas the choice is often based on the product at hand. This is also the case for selection of an appropriate sensor [9], [10], [11], [12], [13]. Using several sensors to improve classification performance for recycling sorting tasks is discussed in [14]. Choosing an illumination is crucial for creating an ideal situation to extract information from the sensor data [15]. Due to the already discussed real-time criterion, speed is of great importance in processing the sensor data. Therefore, implementations in hardware are sometimes favoured over software solutions [16], [17]. Methods for object classification range from SVMs [17], [18] to Fuzzy Logic [4], [16] and also neural networks [16]. For sorting of cohesive, granular materials, compressed air nozzles provide a suitable separation mechanism [13], [19]. Considering an *accept or reject* sorting task, separation is achieved by either activating a nozzle or not. Alternative separation mechanisms for instance include robot arms [20].

Recently, application of multitarget tracking in the context of sensor-based sorting has been proposed [21]. Deriving enhanced motion models using simulations based on the Discrete Element Method (DEM) has been introduced in [22].

### III. MULTITARGET TRACKING IN SENSOR-BASED SORTING

Multitarget tracking is a well-studied problem [23], [24] that still receives a lot of scientific attention. In this section, it is demonstrated how sensor-based sorting systems can benefit from incorporating multitarget tracking in corresponding evaluation systems. Also, details of a corresponding implementation are provided.

#### A. Necessity for Multitarget Tracking

As has been discussed, many state-of-the-art sensor-based sorting systems make use of scanning sensors to retrieve

information about objects contained in the material stream. Also, it has been mentioned that perception and separation happen at different points in time. Therefore, systems alike are making assumptions regarding the point in time as well as the position of an object when reaching the separation stage in case it is to be rejected. Basically, working with such static information is based on the assumption of ideal flow control. In more detail, it is assumed that:

- 1) Each object contained in the material stream is moving at the same velocity in the transport direction.
- 2) No object has a velocity perpendicular to the transport direction.

Obviously, whether truth holds to these assumptions heavily depends on the quality of flow control. For instance, in case of a conveyor belt, the material of the belt as well as its length are crucial. For certain products, designing ideal conditions in terms of flow control is a very hard problem and, in the worst case, may be infeasible. In order to compensate for deviations from the above mentioned assumptions, systems utilizing compressed air nozzles as a separation mechanism deliberately expand the activation time window as well as the area for which nozzles are to be activated, to ensure rejecting the desired object. This comes at the cost of producing by-catch, i.e. falsely co-ejected objects located nearby the targeted object, and eventually increases loss of the product to be accepted.

Furthermore, even if achieving ideal flow control for a specific product may be possible, the same system setup may fail when changing the product. Intuitively, one would expect that the geometric shape of the product under inspection has a significant impact on its motion. More precisely, it might be assumed that plates and spheres would show rather different behavior in this regard. This assumption is supported by experiments presented in [25]. To provide for systems with a high degree of flexibility towards the products to be processed, it is crucial to respect the motion of the objects. By using predictive tracking, more precise triggering of the nozzles can be performed, thus reducing by-catch and eventually resulting in economic benefits.

#### B. Tracking Objects during Evaluation

In the course of this work, integration of multitarget tracking into a software evaluation system used for sensor-based sorting is considered. Furthermore, it is assumed that

information retrieved from the sensor can be represented as an image. For the purpose of detection of measurements, subsequent to image pre-processing and segmentation, connected component analysis is performed to identify individual objects in the image data. This yields a set of unlabeled measurements, for instance represented by the centroids of the extracted connected components. The main remaining tasks of the multitarget tracking system can then be summarized as:

- **State estimation** to predict the position of the next measurement for each track
- **Gating** to subdivide the search space for subsequent association (optional)
- **Association** between current measurements and predictions
- **Internal state management** to update existing tracks, create new tracks and delete disappeared tracks

A standard Kalman filter with a constant velocity model is used for the task of estimation. State variables are given by the  $x$  and  $y$  coordinate of the centroid of the object as obtained from connected component analysis as well as velocities in  $x$  and  $y$  direction,  $v_x$  and  $v_y$ , respectively. Its complexity lies in  $O(n)$  where  $n$  denotes the number of current target tracks, which is expected to be close to the number of extracted measurements.

