Fraunhofer ITWM

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Berichte des Fraunhofer ITWM, Nr. 218 (2012)

ISSN 1434-9973

Bericht 218 (2012)

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

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Vorwort

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Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

The 2nd Joint International Conference on Multibody System Dynamics
May 29–June 1, 2012, Stuttgart, Germany

Geometrically exact Cosserat rods with Kelvin–Voigt type viscous damping

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ABSTRACT

We present the derivation of a simple viscous damping model of Kelvin–Voigt type for geometrically exact Cosserat rods from three–dimensional continuum theory. Assuming a homogeneous and isotropic material, we obtain explicit formulas for the damping parameters of the model in terms of the well known stiffness parameters of the rod and the retardation time constants defined as the ratios of bulk and shear viscosities to the respective elastic moduli. We briefly discuss the range of validity of our damping model and illustrate its behaviour with a numerical example.

1 INTRODUCTION

Simulation models for computing the transient response of structural members to dynamic excitations [4] should contain a good approach to account for *dissipative effects* in order to be useful in realistic applications. If the structure considered may be treated within the range of linear dynamics with small vibration amplitudes, there is a well established set of standard approaches, e.g. Rayleigh damping, or a more general modal damping ansatz, to add such effects on the level of discretized versions of linear elastic structural models. In the case of geometrically exact structure models for rods and shells, such linear approaches are not applicable. Here the proper approach to model viscous damping requires the inclusion of a *frame–indifferent viscoelastic constitutive model* into the continuum formulation of the structure model that is capable of dealing with large displacements and finite rotations.

Viscous Kelvin–Voigt damping for Cosserat rods

In our recent work [7], following general considerations of Antman [3] about the functional form of viscoelatic constitutive laws for Cosserat rods, we suggested the possibly simplest model of this kind to introduce *viscous material damping* in our quaternionic reformulation of Simo's dynamic continuum model for Cosserat rods [10] by adding viscous contributions to the material stress resultants $\mathbf{F}$ and stress couples $\mathbf{M}$, which we assumed to be proportional to the *rates* of the material strain measures $\mathbf{U}$ and $\mathbf{V}$ of the rod:

$$\mathbf{F} \;=\; \hat{\mathbb{C}}_F \cdot (\mathbf{V} - \mathbf{V}_0) \;+\; \hat{\mathbb{V}}_F \cdot \partial_t \mathbf{V} \;\;,\quad \mathbf{M} \;=\; \hat{\mathbb{C}}_M \cdot (\mathbf{U} - \mathbf{U}_0) \;+\; \hat{\mathbb{V}}_M \cdot \partial_t \mathbf{U}\;. \tag{1}$$

The configuration variables of a Cosserat rod [3] are its *centerline* curve $\boldsymbol{\varphi}(s,t)$ and *"moving frame"* $\hat{\mathbf{R}}(s,t) \,=\, \mathbf{a}^{(k)}(s,t) \otimes \mathbf{e}_k$ of orthonormal director vectors, both smooth functions of the curve parameter s and the time t, with the pair $\{\mathbf{a}^{(1)}, \mathbf{a}^{(2)}\}$ of directors spanning the local (plane and rigid) cross sections with normals $\mathbf{a}^{(3)}$ along the rod. The material strain measures associated to this configuration are given by *(i)* the components $V_k = \mathbf{a}^{(k)} \cdot \partial_s \boldsymbol{\varphi}$ of the tangent vector in the local frame (i.e.: $\mathbf{V} = \hat{\mathbf{R}}^T \cdot \partial_s \boldsymbol{\varphi} = V_k \mathbf{e}_k$), with V_1, V_2 measuring transverse shear deformation and V_3 measuring (extensional) dilatation, and *(ii)* the *material Darboux vector* $\mathbf{U} \,=\, \hat{\mathbf{R}}^T \cdot \mathbf{u} \,=\, U_k \mathbf{e}_k$, obtained from its spatial counterpart $\mathbf{u} \,=\, U_k \mathbf{a}^{(k)}$ governing the Frénet equations $\partial_s \mathbf{a}^{(k)} \;=\; \mathbf{u} \times \mathbf{a}^{(k)}$ of the frame directors, with U_1, U_2 measuring bending curvature w.r.t. the director axes $\{\mathbf{a}^{(1)}, \mathbf{a}^{(2)}\}$, and U_3 measuring torsional twist around the cross section normal. In general, the *reference configuration* of the rod, given by its centerline $\boldsymbol{\varphi}_0(s)$ and frame

$\hat{\mathbf{R}}_0(s) = \mathbf{a}_0^{(k)}(s) \otimes \mathbf{e}_k$, may have non–zero curvature and twist (i.e. $\mathbf{U}_0 \neq \mathbf{0}$). However we may assume zero initial shear ($V_{01} = V_{02} = 0$), such that all cross sections of the reference configuration are orthogonal to the centerline tangent vector, which coincides with the cross section normal (i.e.: $\partial_s \boldsymbol{\varphi}_0 = \mathbf{a}_0^{(3)} \Leftrightarrow V_{03} = 1$) if we choose the arc–length of the reference centerline as curve parameter s.

In the material constitutive equations (1) the elastic properties of the rod are determined by the effective stiffness parameters contained in the symmetric 3×3 matrices $\hat{\mathbb{C}}_F$ and $\hat{\mathbb{C}}_M$. For homogeneous isotropic materials, both matrices are diagonal and given by:

$$\hat{\mathbb{C}}_F \;=\; \text{diag}\,(GA\kappa_1, GA\kappa_2, EA) \quad , \quad \hat{\mathbb{C}}_M \;=\; \text{diag}\,(EI_1, EI_2, GI_3) \;\; ,$$

with stiffness parameters given by the elastic moduli E and G, geometric parameters of the cross section (area A and geometric moments I_k), and dimensionless shear correction factors κ_j of $\mathcal{O}(1)$ that also depend on the cross section geometry[1]. In [7] we assumed a similar structure for the matrices $\hat{\mathbb{V}}_F$ and $\hat{\mathbb{V}}_M$, which determine the viscous response:

$$\hat{\mathbb{V}}_F = \text{diag}\,(\gamma_{S1}, \gamma_{S2}, \gamma_E) \quad , \quad \hat{\mathbb{V}}_M = \text{diag}\,(\gamma_{B1}, \gamma_{B2}, \gamma_T) \;\; . \tag{2}$$

The set of six *effective viscosity parameters* γ_{xx} introduced in (2) represents the *integrated cross-sectional viscous damping behaviour* associated to the basic deformation modes (bending, twisting, shearing and extension) of the rod, in the same way as the well known set of stiffness parameters given above determines the corresponding elastic response. Apart of such considerations, Antman [2] suggested a damping model as given by eqns. (1) with positive parameters (2) from a different viewpoint, namely as a simple possibility to introduce dissipative terms (*artificial viscosity*) into the dynamic balance equations of a Cosserat rod, which are a nonlinear coupled hyperbolic system of PDEs, and thereby achieve a regularization effect in view of the possible formation of shock waves that might appear in the undamped hyperbolic equations.

