# Simulation of a neutron multiplicity counter and comparison to validation experiments

**Olaf Schumann, Theo Köble, Wolfram Berky, Monika Risse**

Fraunhofer Institute for Technological Trend Analysis INT
Euskirchen, Germany

***Abstract:***

*Neutron coincident counting is a useful tool, both to determine the nature of a neutron source and to extract parameters like the multiplicity, α-ratio and ultimately the mass. For the latter, well characterized detectors, like the Active Well Coincident Counter (AWCC), enable the measurement of uranium or plutonium content in the order of several grams.*
*The multiplicity analysis also allows determining if an unknown neutron source emits fission neutrons and thus possibly contains special nuclear material. The Ortec Fission Meter is an instrument designed exactly for this purpose. It consists of a highly efficient moderated $^{3}$He neutron detector and a Windows Mobile handheld computer with dedicated software. It is powered by batteries and intended for field use. In order to get a deeper understanding of the measured data and to predict the dependence of the analysis on different parameters like additional shielding, Fraunhofer INT performed a Monte-Carlo simulation of the instrument. A MCNP simulation of the source assembly and the instrument results in the arrival times of the neutrons for one single source event. Further software modules allow to generate a pulse train and to perform the same analysis as the Fission Meter hard- and software does. While the count rate of the simulation and a validation experiment were in agreement, the calculated Feynman-Variance showed a significant deviation. The main cause is presumably a small fraction of double pulsing from the discriminator. The inclusion of this effect in the post-processing results in a very good agreement of measured and simulated data.*

**Keywords:** neutron multiplicity; MCNP simulation; validation experiment; double pulsing

## 1. Introduction

Neutron multiplicity counting is a useful tool, both to determine the nature of a neutron source and to extract parameters like the multiplicity, α-ratio and ultimately the mass [1]. A multitude of different instruments have been developed to characterize neutron sources and determine the mass of fissile isotopes in different containers, e.g. the Active Well Coincident Counter (AWCC) or the Plutonium Scap Multiplicity Counter (PSMC) [2]. The technique has found applications for safeguard purposes, nuclear material holdup determination and waste characterization.
But it could also be employed to determine the nature of an unknown source for nuclear search and emergency response in a homeland security context. Here the focus lies on the detection of radioactive material that might be potentially used for a radioactive dispersal device or even an improvised nuclear device. A quick and reliable measurement under field conditions is an important characteristic for such a task. The Ortec Fission Meter, which was developed at the Lawrence Livermore National Laboratory, is an instrument which is applicable for such a measurement and is commercially available [3]. It is a highly efficient moderated $^{3}$He neutron detector with integrated electronics for multiplicity counting and a Windows Mobile handheld computer for measurement control and data evaluation. The detector unit consists of two connected panels, each containing 15 $^{3}$He-tubes. The tubes have an active length of 19 inch and are filled with 7.5 bar of $^{3}$He. One side of each panel is covered by a thin HDPE-moderator. This side is designated the thick side, while the other one is called the thin side. The device is battery-powered and with approx. 27 kg weight it can be considered as portable. It is advertised as a “proof positive” identification device for fission neutron sources through multiplicity analysis and to have the highest sensitivity in a portable package. The Fission Meter determines simultaneously 512 multiplicity distributions with gate lengths von 1 µs up to

512 µs and the Rossi-α distribution. From the different multiplicities, a plot of the Feynman variance as function of the gate time is generated.
In order to get a better understanding of the measurement results obtained with the Fission Meter and to predict the outcome of hypothetical measured objects, we decided to perform a numerical modelling of the Fission Meter. As it is the case with every simulation, a validation of the obtained results with a real experiment is of uttermost importance. Only then one can have some confidence that the obtained results from a simulation have some value in examining the detector response to otherwise inaccessible objects. For this we measured a simple $^{252}$Cf-source in a controlled geometry with the Fission Meter and tried to obtain simulation results which describe the measurements as well as possible.