Solving the association problem, which is also required for each frame, typically yields higher complexity. The task is to associate each extracted measurement with an existing track or declare it to potentially belong to a new track. Various algorithms exist to solve this problem which differ in terms of computational complexity and whether they guarantee optimal results. In the remainder, local nearest neighbor (LNN) as well as global nearest neighbor (GNN) are considered for this purpose. To solve the association problem for GNN, the shortest augmenting path algorithm LAPJV [26] is considered. In terms of performance, it is important to note that LNN does not yield optimal solutions but has a comparably low complexity of  $O(n^2)$ , while LAPJV does guarantee optimal solutions at the cost of a complexity of  $O(n^3)$ . Further information regarding the handling of newly discovered objects as well as those disappearing from the observable area is provided in [21].

#### IV. HANDLING THE REAL-TIME REQUIREMENT

In Sec. III-B, core components of a multitarget tracking system for sensor-based sorting were discussed. State estimation, in this case implemented by means of a Kalman filter, has linear complexity with respect to the number of update and filter steps. However, algorithms for solving the association problem differ in complexity which generally is higher compared with state estimation. Also, corresponding algorithms differ in terms of the quality of their results. In the course of this work, exactly these properties are exploited to support fast multitarget tracking and hence increase the chance of meeting the deadlines to derive a sorting decision for each object. More precisely, the proposed system consists of several algorithms capable of solving the association

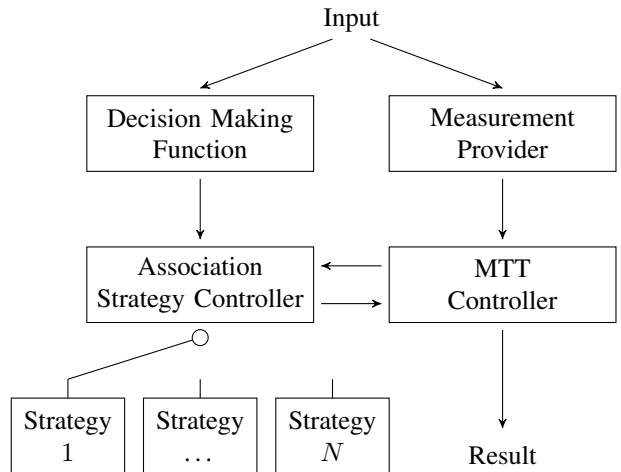


Fig. 2. Architecture of the proposed system to support meeting the real-time requirement.

problem. The system further implements a decision making function which selects the appropriate algorithm for each frame. The architecture of the system is illustrated in Fig. 2.

The purpose of the decision making function as depicted in Fig. 2 is to estimate the expected workload of the system for each frame. Based upon this estimation, the association strategy controller is configured to select an appropriate strategy. It should be noted that this on the one hand, as is the case in the remainder of this work, can imply selection of a solver algorithm, but on the other hand may also activate a gating to be performed prior to association problem solving. The implemented system, for which experimental results are presented in the next section, derives this decision by means of the number of measurements extracted per frame as well as the number of active tracks. Although the scheme is rather simple, it hence is based upon the key figures having a significant impact on the required time.

#### V. EXPERIMENTAL RESULTS

In this section, results for a multitarget tracking system switching between two association strategies, namely GNN with LAPJV and LNN, are presented. Switching occurs whenever the number of measurements obtained from a frame and / or the number of active tracks reaches a predefined threshold. More precisely, the LAPJV algorithm which is supposed to yield superior results in terms of association quality is only used whenever load is expected to be low. For the LNN solver, only assignments of measurements to predictions with a distance that does not exceed the expected average distance between two frames are considered valid.