Damping parameter formulas

However, in [2] as well as in [7] the damping parameters γ_{xx} were left undetermined w.r.t. their specific dependence on material and geometric properties. In the case of homogeneous and isotropic material properties, they certainly cannot be independent, but rather should be mutually related in a similar way as the stiffness parameters of the rod in terms of two material parameters (E,G) and the geometrical quantities (A,I_k) associated to the cross section. In fact, we will show that they are given by

$$\frac{\gamma_{S1/2}}{A\kappa_{1/2}} \;=\; \frac{\gamma_T}{I_3} \;=\; \eta \; , \;\; \frac{\gamma_E}{A} \;=\; \frac{\gamma_{B1/2}}{I_{1/2}} \;=\; \zeta(1-2\nu)^2 + \frac{4}{3}\eta(1+\nu)^2 \;\; =: \; \eta_E \; , \tag{3}$$

where ζ and η are the *bulk and shear viscosities* of a viscoelastic *Kelvin–Voigt solid* [8] with elastic moduli G and $E = 2G(1+\nu)$. While the viscous damping of the deformation modes of pure shear type is solely affected by shear viscosity η, extensional and bending deformations are both associated to normal stresses in the direction orthogonal to the cross section, which are damped by a specific combination of both bulk and shear viscosity that depends on the compressibilty of the material and may be interpreted as *extensional viscosity* parameter η_E. Introducing the *retardation time* constants $\tau_S = \eta/G$ and $\tau_B = \zeta/K$, which relate the viscosities η and ζ to the shear and bulk moduli G and $3K = E/(1-2\nu)$, as well as the time constant $\tau_E = \eta_E/E$, which analogously relates the extensional viscosity to Young's modulus, the formulas (3) may be equivalently rewritten as

$$\frac{\gamma_{S1/2}}{GA\kappa_{1/2}} \;=\; \frac{\gamma_T}{GI_3} \;=\; \tau_S \; , \;\; \frac{\gamma_E}{EA} \;=\; \frac{\gamma_{B1/2}}{EI_{1/2}} \;=\; \frac{1}{3}\left[(1-2\nu)\,\tau_B \;+\; 2(1+\nu)\,\tau_S\right] \;=\; \tau_E \tag{4}$$

in terms of the stiffness parameters of the rod and the retardation time constants. For a given rod consisting of homogeneous and isotropic material, one may first determine the elastic moduli G and E by standard tests (i.e.: twisting, uniaxial extension of 3–point–bending of the rod with small deformation amplitudes), and after that measure the time constants by observing relaxation oscillations or cylic data at sufficiently low frequencies (see [8]). Interesting special cases of (4) are the simplified expressions $\eta_E = \zeta + \frac{4}{3}\eta$, $\tau_E = \frac{1}{3}(\tau_B + 2\tau_S)$ for completely compressible materials[2] (i.e. $\nu = 0$), and $\eta_E = 3\eta$, $\tau_E = \tau_S$ for incompressible materials[3] ($\nu = \frac{1}{2}$). For $\tau_B = \tau_S$ we obtain $\tau_E = \tau_{B/S}$ independent of the value of ν.

[1] If out–of–plane cross section warping is not negligible, GI_3 should be replaced by an improved torsional stiffness parameter.
[2] The formulas given in the extended abstract are valid only for this case.
[3] Note that the latter relation between shear and extensional viscosity is well known for incompressible Newtonian fluids.

Overview of the remaining sections of the paper

In the following section, we will present a derivation of the formulas (3) from 3D continuum theory in analogy to the derivation of the elastic energy (or *stored energy function*)

$$W_{el} \;=\; \int_0^L ds\,\frac{1}{2}\,\Big[(\mathbf{V}-\mathbf{V}_0)^T\cdot\hat{\mathbb{C}}_F\cdot(\mathbf{V}-\mathbf{V}_0)\;+\;(\mathbf{U}-\mathbf{U}_0)^T\cdot\hat{\mathbb{C}}_M\cdot(\mathbf{U}-\mathbf{U}_0)\Big] \tag{5}$$

of a Cosserat rod. The analogous procedure for the viscous part results in the derivation of the *dissipation function* of a Cosserat rod as introduced[4] in [7]

$$W_{visc} \;=\; \int_0^L ds\,\frac{1}{2}\,\Big[\partial_t\mathbf{V}^T\cdot\hat{\mathbb{V}}_F\cdot\partial_t\mathbf{V}\;+\;\partial_t\mathbf{U}^T\cdot\hat{\mathbb{V}}_M\cdot\partial_t\mathbf{U}\Big] \tag{6}$$

from the 3D (volumetric) dissipation function of a Kelvin–Voigt solid [6, 8]. At the end of the section we briefly discuss the validity of the Kelvin–Voigt model. We also comment on the relation of our continuum model to the Kelvin–Voigt type model recently proposed in [1] for usage within an alternative discretized approach to geometrically nonlinear rods using the *absolute nodal coordinate formulation* (ANCF).

In the third section, we will illustrate the behaviour of our Kelvin–Voigt type viscous damping model (1) by simple numerical experiments. We conclude our paper with a short summary.

2 DERIVATION OF THE KELVIN–VOIGT MODEL FOR COSSERAT RODS

In order to set the notational and conceptional framework for the derivation of the viscous part of our damping model, we first give a brief account of the derivation of its elastic part, i.e.: the stored energy function (5) of a Cosserat rod. Within this derivation we will encounter a variety of smallness assumptions w.r.t. the curvatures describing the reference geometry of the rod as well as the local strains occuring in its deformed configurations. In our subsequent derivation of the viscous dissipation function (6) we will use the same assumptions and thereby remain consistent with the derivation of the elastic part.

2.1 Basic Cosserat rod kinematics

Introducing cartesian coordinates (ξ_1,ξ_2) w.r.t. the director basis $\{\mathbf{a}_0^{(1)}(s),\mathbf{a}_0^{(2)}(s)\}$ chosen in the cross section located at the centerline point $\boldsymbol{\varphi}_0(s)$, the spatial positions of material points in the reference configuration of the rod are given by[5]

$$\mathbf{X}(\xi_1,\xi_2,s) \;=\; \boldsymbol{\varphi}_0(s)\;+\;\xi_\alpha\,\mathbf{a}_0^{(\alpha)}(s)\;. \tag{7}$$

The positions of the same material points in the current (deformed) configuration are then given by

$$\mathbf{x}(\xi_1,\xi_2,s,t) \;=\; \boldsymbol{\varphi}(s,t)\;+\;\xi_\alpha\,\mathbf{a}^{(\alpha)}(s,t)\;+\;\mathbf{w}(\xi_1,\xi_2,s,t) \tag{8}$$

in terms of the deformed centerline curve $\boldsymbol{\varphi}(s,t)$, the rotated orthonormal cross section basis vectors $\{\mathbf{a}^{(1)}(s,t),\mathbf{a}^{(2)}(s,t)\}$, the same pair of cartesian cross section coordinates (ξ_1,ξ_2), and an additional displacement vector field $\mathbf{w}(\xi_1,\xi_2,s,t)$, which by definition describes the (in–plane and out–of–plane) *warping* deformations of the cross sections along the deformed rod.

The kinematic assumption that the cross sections of a rod remain plane and rigid in a configuration is equivalent to the assumption that the displacement field $\mathbf{w}$ vanishes identically. Although we will initially adhere to this very common assumption for rod models, we will later admit some specific form of in–plane deformation of cross sections — namely: a *uniform lateral contraction* — to correct a deficiency caused by the overly rigid kinematical ansatz (8) with $\mathbf{w}\equiv\mathbf{0}$.

[4] Note that in [7] (compare eqns. (9) and (10) in sec. 2.2) we absorbed the prefactor $1/2$ into the definition (2) of the damping parameters, which leads to an additional factor of 2 multiplying $\widehat{\mathbb{V}}_F$ and $\widehat{\mathbb{V}}_M$ in the constitutive equations (1) of the rod model.