## 2. Measurements and Numerical Modeling

Modeling and experimental measurement of a simple geometry was performed, in order to validate the ability of our modeling approach to predict the Fission Meter results for different measurement conditions. This will ultimately lead to a better understanding of the influence which different measurement conditions, such as additional moderating material, have on the result. These influences can be hard to control in an experimental measurement, but are easier to handle in a numerical simulation.

### 2.1. Experimental results to model

The experimental results we want to obtain are the count rate, the multiplicity distribution, and the Feynman variance. The multiplicity distribution is a histogram, whose n-th bin gives the number of times $M_n$, when within a specific time interval Δ, the gate time, exactly $n$ neutrons are counted. If for instance within the gate time 4 neutrons are counted, the value of $M_4$ is increased by one. When repeating this process many times, the multiplicity distribution is obtained. The sum of all entries from the histogram is the total number of measurement cycles. Multiplied by the gate time this gives the measurement time. The number of neutrons measured can be obtained by the fact that $M_1$ times exactly 1 neutron was measured, $M_2$ times 2 neutrons were measured, etc. The number of measurement cycles $C$ and the number of registered neutrons $N$ can be obtained by:

$$C = \sum M_n \qquad N = \sum n \cdot M_n \tag{1}$$

These relations can be generalized by the moments of the distribution

$$\mathcal{M}_i = \frac{1}{i!}\sum_{n=i}^{\infty} \binom{n}{i} M_n \tag{2}$$

If the source were to emit neutrons in a purely random fashion, the distribution would be a Poisson distribution. The defining property of this distribution is that its standard variance equals the mean value. The deviation of the obtained distribution from the Poisson distribution and thus the deviation of the source from a purely random and uncorrelated one can be quantified by the variance to mean ratio. This is the well-known Feynman variance $Y_{2f}$, which can be expressed in terms of the moments as follows:

$$Y_{2f} = \frac{\mathcal{M}_2}{\mathcal{M}_1} - \frac{1}{2}\frac{\mathcal{M}_1}{\mathcal{M}_0} \tag{3}$$

The Fission Meter determines simultaneously 512 multiplicity distributions with gate length from 1 µs up to 512 µs and generates a plot of the Feynman Variance $Y_{2f}$ as function of the gate time. From a point kinetic reactor model, Feynman has derived [4] the theoretical dependence of this function as:

$$Y_{2f}(T) = R_{2f} \cdot \left(1 - \frac{1 - e^{-\lambda T}}{\lambda T}\right) \tag{4}$$

Where $\lambda$ is the inverse of the neutron lifetime; the mean time it takes from the creation of a neutron to its detection. The time from creation of an individual neutron to its detection itself is exponentially distributed. The neutron life time determines the shape of the curve. If it is small, the curve starts much steeper and reaches the limit earlier. If the neutron life time is large, the curve is shallow and reaches its limit later. The value $R_{2f}$ is the limiting value, where eventually the curve will tend to.

## 2.2. Experimental Setup

Experimental data have been recorded in a laboratory used for neutron irradiation experiments by a neutron generator. In this lab, we could ensure that the walls and the ceiling had a distance of a minimum of 2 m to the Fission Meter and the source in order to reduce the backscattering of the neutrons. Concerning the floor, the Fission Meter was positioned above a pit that is covered with open mesh flooring covered by a thin plywood layer, so that the concrete floor also was 2 m below the Fission Meter. The walls and the roof consist of concrete with a thickness of 1 m to 2 m. A $^{252}$Cf-source with approx. 240 kBq was used in these experiments and placed in different distances from the center of the Fission Meter.

**Figure 1:** Picture of the experimental setup used in the measurements. The Fission Meter is placed on an open mesh flooring which is covered by plywood. It is used in its closed configuration and the white moderator side is facing outwards. The source is placed on the edge of a scissor lift and positioned in the center of the Fission Meter. All walls, ceiling and the concrete floor under the open mesh flooring are more than 2 m away from the source and the detector in order to reduce scattering.