##### A. Test methodology

Two key figures are of particular interest, namely the time required to solve the association problem and the correctness of its results. Regarding the first, the time elapsed during solving the association problem is taken for each frame. This also allows determining the overall time spent on solving the problem for the entire data-set. With respect to the

correctness of results, a ground truth needs to be available to draw any conclusion which typically is not available when using real-world data obtained from a sorting system. Therefore, results are presented for two simulated data-sets which were obtained using the simulation approach described in [22]. In this regard, the Discrete Element Method (DEM) is used to model the sorting system and calculate the particle movement, also respecting particle–particle and particle–wall interactions. As a measure of performance, it was evaluated how many actual objects existed according to the performed tracking in more than one track. More precisely, each resulting track was saved including the associated measurements and their label as was assigned during simulation. Then, for each object, the number of tracks including the corresponding label is counted. Whenever this yields a result greater 1, an object could either not be tracked without interruption, or it has been falsely associated to a different track. Equation (1) depicts the calculation of this error assuming a data-set containing actual objects  $\mathcal{O} = \{o_1, \dots, o_n\}$ , resulting in tracks  $\mathcal{T} = \{t_1, \dots, t_m\}$ .

$$e = \sum_{i=1}^n \begin{cases} 1 & \left( \sum_{j=1}^m \begin{cases} 1 & o_i \in t_j \\ 0 & \text{otherwise} \end{cases} \right) > 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

All experiments were run on an Intel Core i7-3770 with 8 GB RAM.

### B. Trade-off between Runtime and Association Quality

The first data-set was generated by simulating the sorting process with 3713 wooden spheres. Fig. 3 illustrates the time required to solve the association problem when using the LAPJV and the LNN algorithm. As can be seen, the LNN algorithm does not exceed  $122\mu\text{s}$  while the LAPJV algorithm requires up to  $543\mu\text{s}$  for certain frames. Assuming a camera operating at 200 Hz, this means that solving the association problem may take up to  $\sim 11\%$  of available processing time which also includes image processing and classification tasks, while LNN has its peak at  $\sim 2\%$ . Using the LNN algorithm, 75 objects existed in more than one track. Employing the LAPJV algorithm, this number drops to 2. In a second step, the system was configured to switch from LAPJV to LNN whenever more than 80 measurements were obtained and / or more than 80 tracks were active. The results are presented in Fig. 4. The maximum time spent per frame in this case is  $\sim 217\mu\text{s}$  which corresponds to  $\sim 4\%$  of available processing time under described conditions. Furthermore, 46 objects were contained in more than one track.

The same experiment was conducted using a data-set with 4412 simulated wooden cylinders. The required processing time when using either the LAPJV or the LNN algorithm is presented in Fig. 5. Assuming a camera running at 200 Hz, solving of the association problem takes up to  $\sim 17\%$  using LAPJV and  $\sim 5\%$  with LNN. Regarding the assessment of the association quality, the number of objects contained in more than one track is significantly higher compared with

the previously discussed data-set. This is most likely to be explained by the rather irregular movement of the cylinders resulting in a harder tracking problem. Using the LAPJV algorithm, 10 objects were found in more than one track, and using the LNN algorithm in 189. A run switching the strategy whenever more than 80 measurements were obtained and / or more than 80 tracks were active was also performed. Results are illustrated in Fig. 6. The number of objects that were contained in more than one track resulted in 166 while taking at most  $\sim 5\%$  of available processing time.

For both data-sets, experiments were conducted to formulate a performance profile in terms of required time and errors made. Results for wooden spheres are presented in Fig. 7 and for wooden cylinders in Fig. 8, respectively. As a reference, the time required by exclusively using LNN and the corresponding occurring errors were used. Fig. 7 and Fig. 8 hence illustrate the ratio of required time and errors avoided for different thresholds for strategy switching. For this purpose, simulations for thresholds ranging from 0, i.e. using exclusively LNN, to the maximum number of occurring measurements per frame were performed. As can be seen, granting the system twice the time required by LNN results in avoiding  $\sim 65\% - 70\%$  of the errors for the wooden spheres data-set and avoiding  $\sim 38\% - 39\%$  of the errors for the wooden cylinders data-set.

## VI. CONCLUSION

In this paper, multitarget tracking in sensor-based sorting was discussed. Having demonstrated how systems can benefit, it was shown how tracking can be included into a software-based evaluation framework. Emphasis was given to the limited computation time available. To tackle this challenge, a system including a decision making function to dynamically select an appropriate strategy for solving the association problem for each frame was introduced. By providing an example implementation of this function it was demonstrated that the trade-off between time spent and quality of predictions can effectively be steered. Eventually, this enables limiting the computation time spent on solving the problem and consequently supports real-time capabilities of an evaluation system for sensor-based sorting.