[5] Within this paper we make use of Einstein's summation convention — as the reader may have observed already — w.r.t. all indices occuring twice within *product* terms, with greek indices $\alpha,\beta,\ldots$ running from 1 to 2 and latin ones $i,j,k,\ldots$ from 1 to 3.

For simplicity we assume the rod to be *prismatic*, such that all cross sections along the rod are identical, and the domain of the cartesian coordinates (ξ_1, ξ_2) coincides with one fixed domain $\mathcal{A} \subset \mathbb{R}^2$. As usual we choose the geometrical center of the domain $\mathcal{A}$ to coincide with the origin of $\mathbb{R}^2$ such that $\langle \xi_\alpha \rangle_\mathcal{A} = 0$ holds, where we introduced the shorthand notation $\langle f \rangle_\mathcal{A} := \int_\mathcal{A} f(\xi_1, \xi_2)\, d\xi_1 d\xi_2$ for the *cross section integral* of functions. In addition we choose the orientation of the orthonormal director pairs $\{\mathbf{a}_0^{(1)}(s), \mathbf{a}_0^{(2)}(s)\}$ as well as $\{\mathbf{a}^{(1)}(s,t), \mathbf{a}^{(2)}(s,t)\}$ to coincide with the principle geometrical axes of $\mathcal{A}$, such that $\langle \xi_1 \xi_2 \rangle_\mathcal{A} = 0$ holds. The quantities that characterize the geometric properties of the cross section in the Cosserat rod model are the *cross section area* $A = \langle 1 \rangle_\mathcal{A}$, the two *area moments* $I_1 = \langle \xi_2^2 \rangle_\mathcal{A}$, $I_2 = \langle \xi_1^2 \rangle_\mathcal{A}$ and the *polar area moment* $I_3 = \langle \xi_1^2 + \xi_2^2 \rangle_\mathcal{A} = I_1 + I_2$.

2.2 Three–dimensional strain measures of a Cosserat rod

In the next step we compute the *deformation gradient* $\hat{\mathbf{F}} = \mathbf{g}_k \otimes \mathbf{G}^k$, the *right Cauchy–Green tensor* $\hat{\mathbf{C}} = \hat{\mathbf{F}}^T \cdot \hat{\mathbf{F}}$ and the *Green–Lagrange strain tensor* $\hat{\mathbf{E}} = \frac{1}{2}(\hat{\mathbf{C}} - \hat{\mathbf{I}})$ from the basis vectors $\mathbf{G}_k = \partial_k \mathbf{X}$ and $\mathbf{g}_k = \partial_k \mathbf{x}$ associated to the curvilinear coordinates of the rod configurations given by (7) and (8), with $\partial_k = \frac{\partial}{\partial \xi_k}$ for $k = 1, 2$ and $\partial_3 = \partial_s$ for $\xi_3 = s$. The dual basis vectors $\mathbf{G}^j$ and $\mathbf{g}^j$ are defined by the relations $\mathbf{G}_i \cdot \mathbf{G}^j = \delta_{ij}$ and $\mathbf{g}_i \cdot \mathbf{g}^j = \delta_{ij}$ respectively. Proceeding in this way we obtain the basis vectors of the reference configuration (7) as $\mathbf{G}_\alpha = \mathbf{a}_0^{(\alpha)}(s)$ and $\mathbf{G}_3 = \mathbf{a}_0^{(3)}(s) + \xi_\alpha U_{0\alpha}(s) \mathbf{a}_0^{(\alpha)}(s)$. Their duals may be computed from the general formula $\mathbf{G}^i = \mathbf{G}^j \times \mathbf{G}^k / J_0$ with $J_0 := (\mathbf{G}_1 \times \mathbf{G}_2) \cdot \mathbf{G}_3$, where (ijk) is a cyclic permutation of the indices (123), with the result:

$$\mathbf{G}^1 \;=\; \mathbf{a}_0^{(1)}(s) \,+\, \xi_2 \frac{U_{03}(s)}{J_0(s)} \mathbf{a}_0^{(3)}(s) \;,\; \mathbf{G}^2 \;=\; \mathbf{a}_0^{(2)}(s) \,-\, \xi_1 \frac{U_{03}(s)}{J_0(s)} \mathbf{a}_0^{(3)}(s) \;,\; \mathbf{G}^3 \;=\; \frac{1}{J_0(s)} \mathbf{a}_0^{(3)}(s) \;.$$

The inital curvatures $U_{0\alpha}(s)$ contained in the determinant $J_0(s) = 1 + \xi_2 U_{01}(s) - \xi_1 U_{02}(s)$ and the initial twist $U_{03}(s)$ of the reference configuration (7) influence the deviation of the dual vectors $\mathbf{G}^k$ from the frame directors $\mathbf{a}_0^{(k)}(s)$ within the cross section. Both vectors coincide if the reference configuration of the rod is straight and untwisted (i.e.: $\mathbf{U}_0 = \mathbf{0}$). We have approximate coincidence $\mathbf{G}^k \approx \mathbf{a}_0^{(k)}(s)$ if curvature and twist of the reference configuration are sufficiently weak, in the sense that for the curvature radii given by $R_k = 1/|U_{0k}|$ the estimates $|\xi_\alpha|/R_3 \ll 1$ and $|\xi_\alpha|/R_\beta \ll 1 \Rightarrow J_0 \approx 1$ hold throughout each cross section along the rod, such that all initial curvature radii R_α are large compared to the cross section diameter. The geometric approximation $J_0(s) \approx 1$ will occur repeatedly and therefore play an important role in the derivation of the elastic energy and dissipation function of a Cosserat rod. To compute the deformation gradient we also need the basis vectors $\mathbf{g}_\alpha = \mathbf{a}^{(\alpha)}(s,t)$ and $\mathbf{g}_3 = \mathbf{a}^{(3)}(s,t) + \xi_\alpha U_\alpha(s,t) \mathbf{a}^{(\alpha)}(s,t)$ of the deformed configuration (8) with vanishing warping vector field ($\mathbf{w} = 0$). For the dual vectors $\mathbf{g}^k$ one obtaines analogous expressions as those for the dual vectors $\mathbf{G}^k$ given above, which we omit here.

For the special kinematical relations of a Cosserat rod, the deformation gradient $\hat{\mathbf{F}} = \mathbf{g}_k \otimes \mathbf{G}^k$ may be expressed in terms of a *pseudo–polar decomposition* [5] by a factorization of the *relative rotation* $\hat{\mathbf{R}}_{rel}(s,t) := \hat{\mathbf{R}}(s,t) \cdot \hat{\mathbf{R}}_0^T(s) = \mathbf{a}^{(k)}(s,t) \otimes \mathbf{a}_0^{(k)}(s)$ connecting the moving frames of the reference and deformed configurations of the rod. The resulting formula

$$\hat{\mathbf{F}}(\xi_1, \xi_2, s, t) \;=\; \hat{\mathbf{R}}_{rel}(s,t) \left[\hat{\mathbf{I}} \,+\, \frac{1}{J_0(s)} \mathbf{H}(\xi_1, \xi_2, s, t) \otimes \mathbf{a}_0^{(3)}(s) \right] \tag{9}$$

depends on the *absolute values* of the curvatures of the reference configuration (7) through $J_0(s)$, and on the *change of the strain measures* of the Cosserat rod given by the difference vectors $\mathbf{U} - \mathbf{U}_0$ and $\mathbf{V} - \mathbf{V}_0 = (V_1, V_2, V_3 - 1)^T$ in terms of the *material strain vector* $\mathbf{H} = H_k \mathbf{a}_0^{(k)}$ with components

$$\begin{aligned} H_1 &= V_1(s,t) - \xi_2 \left[U_3(s,t) - U_{03}(s)\right] \;,\; H_2 \;=\; V_2(s,t) + \xi_1 \left[U_3(s,t) - U_{03}(s)\right] \\ H_3 &= \left[V_3(s,t) - 1\right] + \xi_2 \left[U_1(s,t) - U_{01}(s)\right] - \xi_1 \left[U_2(s,t) - U_{02}(s)\right] \;. \end{aligned} \tag{10}$$