The Fission Meter was used in its closed configuration, that is, the two panels were folded together with both their thick side facing outwards. The background count rate with the Fission Meter without any source was quite low, in the order of 0.8 cps. During the measurements, the Fission Meters position was fixed and the source was positioned at different distances from the front surface. The results of the measurements are given in table 1, an overview of the setup is shown in figure 1. As common with neutron measurements, the data show no clear $1/r^2$ dependence on the distance but vary more like 1/r. This effect is likely due to the backscattering from the room. The count rate for all distances is well above the background rate of 0.8 cps.
The Feynman variance also shows a decrease with increasing distance but seems to saturate at a value of approximately 3.7 %. As it should be proportional to the efficiency of the detector, it is expected that is also shows a $1/r^2$ dependence, which was definitely not seen here.
These experimental data were the reference data that were attempted to reproduce with numerical methods described in the next section.

| Distance (cm) | Time (s) | Count rate (cps) | Feynman Variance (%) | | Neutron die away (µs) | |
|---|---|---|---|---|---|---|
| | | | exp. | sim. | exp. | sim. |
| 5 | 8418 | 1012.55(35) | 8.616(23) | 4.998(31) | 12.92(18) | 27.76(12) |
| 10 | 3177 | 715.95(47) | 7.066(37) | 3.475(25) | 10.92(17) | 27.04(11) |
| 20 | 3933 | 408.48(32) | 5.580(34) | 1.851(18) | 7.59(14) | 27.86(12) |
| 50 | 7307 | 139.33(14) | 4.264(28) | 0.546(10) | 3.518(86) | 29.76(18) |
| 100 | 841 | 60.74(27) | 3.72(10) | 0.107(33) | 3.40(11) | 43.9(11) |
| 200 | 2576 | 31.47(11) | 3.768(72) | 0.0562(47) | 3.16(14) | 43.05(41) |

**Table 1:** Specifications of the measurements taken with the Fission Meter. Shown is the distance from the source to the surface of the Fission Meter, the acquired measurement time, the obtained count rate, the Feynman Variance, calculated from the longest gate time of 512 µs, and the neutron die away time. The latter two values are also given for the simulated results. The measurements at 5 cm and 50 cm were performed for more than 2 h in order to obtain good statistics for the multiplicity counting.

## 2.3. Numerical Modeling and Post-Processing

Numerical modeling was performed by using MCNPX Version 2.7.0 [5] and tailor made scripts for post-processing of the output. A MCNP geometry description of the Fission Meter was prepared based on data provided by Ortec on the geometry of the moderator assembly and by GE Energy / Reuter-Stokes for the used $^{3}$He-tubes. The model includes the $^{3}$He-tubes, the detailed moderator assembly and the outer aluminum case, while batteries, the electronic components and other components have been omitted.

In a first simulation, the response of a single $^{3}$He-tube to thermal neutrons was determined. The tube is positioned with its axial axis in the z-direction. A monoenergetic thermal neutron beam with 25 meV and rectangular cross section is directed at the tube in x-direction. The cross section of the beam encompasses the complete tube assembly. The number of capture reactions of $^{3}$He is recorded by a f8-tally with the "ft8 cap 2003" special treatment. Due to this, a variance reduction is not possible and was thus not employed. The density of the $^{3}$He-gas is calculated without any quench gas from its nominal pressure of 7.5 bar and is equal to $9.25 \cdot 10^{-4}$ g/cm$^{3}$. The neutron flux per unit time is given by the number of histories normalized by the cross section of the simulated beam, the number of captures per unit time results from the f8-tally. This simulation yields a thermal neutron sensitivity of 88.90 cps/nv while manufacturer specification from the spec sheet is 71.81 cps/nv. The quotient of this value of 80.1% is interpreted as an intrinsic efficiency reduction where a neutron capture for some reasons does not lead to a signal on the output of the tube. This effect is accounted for in the following simulations.

The second set of simulations is the calculation of the predicted count rate. Beside the Fission Meter, the room of the laboratory including the pit underneath the Fission Meter and the wooden layer on the open mesh flooring were modeled. All other items in the laboratory were neglected. The source was represented by a minute sphere filled with pure $^{252}$Cf and the source definition was a spontaneous fission source (PAR=SF). The number of captured neutrons was again recorded by an f8-tally with capture treatment. This yielded finally the number of captures within the Fission Meter per source spontaneous fission. Combined with the intrinsic tube efficiency of 80.1% and the number of spontaneous fissions of the used $^{252}$Cf-source, this could be converted to an absolute count rate. No other corrections have been made to the data.