Intended next steps of our research include further utilization of performance profiles as have been presented. For instance, such profiles can be used to develop contract algorithms which guarantee termination for a given deadline while providing a known quality. Also, the scenario vice versa is promising, namely to require a minimum quality under the constraint to use as little computation time as possible. Another approach to make sensor-based sorting systems benefit even more from multitarget tracking is to exploit information made available by multitarget tracking for classification of objects. As objects are observed at several time points, features, for instance geometric or color-based, can also be determined for several observations. Furthermore, motion-based features, such as velocity, can be calculated and utilized during classification. Also, it is

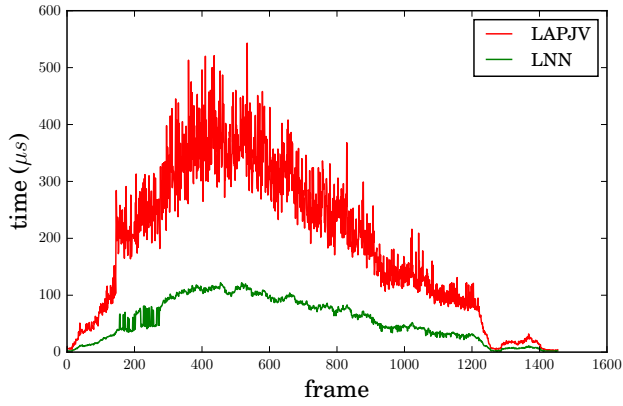


Fig. 3. Required processing time of the LNN and LAPJV algorithm per frame for a simulated data-set with wooden spheres.

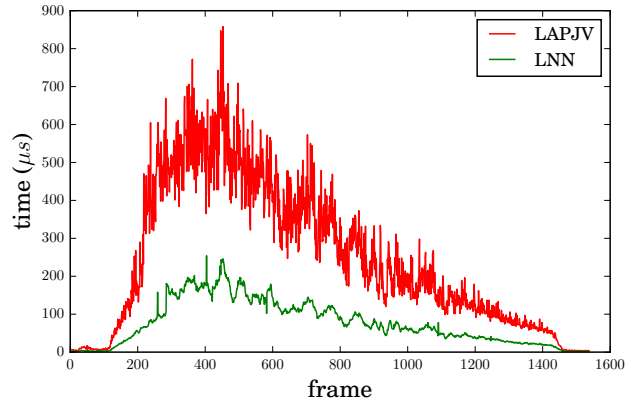
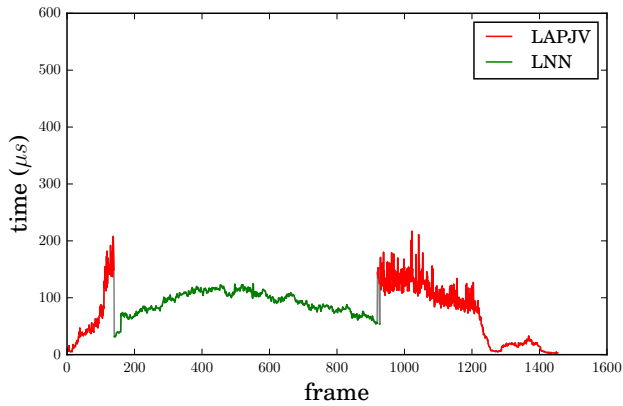
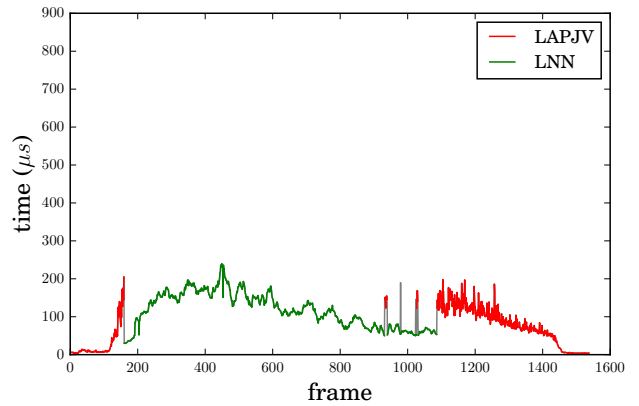


