Electrical Modeling of the Power Delivery to an LED Array Packaged in a Textile

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Abstract

Recent technologies enable large arrays of LEDs to be integrated into textiles, which has applications in the lighting industry. Two analytical modeling techniques are proposed that calculate the supply power at a source necessary to provide the LED drivers with a minimum voltage. The modeling techniques show a correlation with numerical simulations to within 10% but can be implemented for very large LED arrays where, for time constraints, numerical simulations become impractical. The modeling allows a designer to optimize the location of the supply voltage, which can reduce the increase in voltage over the ideal voltage by 40%.

1. Introduction

In recent years, new technology has allowed electronics to be integrated into textiles. This technology is being exploited in applications related to body-area networks, medical, and sensors [1-2]. This technology is also being examined for lighting. An LED array could be integrated into a textile and used to light large areas.

Until now, electrical modeling of electronics in textiles has been limited to miniaturized electronic devices. Some investigations have examined reliability in textile integrated electronics for networks, for example Marculescu et. al. [3-4]. Electrical modeling of antennas in textiles was examined in [5]. Graumann has modeled a large surface area electronic textile in [6] with, again, a focus on network modeling.

Until now, the electrical modeling of textiles used for lighting has not yet been examined. Furthermore, most modeling, until now, has examined electronics with transmission lines woven into a textile, rather than the use of a conductive textile. A large area textile of several square meters in area, used in a lighting application, poses the problem that; (1) the LEDs will have uneven voltage drops across the textile, relative to the source, depending on their distance from the voltage source, and (2) the current density of the textile will be much higher very close to the voltage source, creating heat that must be dissipated.

In this paper, a modeling technique will be introduced to model the voltage on a large area conductive textile with a single source. This modeling technique can then be used to optimize the location of the source and to determine, for a given source location, the necessary voltage to power an array of LEDs driven by microcontrollers. The modeling technique can also be used to estimate the maximum power density on the textile, which can then be reduced below a necessary levels by adjusting the source location and/or size.

2. Problem Description

LEDs integrated into a large area textile with a single source will all have different distances from the source. We approximate an LED as a resistance, as in Fig. 1 (left). For a large area conductive textile, because the distances to the source can vary greatly and the textile is not an ideal conductor, the LEDs farthest away from the source will have a significantly lower current than those close to the source due to the voltage drop over the textile.

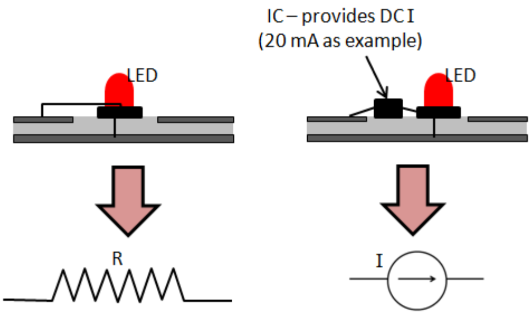


Fig. 1. LED connection to textile.

To solve this problem, a small driver chip that provides a constant current, is integrated with each LED. In this case, each LED and driver can be approximated as a current source, as in Fig. 1 (right). Each driver must be supplied with a minimum voltage to function properly. This means that the source voltage must increase above this minimum source voltage to supply the farthest LEDs with this minimum voltage, due again to the voltage drop over the textile.

Published at *IEEE Electrical Design of Advanced Packaging Symposium (EDAPS)*, Nara, Japan, December 2013.

A large and non-ideally conductive textile can require a significantly higher voltage than the minimum required to drive the microcontrollers. This can pose problems for the driver chips closest to the source voltage, which may dissipate a larger amounts of power or, in extreme cases, have a supply voltage that exceeds their maximum leading to failure.

There is, however, no simple analytical modeling technique to calculate the ideal supply voltage at the source that includes multiple LEDs and the boundaries of the textile. Numerical techniques like the boundary element method, for tens or even hundreds of LEDs integrated into a very large area but very thin conductive textile, would require unreasonably long simulation times.