The third and final set of simulations was done in order to generate the multiplicity distributions, as measured by the Fission Meter. The geometry description and general setup was the same as for the count rate determination. That is, the Fission Meter, the room and the plywood were modeled and all other items of the room were neglected. The source was likewise a spontaneous fission source, which emitted the neutrons for each history at a time of 0. All termination events within the $^{3}$He-Volume of the Fission Meter were recorded to a binary PTRAC-file with the following MCNP input line:

```
PTRAC FILE bin WRITE all EVENT=ter FILTER=900,ICL
```

The PTRAC-file was then post-processed to give the multiplicity distribution. This was done by a set of python scripts. First the PTRAC-file was converted to a format[1], where for each history the arrival times of the neutrons in the detector are listed. From this file a pulse train was created by generating a sequence of exponentially distributed random variables. The sum of this sequence gives the points in time where a spontaneous fission occurs. For each fission event, one of the histories from the PTRAC-file was randomly selected. For each neutron in that history a Russian roulette is played with the intrinsic efficiency of the tube (80.1%) and if it survives, the arrival time is added to the time of the fission. As a neutron from a preceding fission event might arrive later than from the actual fission event, the times have to be sorted. We choose to do this on-line with a priority queue instead of generating all fission times first and then sorting a possible very large set of "almost sorted" neutron arrival times. With this process, the sorting efficiency is greater, as only few arrival times have to be present in the priority queue, as all neutron detection times smaller than the current fission time could be extracted from the queue. In addition a continuous and possibly endless stream of detection times can be generated in this way and feed to the analyzing algorithm. This allows collecting intermediate results and obtaining multiple evaluations with increasing "measurement time". This pulse train is then analyzed by a code that is equivalent to the procedure that has been published for the Fission Meter [6]. Finally the multiplicity distribution and Rossi-α data is saved in the same format as the experimental data from the Fission Meter.

[1] This was implemented with a HDF5-file containing one array of all arrival times, and one an index of which times belong to a which source event. Especially for large number of histories, it is much faster to read the HDF5-file compared to parsing the PTRAC file or to read a text based format.

## 3. Results

Figure 2 shows the results of the simulation of the count rate, compared to the experimental data. As explained in section 2, the results of the simulations were only scaled by the source intensity and the tube efficiency; no other corrections were made. The experimental data can be very well described by the simulation. Only at large distances the simulations tend to overestimate the count rate by approximately 20%. Nevertheless, this shows how well the detector is described by the numerical modeling.

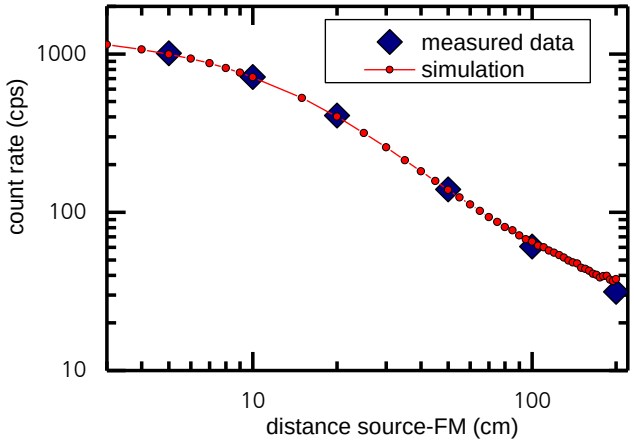


**Figure 2:** Comparison of the measured and simulated count rate dependence on the distance between source and Fission Meter. The simulation results were scaled by the tube efficiency and the actual neutron source strength. No other corrections were made. The agreement between experimental data and numerical results is very good.