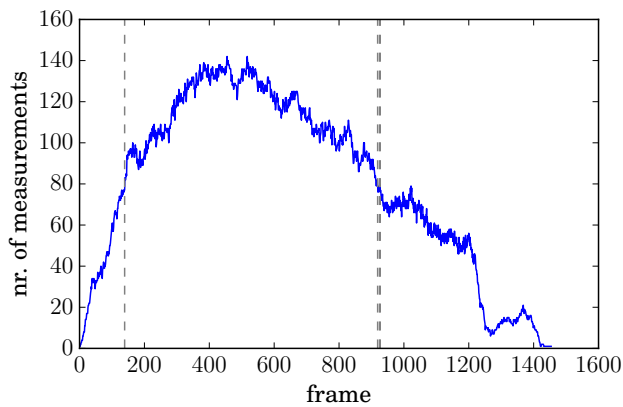
Fig. 5. Required processing time of the LNN and LAPJV algorithm per frame for a simulated data-set with wooden cylinders.



(a) Required processing time when switching between the LNN and LAPJV algorithm.

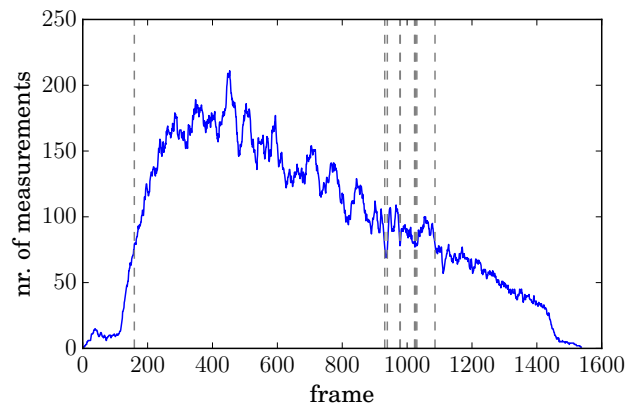


(a) Required processing time when switching between the LNN and LAPJV algorithm.



(b) Measurements obtained per frame. Dashed lines indicate that a strategy switch was triggered.

Fig. 4. Results when switching the association problem solving strategy for the simulated wooden spheres data-set.



(b) Measurements obtained per frame. Dashed lines indicate that a strategy switch was triggered.

Fig. 6. Results when switching the association problem solving strategy for the simulated wooden cylinders data-set.

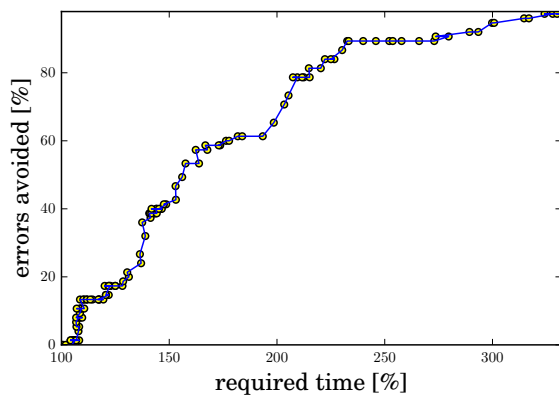


Fig. 7. Performance profile for the data-set with simulated wooden spheres. Results using exclusively LNN serve as the reference.

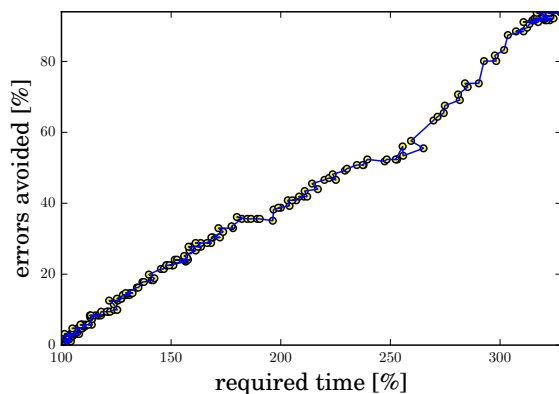


Fig. 8. Performance profile for the data-set with simulated wooden cylinders. Results using exclusively LNN serve as the reference.

intended to extend the pool of association solving strategies, for example by approaches utilizing accelerators, e.g. GPUs.

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