Our derivation generalizes the treatment given in [5] for the simpler case of a straight and untwisted reference configuration of the rod (i.e. $\mathbf{U}_0 = 0$). Computing the right Cauchy–Green tensor field $\hat{\mathbf{C}} = \hat{\mathbf{F}}^T \cdot \hat{\mathbf{F}}$ with the deformation gradient given by (9) results in the following exact expression[6] for the Green–Lagrange

[6]To simplify the notation, we omitted all function arguments here.

strain tensor:

$$\hat{\mathbf{E}} \;=\; \frac{1}{2J_0}\left[\mathbf{H}\otimes\mathbf{a}_0^{(3)} + \mathbf{a}_0^{(3)}\otimes\mathbf{H}\right] \;+\; \frac{\mathbf{H}^2}{2J_0^2}\,\mathbf{a}_0^{(3)}\otimes\mathbf{a}_0^{(3)}\ . \tag{11}$$

The approximate expression

$$\hat{\mathbf{E}} \;\approx\; \frac{1}{2}\left[\mathbf{H}\otimes\mathbf{a}_0^{(3)} + \mathbf{a}_0^{(3)}\otimes\mathbf{H}\right] \tag{12}$$

may be obtained from (11) by the assumption $\|\mathbf{H}\| \ll 1$ of a small material strain vector and the assumption $J_0 \approx 1$. Later we will make use of this approximate strain tensor, which is *linear* in the vector field $\mathbf{H}$ and therefore also in the change of the strain measures of the rod, to obtain the stored energy function (5), which becomes a quadratic form in the change of the strain measures. Likewise we will use (12) to obtain an approximation of the strain rate $\partial_t\mathbf{E}$ in terms of the rate $\partial_t\mathbf{H}$ of the strain vector.

For deformed configurations of a slender rod one observes large displacements and rotations, but local strains remain small. To estimate the size of the strain tensor it is useful to compute its components $E_{ij} = \mathbf{a}_0^{(i)}\cdot(\hat{\mathbf{E}}\cdot\mathbf{a}_0^{(j)})$ w.r.t. the tensor basis $\mathbf{a}_0^{(i)}\otimes\mathbf{a}_0^{(j)}$ obtained from the directors of the reference frame $\hat{\mathbf{R}}_0(s)$. From (11) and (12) we obtain the following exact and approximate expressions:

$$E_{\alpha\beta} = E_{\beta\alpha} \equiv 0\ \ ,\quad E_{\alpha 3} = E_{3\alpha} = \frac{H_\alpha}{2J_0} \approx \frac{H_\alpha}{2}\ \ ,\quad E_{33} = \frac{H_3}{J_0} + \frac{\mathbf{H}^2}{2J_0^2} \approx H_3\ \ . \tag{13}$$

Introducing the the quantity $|\xi|_{max} = \max_{(\xi_1,\xi_2)\in\mathcal{A}}(|\xi_1|,|\xi_2|)$ to estimate the maximal linear extension of the cross section $\mathcal{A}$, one may estimate the deviation of the determinant $J_0(s)$ from unity by $|J_0(s)-1| \leq |\xi|_{max}(1/R_1 + 1/R_2)$ as a coarse check of the validity of the approximation $J_0 \approx 1$. Otherwise the smallness of the components of $\hat{\mathbf{E}}$ is implied by the smallness of the components H_k of the strain vector. According to (10) these components in turn become small if the change of the strain measures of the Cosserat rod is small, i.e. if the estimates $|V_\alpha| \ll 1$, $|V_3 - 1| \ll 1$, $|U_k - U_{0k}| \ll 1/|\xi|_{max}$ hold. For slender rods with moderately curved undeformed geometry these estimates are obviously easily satisfiable, except for extreme deformations of the rod that produce large curvatures or twists of the order of the inverse cross section diameter. In this case, the assumption of small strains obviously would be invalid.

2.3 Elastic constitutive behaviour of rods at small strains

If we assume the rod material to behave hyperelastically with a stored energy density function $\Psi_{el}(\hat{\mathbf{E}})$, a simple Taylor expansion argument[7] shows that the behaviour of the energy density within the range of small strains may be well approximated by the quadratic function $\Psi_{el}(\hat{\mathbf{E}}) \approx \frac{1}{2}\hat{\mathbf{E}} : \widehat{\mathbb{H}} : \hat{\mathbf{E}}$, where $\widehat{\mathbb{H}} = \partial^2_{\hat{\mathbf{E}}}\Psi_{el}(\hat{\mathbf{0}})$ is the fourth order *Hookean material tensor* known from linear elasticity. This quadratic approximation yields a well defined frame–indifferent elastic energy density that is suitable for structure deformations at small local strains, but arbitrary large displacements and rotations, and therefore serves as a proper basis for the derivation of the stored energy function of a Cosserat rod.

The corresponding approximation of the stress–strain relation yields the 2^{nd} *Piola–Kirchhoff stress* tensor $\hat{\mathbf{S}} = \partial_{\hat{\mathbf{E}}}\Psi_{el}(\hat{\mathbf{E}}) \approx \widehat{\mathbb{H}} : \hat{\mathbf{E}}$ for small strains. The 1^{st} *Piola–Kirchhoff stress* tensor $\hat{\mathbf{P}}$, which is used to define the stress resultants and stress couples of the Cosserat rod model [10], is obtained by the transformation $\hat{\mathbf{P}} = \hat{\mathbf{F}}\cdot\hat{\mathbf{S}}$ using the deformation gradient, and the Cauchy stress tensor by yet another transformation $\hat{\sigma} = J^{-1}\hat{\mathbf{P}}\cdot\hat{\mathbf{F}}^T$, which contains also $J = \det(\hat{\mathbf{F}})$. If we approximate the strain tensor $\hat{\mathbf{E}}$ by (12) and consistently discard all terms that are of second order in $\|\mathbf{H}\|$ in accordance with our assumption of small strains, we have to use the approximation $\hat{\mathbf{F}} \approx \hat{\mathbf{R}}_{rel}(s)$ (which implies $J \approx 1$) for the deformation gradient in all stress tensor transformations. This means that all pull back or push forward transformations are carried out approximately as a simple (relative) rotation connecting corresponding frames $\hat{\mathbf{R}}_0(s)$ and $\hat{\mathbf{R}}(s,t)$ of the undeformed and deformed configurations of a Cosserat rod. Alltogether we obtain the approximate expressions $\hat{\mathbf{S}} \approx \widehat{\mathbb{H}} : \hat{\mathbf{E}} \ \rightarrow\ \hat{\mathbf{P}} \approx \hat{\mathbf{R}}_{rel}\cdot\hat{\mathbf{S}}\,,\ \hat{\sigma} \approx \hat{\mathbf{R}}_{rel}\cdot\hat{\mathbf{S}}\cdot\hat{\mathbf{R}}_{rel}^T$ for the various stress tensors, which are valid for the specific type of small strain assumptions for Cosserat rods as previously discussed.