Figure 3 shows the results of the measured and the simulated Feynman variances as function of the gate time. In contrast to the count rates, the agreement between these curves is poor. Neither the limiting value nor the shape is reproduced in the simulations. The values of $Y_{2f}$ for long gate length are systematically about 3.7% larger for the experimental values than in the simulations, as seen in table 1. This is very prominent for the larger distances. While the simulation predicts a quite small overall value for $Y_{2f}$ with a shape that conforms to that of formula 4 for the distances 100 cm and 200 cm, the experimental results show an abrupt increase to approx. 4% within the first few microseconds and can't be reasonable described by formula 4. This step like jump would result in a very short neutron die-away time of some few microseconds, which is quite unphysical.

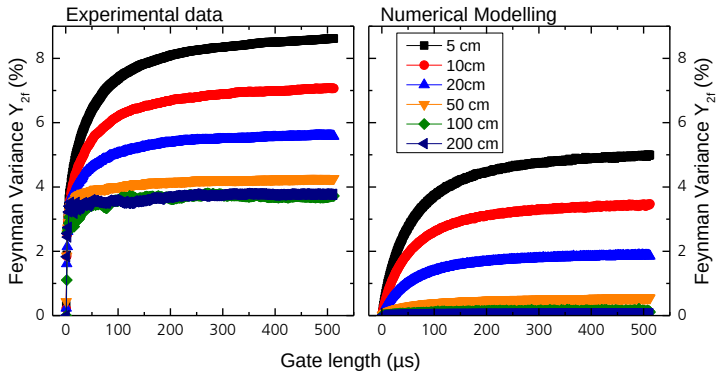


**Figure 3:** Comparison of the measured (left) and simulated (right) Feynman Variance for different distances. The experimental data could not be reproduced by the simulation.

Furthermore, the Feynman variance should show a linear dependence on the detector efficiency, which would result in a vanishing value for increasing distances, but that is not seen in the data. Thus there seems to be an additional "background" correlation in the data that has a characteristic time of at most a few microseconds, that seems to be independent on the distance, and that is absolutely not reproduced by the numerical modeling.

### 3.1 Double pulsing contribution

A hint on the cause of this effect comes from the Rossi-α measurement that is shown in figure 4. The plot shows a prominent sharp peak in the experimental data that occurs at a time difference of 1 µs. This peak is not reproduced at all in the simulated data. A peak structure in the Rossi-α data is described in the literature as due to double pulsing. Double pulsing could occur, if the charge signal on the input of the amplifier/discriminator circuit shows a dip, e.g. due to different charge collection times from the two charged ejectiles from the $^3$He(n,p)t reaction. Other causes might be reflections on signal lines due to inadequate termination, wrong impedance or cable breaks.

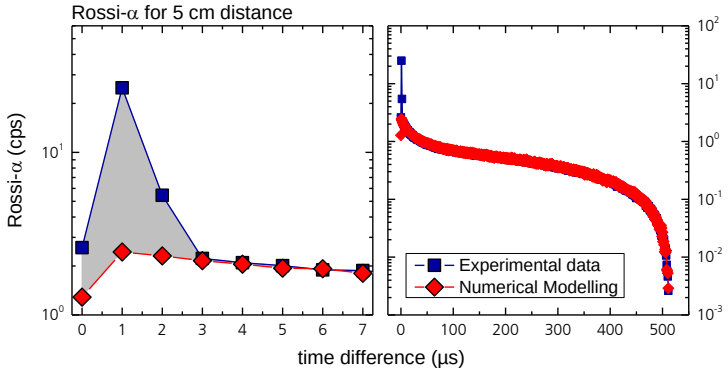


**Figure 4:** Rossi-α plot of the experimental and simulated data for a distance of 5 cm. The peak in the experimental data is not reproduced in the simulations. The left graph is a magnification of the right one.