3. Modeling

For the modeling, we begin by using Ohm's Law to determine the potentials of the contacts and the source. Each LED driver chip is connected with a copper wire, which will be approximated as a perfect conductor. The source voltage is connected with a copper disk, again approximated as a perfect conductor. The derivation for this is shown in Fig. 2.

$$\vec{J} = \kappa \vec{E}$$
$$\vec{E} = -\nabla \Phi$$
$$\vec{J} = -\kappa \nabla \Phi$$
$$J\vec{e}_r = -\kappa \frac{\partial \Phi}{\partial r}\vec{e}_r$$
$$J\vec{e}_r = \frac{I}{2\pi r d}\vec{e}_r = -\kappa \frac{\partial \Phi}{\partial r}\vec{e}_r$$
$$\Phi(r) = -\frac{I}{2\pi d \kappa}\ln(r)$$

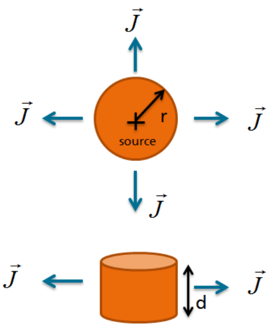
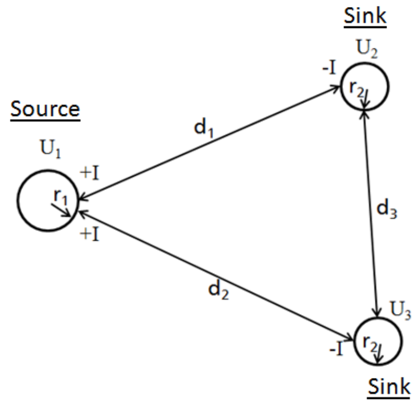


Fig. 2. Using Ohm's law to determine the potentials of the source and LED contacts.

Using this simple derivation, it is easy to find the voltage potential between multiple different contacts on a conductive plane, like a conductive textile. An example is given in Fig. 3 for three different contact points.

The voltage drop between the three points, U_{XY}, is calculated simply as the difference in the two voltage potentials.



$$\phi_1 = \frac{I}{2\pi\sigma t}\left[-\ln d_1 + 2\ln r_1 - \ln d_2\right]$$

$$\phi_2 = \frac{I}{2\pi\sigma t}\left[-\ln r_2 + 2\ln d_1 - \ln d_3\right]$$

$$\phi_3 = \frac{I}{2\pi\sigma t}\left[2\ln d_2 - \ln r_2 - \ln d_3\right]$$

$$U_{13} = \phi_1 - \phi_3$$
$$U_{12} = \phi_1 - \phi_2$$
$$U_{23} = \phi_2 - \phi_3$$

Fig. 3. Calculation of potentials of three contacts on a conductive plane.

It is easy to extend this model to account for a LED array with a DC source somewhere on the plane, similar to the structure in Fig. 4 (bottom) and to calculate the potentials with the general equation in Fig. 4 (top), where n is the number of LEDs. The modeling, however, is limited to an infinite plane. It does not account for the boundaries of the textile. When the source is place in the corner of the textile, as it is in Fig. 4, then the boundaries of the textile will significantly increase the voltage drop from the source to each LED.

$$\boldsymbol{\phi_{NM}} = \frac{I}{2\pi\sigma t}\begin{bmatrix} n\ln r_{11} & n\ln r_{12} & \dots & n\ln r_{1n} \\ -\ln r_{21} & -\ln r_{22} & \dots & -\ln r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ -\ln r_{n1} & -\ln r_{n2} & \dots & -\ln r_{nn} \end{bmatrix}$$

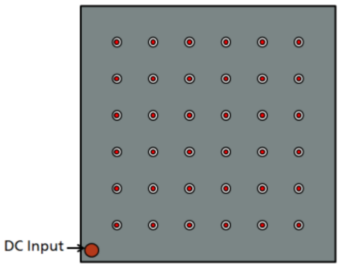


Fig. 4. 6X6 LED array with a DC source.

Two methods have been determined to account for the boundaries of the textile.

A. Image Theory

Image theory states that, when the electric current at a boundary is zero, then it can be approximated by reflecting the electric and magnetic field across the boundary, cancelling it out. Using image theory, the DC source and sinks (LED contacts) will be reflected on each face and on each corner. Those reflections must theoretically be further reflected infinitely. However, a first order calculation could give a good estimate.

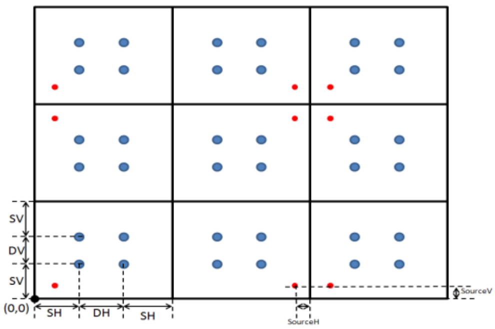


Fig. 5. 1st order image theory representation of a 4x4 LED array (blue) with a single source (red).