In the case of a homogeneous and isotropic material the Hookean tensor acquires the special form of an isotropic fourth order tensor $\widehat{\mathbb{H}}_{SVK} = \lambda\,\hat{\mathbf{I}}\otimes\hat{\mathbf{I}} + 2\mu\,\widehat{\mathbb{I}}$ depending on two constant elastic moduli (the *Lamé*

[7] Additional assumptions are the vanishing of the elastic energy density at zero strain ($\Psi_{el}(\hat{\mathbf{0}}) = 0$), as well as the absence of initial stresses in the undeformed configuration (i.e.: $\hat{\mathbf{S}}_0 = \partial_{\hat{\mathbf{E}}}\Psi_{el}(\hat{\mathbf{0}}) = \hat{\mathbf{0}}$).

parameters λ and μ). Here $\hat{\mathbf{I}}$ and $\widehat{\mathbb{I}}$ are the second and fourth order identity tensors, which act on arbitrary (symmetric) second order tensors $\hat{\mathbf{Q}}$ by double contraction as $\widehat{\mathbb{I}} : \hat{\mathbf{Q}} = \hat{\mathbf{Q}}$ and $\hat{\mathbf{I}} : \hat{\mathbf{Q}} = \mathrm{Tr}(\hat{\mathbf{Q}})$, such that one obtains $\hat{\mathbf{Q}} : (\hat{\mathbf{I}} \otimes \hat{\mathbf{I}}) : \hat{\mathbf{Q}} = \mathrm{Tr}(\hat{\mathbf{Q}})^2$ and $\hat{\mathbf{Q}} : \widehat{\mathbb{I}} : \hat{\mathbf{Q}} = \hat{\mathbf{Q}} : \hat{\mathbf{Q}} = \mathrm{Tr}(\hat{\mathbf{Q}}^2) = \|\hat{\mathbf{Q}}\|_F^2$, where $\| \ldots \|_F$ is the Frobenius norm. The corresponding quadratic energy function is the *Saint–Venant Kirchhoff* potential

$$\Psi_{SVK}(\hat{\mathbf{E}}) \;=\; \frac{1}{2}\,\hat{\mathbf{E}} : \widehat{\mathbb{H}}_{SVK} : \hat{\mathbf{E}} \;=\; \frac{\lambda}{2}\,\mathrm{Tr}(\hat{\mathbf{E}})^2 \;+\; \mu\,\|\hat{\mathbf{E}}\|_F^2 \;=\; \frac{K}{2}\,\mathrm{Tr}(\hat{\mathbf{E}})^2 \;+\; \mu\,\|\widehat{\mathbb{P}} : \hat{\mathbf{E}}\|_F^2 \,, \tag{14}$$

where $\widehat{\mathbb{P}} = \widehat{\mathbb{I}} - \frac{1}{3}\hat{\mathbf{I}} \otimes \hat{\mathbf{I}}$ is the orthogonal projection operator on the subspace of traceless second order tensors, such that $\widehat{\mathbb{P}} : \hat{\mathbf{E}} = \hat{\mathbf{E}} - \frac{1}{3}\mathrm{Tr}(\hat{\mathbf{E}})\hat{\mathbf{I}}$ is the traceless (deviatoric) part of the strain tensor, and $K = \lambda + \frac{2}{3}\mu$ is the *bulk modulus*. The well known stress–strain relation obtained from (14) is given by

$$\hat{\mathbf{S}}_{SVK} \;=\; \partial_{\hat{\mathbf{E}}}\,\Psi_{SVK}(\hat{\mathbf{E}}) \;=\; \lambda\,\mathrm{Tr}(\hat{\mathbf{E}})\,\hat{\mathbf{I}} \,+\, 2\mu\,\hat{\mathbf{E}} \;=\; K\,\mathrm{Tr}(\hat{\mathbf{E}})\,\hat{\mathbf{I}} \,+\, 2\mu\,\widehat{\mathbb{P}} : \hat{\mathbf{E}} \,. \tag{15}$$

2.4 Modified strain tensor including uniform lateral contraction effects

Inserting the approximate expressions[8] (12) and (13) of the strain tensor and its components into (15) yields the small strain approximation $\hat{\mathbf{S}}_{SVK} \approx \lambda H_3 \hat{\mathbf{I}} + \mu[\mathbf{H} \otimes \mathbf{a}_0^{(3)} + \mathbf{a}_0^{(3)} \otimes \mathbf{H}]$ of the stress tensor $\hat{\mathbf{S}}_{SVK}$ for Cosserat rods. The computation of the stress components w.r.t. the basis of $\hat{\mathbf{R}}_0(s)$ directors yields normal stress components $S_{\alpha\alpha} \approx \lambda H_3$ and $S_{33} \approx (\lambda + 2\mu) H_3$, and the shear stress components are given by $S_{12} = S_{21} = 0$ and $S_{\alpha 3} = S_{3\alpha} \approx \mu H_\alpha$ respectively.

As the elastic moduli $\lambda = 2\mu\nu/(1-2\nu)$ and $\lambda + 2\mu = 2\mu(1-\nu)/(1-2\nu)$ appearing in the expressions for the normal stress components, expressed in terms of the shear modulus $\mu = G$ and Poisson's ratio given by $2\nu = \lambda/(\lambda + \mu)$, diverge in the incompressible limit $\nu \to \frac{1}{2}$ (just as the bulk modulus $K = \frac{2}{3}\frac{1+\nu}{1-2\nu}G$ does), the normal stresses would become infinitely large whenever the normal strain $E_{33} \approx H_3$ becomes nonzero. This unphysical behaviour clearly is a direct consequence of the kinematical assumption of plain and *rigid* cross section, which e.g. prevents any lateral contraction of the cross section in the case of a longitudinal extension. Therefore the assumption of a perfectly rigid cross section, as well as the expressions (11) and (12) derived under this assumption, are compatible only with perfectly compressible materials ($\nu = 0$).

The standard procedure to fix this deficiency [11] is based on the plausible requirement that *all* in–plane stress components $S_{\alpha\beta}$ (including the normal stresses $S_{\alpha\alpha}$), which for rods in practice are very small compared to the out of plain normal and shear stresses $S_{\alpha 3}$ and S_{33}, should *vanish* completely. This may be achieved by imposing a *uniform lateral contraction* with in–plane normal strain components $E_{\alpha\alpha} = -\nu E_{33}$ upon the cross section. Allthough this procedure seems to be rather ad hoc, it may be justified by an asymptotic analysis of the local strain field for rods, e.g. in the way as presented by A.E.H. Love in the paragraph §256 on the *"Nature of the strain in a bent and twisted rod"* in Ch. XVIII of his book [9]. Following Love's analysis, we obtain the in–plane normal strains to leading order as $E_{\alpha\alpha} = \partial_\alpha w_\alpha = -\nu E_{33}$ with the additional requirement that $E_{12} = E_{21} = \partial_1 w_2 + \partial_2 w_1 = 0$, which determines the in–plane components w_α of the the warping field $\mathbf{w}$ corresponding to the lateral contraction in terms of E_{33}.