The effect of double pulsing on the Feynman variance can be determined from its formulation in terms of the detection probabilities for a single source event. While formula 3 is valid for a multiplicity distribution from a counting experiment, where counts from different source events can overlap, it can be shown [7], that for a single source event, the Feynman variance has the form:

$$Y_{2f} = \frac{\sum_n n \cdot (n-1) \cdot p_n}{\sum_n n \cdot p_n} = \frac{\mathcal{M}_2}{\mathcal{M}_1} = \frac{p_2 + 3p_3 + 6p_4 + 10p_5 + \cdots}{p_1 + 2p_2 + 3p_3 + 4p_4 + \cdots} \tag{5}$$

Here, the $p_n$ are the probabilities that $n$ neutrons from a single spontaneous fission source event are detected within the time gate. This is a combination of the source multiplicity, the detection efficiency and a factor that relates the gate length to the neutron die away time. For arbitrary long gate times, all neutrons from a single source event are counted in the same gate and $Y_{2f}$ tends to $R_{2f}$.To take double pulsing into account, one introduces the probability $\delta$ that a single neutron generates two counts instead of only one. If $n$ neutrons are detected, each one can result in a double pulse with a probability $\delta$ and $n$ up to $2n$ counts might be registered. The exact number is given by a binomial distribution.
This allows calculating the two moments $\mathcal{M}_1$ and $\mathcal{M}_2$ with the double pulsing included. For long gate times, it is ensured that all pulses are registered within the same gate and the resulting formula for $R_{2f}$ is given by:

$$\hat{R}_{2f} = \frac{\widehat{\mathcal{M}}_2}{\widehat{\mathcal{M}}_1} = \frac{\mathcal{M}_2 \cdot (1+\delta)^2 + \mathcal{M}_1 \cdot \delta}{\mathcal{M}_1 \cdot (1+\delta)} = R_{2f} \cdot (1+\delta) + \frac{\delta}{1+\delta} \approx R_{2f} + \delta \tag{6}$$

As the two pulses of a double pulse might be registered in different gates, it is more challenging to determine an analytic expression for the time dependence of the Feynman variance with double pulsing for different gate length. Nevertheless, this result allows determining the double pulsing probability from the difference of the experimental and simulated Feynman variance data at large gate length. Interestingly, it is basically constant for all distances and has a value of $\delta = 3{,}7\%$.
An analysis of the height of the peak in the Rossi-α data with an analytical model [7] allows an independent determination of this value and yields consistent values. This also allows extracting the time difference between the two signals within a double pulse. A good agreement with the measured data could be reached by a normal distribution with a mean of 1.06 µs and a width of 0.3 µs, for all measured distances.
This is all information that is required to include the double pulsing in the post-processing of the simulated multiplicity data. For every neutron that is inserted into the pulse train, a second signal is inserted with a probability $\delta = 3.7\%$. The time difference between first and second signal is drawn from

a normal distribution with a mean of 1.06 µs and a width of 0.30 µs. The processing of this modified pulse train will then proceed as described above. The results of the Feynman variance calculation with double pulsing included is shown in figure 5.

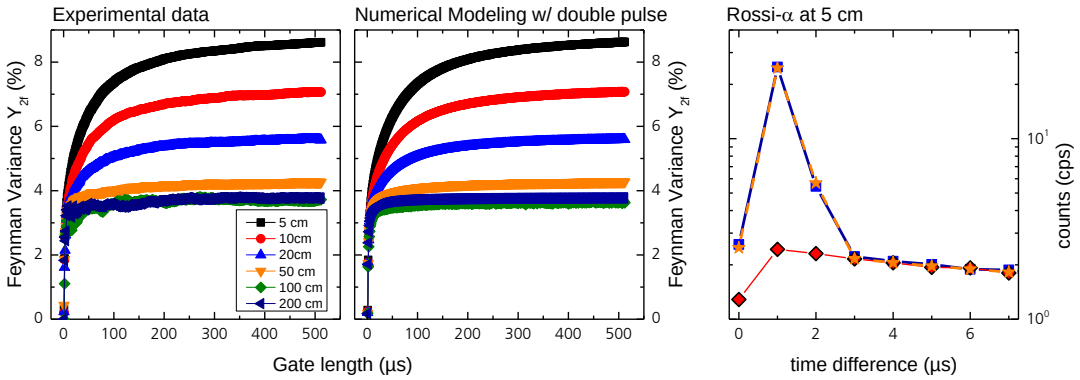


**Figure 5:** Comparison of the measured (left) and simulated (middle) Feynman Variance for different distances. In contrast to figure 3, the experimental data could be reproduced very well by incorporating a double pulsing of 3.7% in the post-processing of the simulation data. On the right, the beginning of the Rossi-α for 5 cm is shown in analogy to figure 4. With the inclusion of double pulsing, the peak could be reproduced very well.