B. Calculation of Potentials

The problem can also be seen as a electrical potential problem.

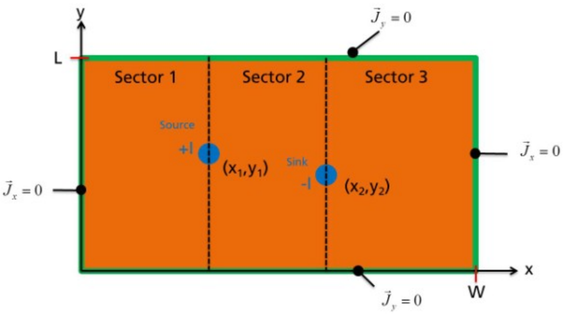


Fig. 6. Rectangular textile with a source and a sink.

The electrical field strength can be calculated as the gradient of a scalar potential. The potential follows the Poisson-Equation:

$$\nabla \times \vec{E} = -\frac{dB}{dt} = 0$$

$$\nabla \times (\nabla \times \phi) = 0$$

$$\vec{E} = -\nabla \phi$$

The current density J is related to the electrical field strength by the resistivity κ.

$$\nabla \times \vec{H} = \hat{J} + \frac{dD}{dt}$$

$$\frac{dD}{dt} << \hat{J}$$

$$\vec{J} = \kappa \vec{E} = -\kappa \nabla \phi$$

Using this, one can find the current density with the gradient potential.

$$\nabla D = q_v = \nabla \varepsilon \vec{E}$$

$$\nabla^2 \phi = q_v$$

The Poisson Equation can be solved by dividing the textile into sections where the source and sinks always lie on the edges, as in Fig. 6. In doing so, the Poisson Equation simplifies to the Laplace Equation:

$$\nabla^2 \phi = 0$$

There will be many LEDs in the problem and, therefore, the textile must be divided into many different regions. We will begin with a single source and sink. We start with the Bernoulli method:

$$\phi(x, y) = X(x)Y(y).$$

Substituting the Bernoulli method into the Laplace Equation one arrives at the following differential equation:

$$\underbrace{\frac{\partial^2 X}{\partial x^2}}_{=p^2} + \underbrace{\frac{\partial^2 Y}{\partial y^2}}_{=-p^2} = 0$$

The differential equation can be divided into two equations, where p is constant:

$$\phi(x,y) = (A_0 + B_0 y)(C_0 + D_0 x) + \sum_{p\neq 0}^{\infty}\big(Acos(py) + Bsin(py)\big)(Ccosh(px) + Dsinh(px))$$

With this, the general current distribution can be calculated:

$$J_x(x,y) = -\kappa\frac{\partial\phi}{\partial x} = -\kappa(A_0 + B_0 y)D_0 - \kappa\sum_{p\neq 0}^{\infty} p\big(Acos(py) + Bsin(py)\big)(Csinh(px) + Dcosh(px))$$

$$J_y(x,y) = -\kappa\frac{\partial\phi}{\partial y} = -\kappa(C_0 + D_0 x)B_0 - \kappa\sum_{p\neq 0}^{\infty} p\big(-Asin(py) + Bcos(py)\big)(Ccosh(px) + Dsinh(px))$$

The Laplace Equation is made unique by fulfilling the boundary conditions. With these boundary conditions, we can solve for the constants in the potentials. These boundary conditions are determined by the current density. The boundary conditions are linked through the first derivative, so that we have a 2nd order boundary value problem. The boundary conditions are as follows:

The boundary conditions in the top and bottom are the same in all sections. These boundary conditions imply that no current can flow:

$$J_{y=0} = \kappa E_{y=0} = -\kappa\frac{\partial\phi}{\partial y}\bigg|_{y=0} = 0$$

$$J_{y=L} = \kappa E_{y=0} = -\kappa\frac{\partial\phi}{\partial y}\bigg|_{y=L} = 0$$

This boundary condition is valid for all 3 sections simultaneously.
The boundary condition for the first section is that no current can flow in the x-direction:

$$J_{x=0} = \kappa E_{x=0} = -\kappa\frac{\partial\phi}{\partial x}\bigg|_{x=0} = 0$$