To obtain the modified value of $E_{\alpha\alpha} = -\nu E_{33}$ one has to add an additional term $-\nu E_{33}\,\mathbf{a}_0^{(\alpha)} \otimes \mathbf{a}_0^{(\alpha)}$ to the exact expression (11) of the strain tensor. Using the identity $\hat{\mathbf{I}} = \mathbf{a}_0^{(k)} \otimes \mathbf{a}_0^{(k)}$, we obtain the following modified expression for the strain tensor:

$$\hat{\mathbf{E}}' \;=\; \hat{\mathbf{E}} \;-\; \nu\,E_{33}\,\left[\hat{\mathbf{I}} - \mathbf{a}_0^{(3)} \otimes \mathbf{a}_0^{(3)}\right] \,. \tag{16}$$

The small strain approximation is obtained by using $E_{33} \approx H_3$ according to (12). Inserting this modified strain tensor (16) into the stress–strain equation of the Saint–Venant–Kirchhoff material with $\mathrm{Tr}(\hat{\mathbf{E}}') = (1-2\nu)E_{33} \approx (1-2\nu)H_3$, and using the relation $\lambda(1-2\nu) = \frac{\nu}{1+\nu}E$ relating the Lamé parameter λ to Young's modulus E, we obtain the following modified expression for the stress of a Cosserat rod:

$$\hat{\mathbf{S}}'_{SVK} \;\approx\; \frac{E\nu}{1+\nu}\,H_3\,\mathbf{a}_0^{(3)} \otimes \mathbf{a}_0^{(3)} \;+\; G\,\left[\mathbf{H} \otimes \mathbf{a}_0^{(3)} + \mathbf{a}_0^{(3)} \otimes \mathbf{H}\right] \,. \tag{17}$$

[8]All of the following considerations are likewise valid if the *exact* expression (11) for the strain tensor and its components (13) were used. However, the seemingly more complete formulas obtained in this way would have only formal meaning and seemingly improved accuracy, as we made use of a constitutive model valid for *small* strains only. Therefore we use the *approximate* expressions not only for simplicity, but also to remain consistent with the small strain assumption.

By construction, we now obtain vanishing in–plane stress components $S'_{12} = S'_{21} = S'_{\alpha\alpha} \equiv 0$, while the transverse shear stresses remain unaffected by the modification (i.e.: $S'_{\alpha 3} = S'_{3\alpha} \approx GH_\alpha$ with $G = \mu$). As $2G = E/(1+\nu)$, we obtain the modified expression $S'_{33} \approx E\,H_3$ for the normal stress component orthogonal to the cross section, which corresponds to the familiar expression from elementary linear beam theory, with the correct modulus E replacing the deficient term $\lambda + 2\mu$.

2.5 Stored energy function of a Cosserat rod

We briefly demonstrate that the modified expressions (16) and (17) immediately lead to the known stored energy function (5) mentioned in the introduction. The stored energy function is obtained by integrating the density (14) over the cross section and along the centerline, i.e.: $W_{el} = \int_0^L ds \left\langle \Psi_{SVK}(\hat{\mathbf{E}}') \right\rangle_{\mathcal{A}}$, using $\hat{\mathbf{E}}'$ from (16). As

$$\Psi_{SVK}(\hat{\mathbf{E}}') = \frac{1}{2}\,\hat{\mathbf{S}}'_{SVK} : \hat{\mathbf{E}}' \approx \frac{1}{2}\left[EH_3^2 + G(H_1^2 + H_2^2)\right] ,$$

the cross section integral of the energy density can be computed in terms of the integrals

$$\left\langle H_1^2 + H_2^2 \right\rangle_{\mathcal{A}} = A(V_1^2 + V_2^2) + I_3(U_3 - U_{03}) , \quad \left\langle H_3^2 \right\rangle_{\mathcal{A}} = A(V_3 - 1)^2 + I_\alpha(U_\alpha - U_{0\alpha}) .$$

This yields the desired result

$$2\left\langle \Psi_{SVK}(\hat{\mathbf{E}}') \right\rangle_{\mathcal{A}} \approx EA(V_3 - 1)^2 + GA(V_1^2 + V_2^2) + EI_\alpha(U_\alpha - U_{0\alpha}) + GI_3(U_3 - U_{03}) .$$

Introducing the shear correction factors ($GA \to GA\kappa_\alpha$) finally yields the stored energy function (5).

2.6 Kelvin–Voigt type dissipation function of a Cosserat rod

Now we have collected all material and approximate results that enable us to derive the dissipation function (6) of a Cosserat rod from a three-dimensional Kelvin–Voigt model in analogy to the derivation of the stored energy function (5) in a consistent way.

In [6] (see Ch. V §33) the *dissipation function* $\int_V dV\, \frac{1}{2}\eta_{ijkl}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{kl}$ was suggested to model dissipative effects within a solid body near thermodynamic equilibrium, with constant fourth order tensor components η_{ijkl} that are the viscous analogon of the Hookean elasticity tensor. Transfering this ansatz to the formalism used in our paper, the dissipation function of [6] becomes that of a *Kelvin–Voigt solid* [8]

$$W_{KV} = \int_0^L ds \left\langle \Psi_{KV}(\partial_t\hat{\mathbf{E}}) \right\rangle_{\mathcal{A}} , \quad \Psi_{KV}(\partial_t\hat{\mathbf{E}}) = \frac{1}{2}\,\partial_t\hat{\mathbf{E}} : \widehat{\mathbb{V}} : \partial_t\hat{\mathbf{E}} , \tag{18}$$

which is a quadratic form in the material strain rate $\partial_t\hat{\mathbf{E}}$ defined as the time derivative of the Green–Lagrange strain tensor. The constant fourth order *viscosity tensor* $\widehat{\mathbb{V}}$ may be assumed to have the same symmetries as the Hookean tensor $\widehat{\mathbb{H}}$, with its components depending on *viscosity parameters* in the same way as the components of $\widehat{\mathbb{H}}$ depend on elastic moduli. The stress–strain relation of the Kelvin–Voigt model is given by $\hat{\mathbf{S}} = \widehat{\mathbb{H}} : \hat{\mathbf{E}} + \widehat{\mathbb{V}} : \partial_t\hat{\mathbf{E}}$, with the viscous stress given by the term $\widehat{\mathbb{V}} : \partial_t\hat{\mathbf{E}} = \partial_{\partial_t\hat{\mathbf{E}}}\Psi_{KV}(\partial_t\hat{\mathbf{E}})$.

The dissipation function for a Cosserat rod results by inserting the rate $\partial_t\hat{\mathbf{E}}'$ of the modified strain tensor (16) into the dissipation density function Ψ_{KV} of the Kelvin–Voigt model. We will compute this dissipation function explicitely in closed form for the special case of a *homogeneous and isotropic* material. In this case, the viscosity tensor assumes the special form $\widehat{\mathbb{V}}_{IKV} = \zeta\,\hat{\mathbf{I}} \otimes \hat{\mathbf{I}} + 2\eta\,\widehat{\mathbb{P}} = (\zeta - \frac{2}{3}\eta)\,\hat{\mathbf{I}} \otimes \hat{\mathbf{I}} + 2\eta\,\widehat{\mathbb{I}}$, depending on two constant parameters: *bulk viscosity* ζ and *shear viscosity* η.