These graphs are based on the same MCNP-simulation results as those in figure 3, but include the effect of a double pulsing probability of 3.7%. This results in a very good agreement between numerical modelling and experimental data. The shape of the Feynman Variance and the limiting value $R_{2f}$ matches very well for all distances. Also the peak in the Rossi-α data could be replicated with this approach.

## 4. Discussion

The findings from section 3 show that the detailed numerical modeling yields results that are in very good agreement with the experimental data. The count rate is easily reproduced to a few percent without any artificial scaling of the data. Only the efficiency of the neutron tube needs to be scaled, so that the simulation results from MCNP are equal to the manufacturer stated value. One contribution of this scaling is the unknown composition of the quenching gas as this is considered proprietary information. Instead, the gas was simulated as pure $^3$He with the nominal pressure. This definitely leads to an overestimation of the sensitivity, which results in the need of the scaling.
For the multiplicity distribution and especially the Feynman variance, the numerical method has to be refined. The experimental data can be reproduced very accurately, when double pulsing is taken into account. This allows replicating the shape and absolute value of the Feynman variance and, at the same time, the peak in the Rossi-α data. The inclusion of double pulsing can be done entirely in the post-processing of the MCNP PTRAC date. Thus the most time consuming step, the Monte Carlo simulation of the detector response, is independent of this step.
The amount of double pulsing of 3.7% in the Fission Meter used in this investigation seems to be quite high. Measurements with a second Fission Meter indicate a much lower value in the order of 0.5%. This might point to a hardware problem of the Fission Meter used in this work, such as a defective preamp, a bad or faulty connection, or HV issues. This will be investigated in more detail in the future.
Nevertheless, the measurement data for unknown configurations of fissile material can be simulated with our approach. This can be done for an ideal Fission Meter that shows no double pulsing at all, and in addition the effect of double pulsing can be included. This work shows also that a small amount of double pulsing can have quite some influence on the evaluation of multiplicity data. Especially, the resulting Feynman variance values are larger, and the change of the shape of the Feynman curve results in fitted neutron die away times that are much smaller than their real values. Sometimes, the die away times can get unphysically small. In the case of the Fission Meter, the neutron die away time serves as an indication of additional moderator, which is thus hindered by the double pulsing.

## 5. Acknowledgements

This work was partly funded by the Federal Ministry for the Environment, Nature Conservation and Nuclear Safety under contract number 3616R01683. The author would like to thank AMETEK/Ortec for providing an engineering drawing of the moderation assembly and GE Electric for providing an engineering drawing and technical information on the used $^{3}$He-tubes.

## 6. References

[1] Reilly D., Ensslin N., Smith H., Kreiner S.; *Passive Nondestructive Assay of Nuclear Materials*; United States Nuclear Regulatory Commission, 1991

[2] Ensslin N., Harker W.C., Krick M.S, Langner D.G., Pickrell M.M., Stewart J.E.; *Application Guide to Neutron Multiplicity Counting;* Los Alamos National Laboratory; 1998; LA-13422-M

[3] Ortec; *Fission Meter Portable Neutron Source Identification System User's Manual;* Rev. B; 2007

[4] Walston S.; *The Idiot's Guide to the Statistical Theory of Fission Chains;* 2009; Lawrence Livermore National Laboratory; LLNL-TR-414245

[5] D.B. Pelowitz, Ed.; *MCNPX Users Manual Version 2.7.0;* 2011; LA-CP-11-00438

[6] Rowland M.S., Alvarez R.A.; *Real-Time Multi-Mode Neutron Multiplicity Counter;* 2013, US Patent 8,384,004 B1

[7] Schumann O., not yet published