The same is valid for the third section, that no current can flow in the x-direction because no conductive material exits.

$$J_{x=W} = \kappa E_{x=W} = -\kappa\frac{\partial\phi}{\partial x}\bigg|_{x=W} = 0$$

Additionally, the continuity between the sections must also be preserved.

$$J_{y2=x1} - J_{y1=x1} = 0$$
$$J_{y3=x2} - J_{y2=x2} = 0$$

Lastly, the current between these sections is determined. The current is approximated as ideal, the source as positive and the sink as negative current sources. These sources are considered area current sources with the help of Dirac functions:

$$J_{x2=x1} - J_{x1=x1} = I\delta(x - x1)$$
$$J_{y3=x2} - J_{y2=x2} = -I\delta(x - x2)$$

Now the unknown constants can be determined with the help of the boundary conditions and the consistency conditions. Several orthogonal trends were used during the determination of the constants. The general solution to the potentials and constants is shown in Appendix 1.

4. Model Validation

To validate the modeling, simulations were conducted with Ansys Q3D and compared to the analytically calculated values. The comparison for the image theory method is shown in Fig. 7. It was necessary for the LED array to be very small, in this case only 4x4, and very large contacts for the source and sink connection because the simulation model would be otherwise too large to calculate.

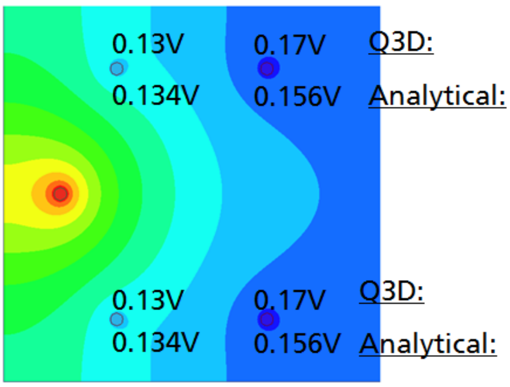


Fig. 7. Comparison of voltages calculated from Ansys Q3D and image theory.

Q3D simulations were also used to validate the potential method. The results are shown in Table 1, where coordinates of the source and sink are given, and analytical and simulation results are compared. In both techniques, results match within 10%.

Source		Sink			
X_1 [mm]	Y_1 [mm]	X_2 [mm]	Y_2 [mm]	Analytical [mΩ]	Q3D [mΩ]
22.5	75	45	145	364	375
22.5	75	45	125	295	296
22.5	75	45	75	168	189
22.5	75	145	75	414	453
22.5	75	145	5	591	623

Table 1. Ansys Q3D Simulation results compared to analytical values.

5. Analysis

Fig. 8 and Fig. 9 show how important placement of the source is, in determining the minimum input voltage. In both cases, a 6x6 array of LEDs with a single source is calculated using image theory. They are spaced 10 cm from each other. They each draw 20mA of current. The textile has a thickness of 100µm and σ = 38 kS/m (values measured from a commercially available conductive textile). The source and sink radii are 1mm and 0.5mm respectively. The array is 20cm from the edge. The source starts at a position of 1cm from the edge.

In Fig. 8, we see that the minimum input voltage, when the microcontroller requires 4V at its input, varies from 4.3V to 4.6V as the source moves from the edge of the material, to the middle.

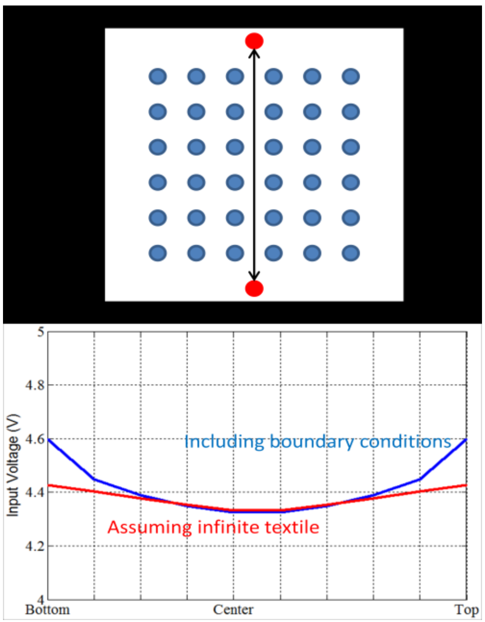


Fig. 8. Minimum input voltage as the source moves from the edge across the middle of an LED array.