To compute the strain rate $\partial_t\hat{\mathbf{E}}'$ we use the expression (16) for the modified Green–Lagrange strain tensor of a Cosserat rod including the small strain approximation (12), with the result

$$\partial_t\hat{\mathbf{E}}' \approx \frac{1}{2}\left[\partial_t\mathbf{H} \otimes \mathbf{a}_0^{(3)} + \mathbf{a}_0^{(3)} \otimes \partial_t\mathbf{H}\right] - \nu\,\partial_t H_3\left[\hat{\mathbf{I}} - \mathbf{a}_0^{(3)} \otimes \mathbf{a}_0^{(3)}\right] \tag{19}$$

depending on the time derivative $\partial_t\mathbf{H} = \partial_t H_k\,\mathbf{a}_0^{(k)}$ of the material strain vector with components

$$\begin{aligned} \partial_t H_1 &= \partial_t V_1(s,t) - \xi_2\,\partial_t U_3(s,t) , \quad \partial_t H_2 = \partial_t V_2(s,t) + \xi_1\,\partial_t U_3(s,t) \\ \partial_t H_3 &= \partial_t V_3(s,t) + \xi_2\,\partial_t U_1(s,t) - \xi_1\,\partial_t U_2(s,t) . \end{aligned} \tag{20}$$

Inserting (19) and (20) into the dissipation density function $\Psi_{IKV}(\partial_t\hat{\mathbf{E}}) = \frac{1}{2}\,\partial_t\hat{\mathbf{E}} : \widehat{\mathbb{V}}_{IKV} : \partial_t\hat{\mathbf{E}}$ of the isotropic Kelvin–Voigt model, analogous computational steps as those done for the derivation of the stored energy $\Psi_{SVK}(\hat{\mathbf{E}}')$ in the previous subsection yield the expression

$$2\Psi_{IKV}(\partial_t\hat{\mathbf{E}}) \;\approx\; \left[\zeta(1-2\nu)^2 + \frac{4}{3}\eta(1+\nu)^2\right](\partial_t H_3)^2 \;+\; \eta\,\left[(\partial_t H_1)^2 + (\partial_t H_2)^2\right]\;,$$

with the *extensional viscosity* parameter η_E defined in (3) appearing as the prefactor of $(\partial_t H_3)^2$. The computation of the cross section integrals of the squared time derivatives $(\partial_t H_k)^2$ yields

$$\left\langle(\partial_t H_3)^2\right\rangle_{\mathcal{A}} \;=\; A(\partial_t V_3)^2 + I_\alpha(\partial_t U_\alpha)^2\,,\;\; \left\langle(\partial_t H_1)^2 + (\partial_t H_2)^2\right\rangle_{\mathcal{A}} \;=\; A\left[(\partial_t V_1)^2 + (\partial_t V_2)^2\right] + I_3(\partial_t U_3)^2\,,$$

from which we obtain the desired cross section integral of the dissipation density function:

$$2\left\langle\Psi_{IKV}(\partial_t\hat{\mathbf{E}})\right\rangle_{\mathcal{A}} \;\approx\; \eta_E A\,(\partial_t V_3)^2 \,+\, \eta_E I_\alpha\,(\partial_t U_\alpha)^2 \,+\, \eta A\,\left[(\partial_t V_1)^2 + (\partial_t V_2)^2\right] \,+\, \eta I_3\,(\partial_t U_3)^2\;. \quad (21)$$

The dissipation function (6) of the Cosserat rod with diagonal damping coefficient matrices (2) and damping parameters (3) is finally obtained as $W_{visc} = \int_0^L ds\,\left\langle\Psi_{IKV}(\partial_t\hat{\mathbf{E}}')\right\rangle_{\mathcal{A}}$ after introducing the shear correction factors (i.e.: $\eta A \to \eta A\kappa_\alpha$). This completes our derivation.

Kelvin–Voigt damping model for geometrically nonlinear ANCF beams
In the recent article [1], a damping model for geometrically nonlinear beams given in the ANCF (absolute nodal coordinates) formulation has been proposed. The authors obtained their model by inserting the 3D isotropic Kelvin–Voigt model as described above into their ANCF element ansatz. They used the Lamé parameters λ and μ as elastic moduli, and introduced corresponding viscosity parameters λ_v and μ_v, which they related to the elastic moduli by *dissipation factors* γ_{v1} and γ_{v2}. From the context it seems clear that in our notation $\gamma_{v2} = \tau_S$, such that $\mu_v = G\tau_S = \eta$. Likewise we may identify $\gamma_{v1} = \tau_B$, such that $\lambda_v = K\tau_B - \frac{2}{3}G\tau_S = \zeta - \frac{2}{3}\eta$, and the viscosities are related by the same relation as the elastic moduli (i.e.: $\lambda = K - \frac{2}{3}G$). As in our understanding the ANCF ansatz handles lateral contraction effects, both models should behave similar, with similar simulation results. However, a detailed comparison remains to be done.

Validity of the Kelvin–Voigt model
As remarked already in [6], the modelling of viscous dissipation for solids by a dissipation function of Kelvin–Voigt type is valid only for relatively slow processes near thermodynamic equilibrium, which means that the temperature within the solid should be approximately constant, and the macroscopic velocities of the material particles of the solid should be sufficiently slow w.r.t. the time scale of all internal relaxation processes. To illustrate this statement, we briefly discuss the one–dimensional example of a linear viscoelastic stress–strain relation $\sigma(t) = \int_0^\infty d\tau\, G(\tau)\dot{\varepsilon}(t-\tau)$ governed by the relaxation function $G(\tau) = G_\infty + \sum_{j=1}^N G_j \exp(-\tau/\tau_j)$ (i.e.: a *Prony series*) of a *generalized Maxwell model*. By Fourier transformation we obtain the relation $\hat{\sigma}(\omega) = \hat{G}(\omega)\hat{\varepsilon}(\omega)$ in the frequency domain, where the real and imaginary parts of the complex modulus function $\hat{G}(\omega) = G_\infty + \sum_{j=1}^N G_j \frac{i\tau_j\omega}{1+i\tau_j\omega}$ model the frequency dependent stiffness and damping properties of the material. Using a 1D Kelvin–Voigt model $\sigma_{KV}(t) = G\varepsilon(t) + \eta\dot{\varepsilon}(t)$ we obtain the simple expression $\hat{\sigma}_{KV}(\omega) = [G + i\eta\omega]\hat{\varepsilon}(\omega)$, which approximates the generalized Maxwell model at sufficiently low frequencies with $G = G_\infty$ and $\eta = \sum_{j=1}^N G_j\tau_j$. The deviation between the generalized Maxwell model and its Kelvin–Voigt approximation may be estimated as $|\sigma(t) - \sigma_{KV}(t)| \leq \frac{1}{2\pi}\int_{-\infty}^\infty d\omega\,|\hat{\varepsilon}(\omega)| \sum_{j=1}^N G_j \frac{(\tau_j\omega)^2}{\sqrt{1+(\tau_j\omega)^2}}$, which may indeed become small, provided that the strain spectrum $\hat{\varepsilon}(\omega)$ contains only frequencies much smaller than the inverse relaxation times $1/\tau_j$.

3 NUMERICAL EXAMPLE

To illustrate the behaviour of our damping model, we show the results of a numerical simulation of nonlinear vibrations of a cantilver beam in Fig. 1 obtained with the discrete Cosserat rod model presented in [7]. The beam is fully clamped at one end, the other end is pulled sideways far beyond the linear deformation range and then released. No gravitation is present. The parameters of the beam are: length $L = 0.3m$, quadratic cross-section area $A = 0.01 \times 0.01m^2$, density $\rho = 10^3 kg/m^3$, Young's modulus $E = 10^6 Pa$, Poisson's ratio $\nu = 0.3$. The tests were performed with three different values of $\tau_E = \tau_{B/S} = 0.01s,\ 0.02s,\ 0.04s$.

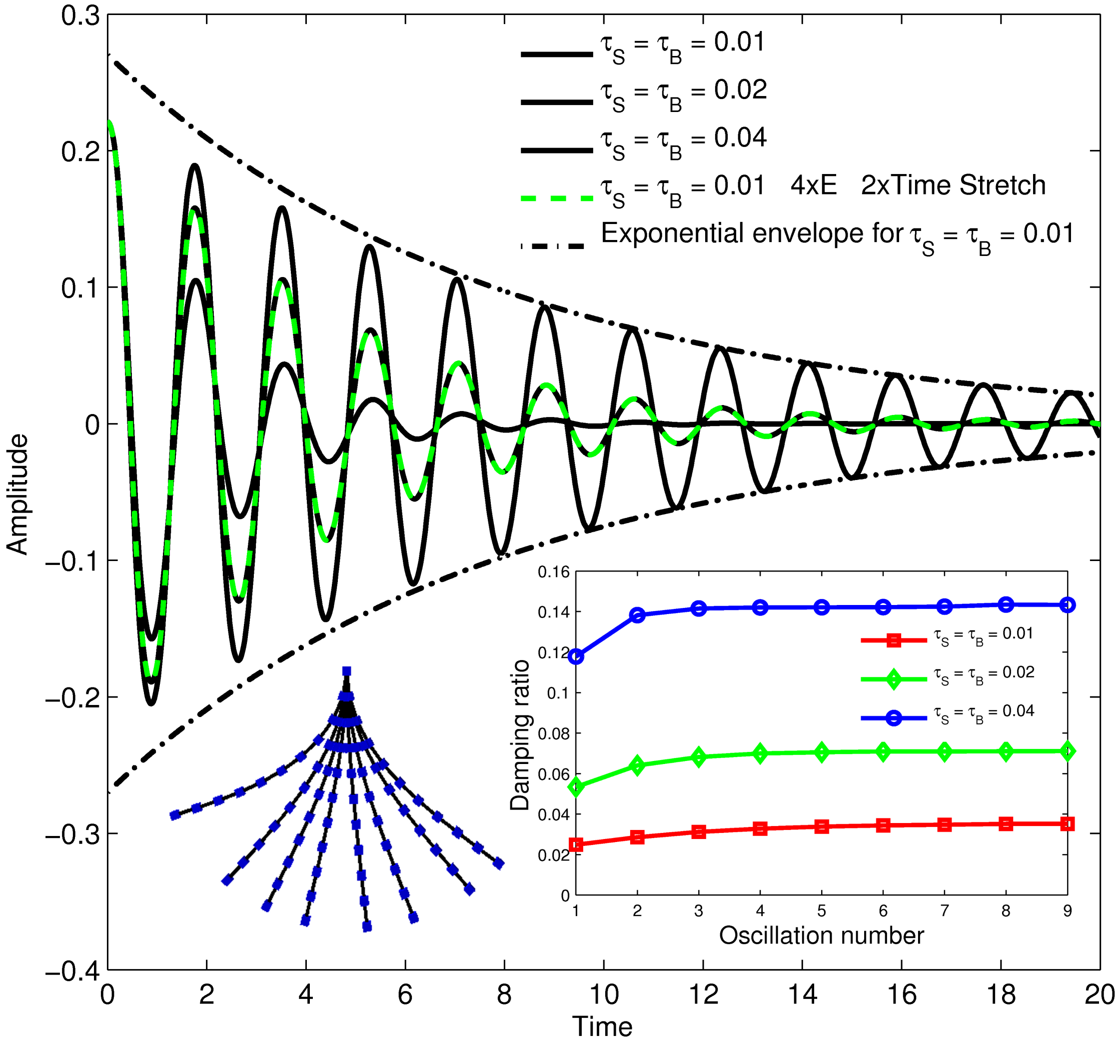

Figure 1. Damped non-linear bending vibrations of a clamped cantilever beam.

The deformations of the beam shown in the inset of Fig. 1 are snapshots taken during the first period of the oscillations which illustrate that in the initial phase of the oscillations substantial geometric nonlinearities are present. During the vibrations the beam remains in the plane of its initial deformation, such that all deformations are of plane bending type, and the extensional viscosity $\eta_E = E\tau_E$ becomes the main damping factor. As expected, the plots of the transverse oscillation amplitude $x(t) = \mathbf{e}_1 \cdot \boldsymbol{\varphi}(L,t)$ recorded at the free end of the beam show an exponential dying out in the range of small amplitudes (linear regime). The deviations from the exponential envelope adapted to the linear regime that are observed during the initial phase clearly show the influence of geometric nonlinearity. The plots also suggest that damping becomes weaker in the nonlinear range. However, linear behaviour seems to start already with the fifth oscillation period, where the amplitude still has a large value of $\approx L/3$.

This may be further analyzed by evaluating the *logarithmic decrements* $\delta_k = \ln(x(t_k)/x(t_{k+1}))$ recorded between succesive maxima $x(t_k)$ of the amplitude as well as the corresponding *damping ratios* ζ_k implicitely defined [4] by $\delta_k = 2\pi\zeta_k/\sqrt{1-\zeta_k^2}$. The plots for the values of ζ_k determined in this way are shown in the inset of Fig. 1. As expected, the ratios approach constant values in the linear regime, which scale as $1:2:4$ proportional to the values of the time constant τ_E used in the simulations. The simulations also show that the decrements become lower in the range of large amplitudes, which confirms the observation that the damping effect of our Kelvin–Voigt model is extenuated by the presence of geometrical nonlinearity. Nevertheless, ζ_k still scales approximately proportional to τ_E also in the nonlinear range.

To investigate the influence of a variation of the bending stiffness on the damping behaviour, an additional test with quadrupled Young's modulus $E = 4 \cdot 10^6$ was performed. In the corresponding amplitude plot shown in Fig. 1 the time axis of the plot with quadrupled E was streched twofold, such that the oscillations could be compared directly. After time stretching the $(E = 4 \cdot 10^6, \tau_E = 0.01)$ plot coincides with the $(E = 10^6, \tau_E = 0.02)$ plot, surprisingly even throughout the whole nonlinear range. Since the oscillation period T of the $E = 4 \cdot 10^6$ beam is two times smaller than that of the $E = 10^6$ beam, this suggests that the damping ratio varies proportional to the ratio τ_E/T. Again this would be the expected behaviour in the linear regime, but is observed here in the nonlinear range as well.

4 CONCLUSION

In our paper we presented the derivation of a viscous Kelvin–Voigt type damping model for geometrically exact Cosserat rods. For homogeneous and isotropic materials we computed the damping parameters explicitely in terms of the stiffness parameters and retardation time constants. In numerical simulations of vibrations of a clamped cantilever beam we observed a weakening influence of geometric nonlinearities on the damping of the oscillation amplitudes. We also found that the variation of retardation time and bending stiffness has a similar effect on the damping ratio as in the linear regime. In view of the limitations of the Kelvin–Voigt model w.r.t. higher frequencies it would be worthwile to develop more complex viscoelastic models (e.g. of generalized Maxwell type) for Cosserat rods. The technical details of our derivation of the Kelvin–Voigt model may be helpful to derive such models from 3D continuum theory in an analogous way.

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