We see a similar effect in Fig. 9 when the source moves from a corner to the other corner. In this case the voltage varies from 4.6V to 5V.

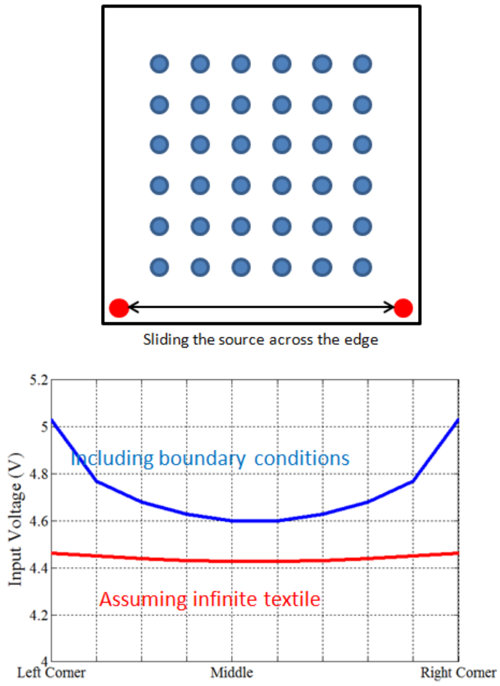


Fig. 9. Minimum input voltage as the source moves from one corner to the other corner of an LED array.

The red lines show what the error in the calculation would have been if the boundaries of the textile were not included in the calculations. This shows a significant error - when the source is in the corner, it more than 100% of the potential difference.

6. Conclusions

Two techniques have been proposed for calculating the voltages necessary to drive an array of LED microcontroller drivers on a conductive textile. These techniques are accurate to within 10% when compared to numerical simulations. The simulations show that, for an example 6x6 LED array, the placement of the source can change the maximum potential between the source and the sinks from 0.3V to more then 1V.

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Appendix 1:

$$\phi_1(x,y) = A_0 + \sum_{p\neq 0}^{\infty} G_1 cos(py) cosh(px)$$

$$\phi_2(x,y) = A_0 + H(x-x_1) + \sum_{p\neq 0}^{\infty} \cos(py)\left(G_1 cosh(px_1) cosh\big(p(x-x_1)\big) + \left(\frac{G_3\cosh\big(p(x_2-W)\big) - G_1\cosh(px_1)\cosh\big(p(x_2-x_1)\big)}{\sinh\big(p(x_2-x_1)\big)}\right)\sinh\,(p(x-x_1))\right)$$

$$\phi_3(x,y) = A_0 + H(x_2-x_1) + \sum_{p\neq 0}^{\infty} G_3 cos(py) cosh(p(x-W))$$

Where:

$$p = \frac{n\pi}{L}, n = 1,2,3\ ...$$

and A_0 is unknown due to the 2nd order differential.

$$H = -\frac{I}{dL\kappa}$$

Where *d* is the thickness of the conductive material.

$$G_1 = \left\{-\frac{I}{d}\cos(py_2) + \frac{I}{d}\cos(py_1)\frac{\sinh\big(p(x_2-x_1)\big)}{cosh\big(p(x_2-W)\big)}\left(\frac{cosh\big(p(x_2-W)\big)}{\sinh\big(p(x_2-x_1)\big)}\cosh\big(p(x_2-x_1)\big) - \sinh\big(p(x_2-W)\big)\right)\right\}\left\{\kappa p\frac{L}{2}\cosh(px_1)\left[\sinh\big(p(x_2-x_1)\big) - \coth\big(p(x_2-x_1)\big)\cosh\big(p(x_2-x_1)\big)\right] + \left[\sinh(px_1) - \cosh(px_1)\coth\big(p(x_2-x_1)\big)\right]\left(\frac{\sinh\,(p(x_2-x_1))}{cosh(p(x_2-W))}\kappa p\frac{L}{2}\right)\left(\frac{cosh(p(x_2-W))}{\sinh\,(p(x_2-x_1))}\cosh\big(p(x_2-x_1)\big) - \sinh\big(p(x_2-W)\big)\right)\right\}^{-1}$$

$$G_3 = \left(G_1\left[\sinh(px_1) + \cosh(px_1)\coth\big(p(x_2-x_1)\big)\right] - \frac{2I}{d\kappa pL}\cos(py_1)\right)\frac{\sinh\,(p(x_2-x_1))}{cosh(p(x_2-W))}$$