
MULTI-SCALE MATERIAL MODELLING OF GLASS-CERAMICS

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September 22, 2015

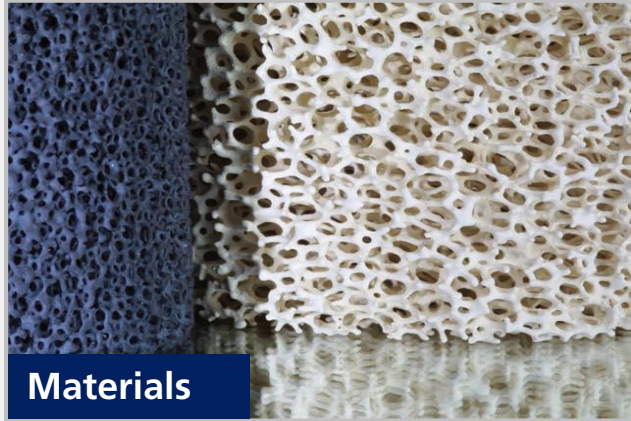
11th International Workshop Direct and Inverse Problems on Piezoelectricity



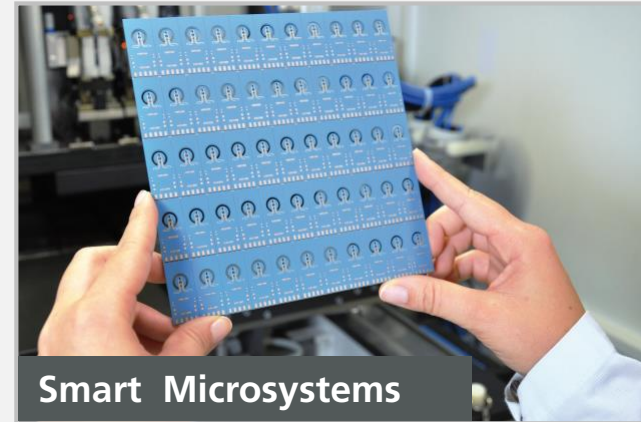
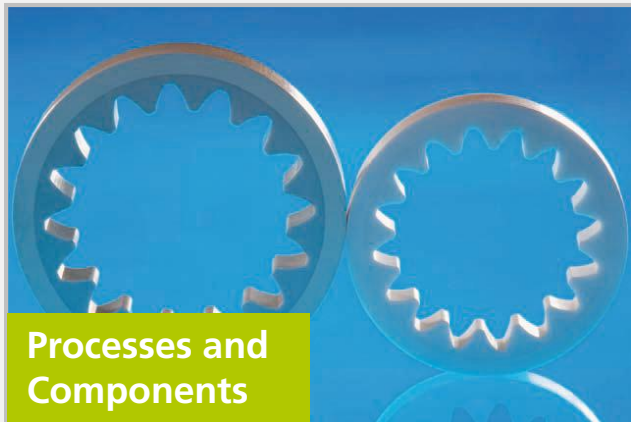
Fraunhofer IKTS in Profile

Research Fields

Structural ceramics



Functional ceramics



Dresden



(1)



(2)

Dresden

CONTENT

- Motivation
- Basics
- Simulation
 - Multi-Scale Model
 - Material Description
 - Geometric Representation
- Results
- Fabrication of Glass-Ceramic
- Summary

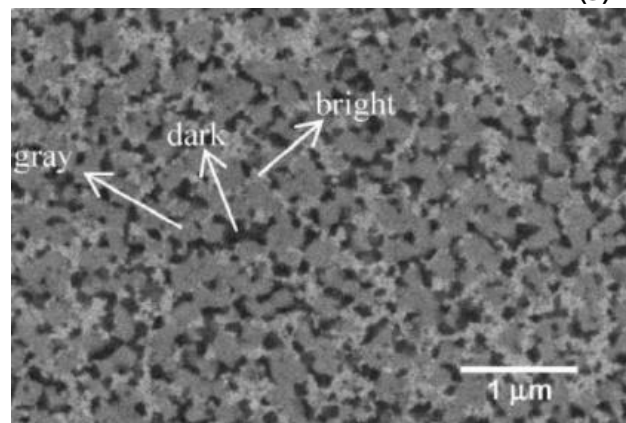
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Motivation



(1)



(3)

Exit from nuclear and fossil-fuel energy

Elevated demand for high voltage ceramic capacitors

Increase of dielectric medium

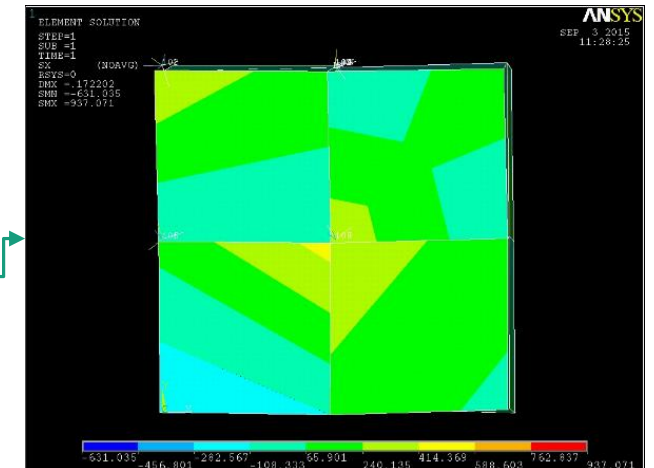
New material:
Glass-Ceramic

Understand microstructure-properties-
correlation

Improve material properties



(2)

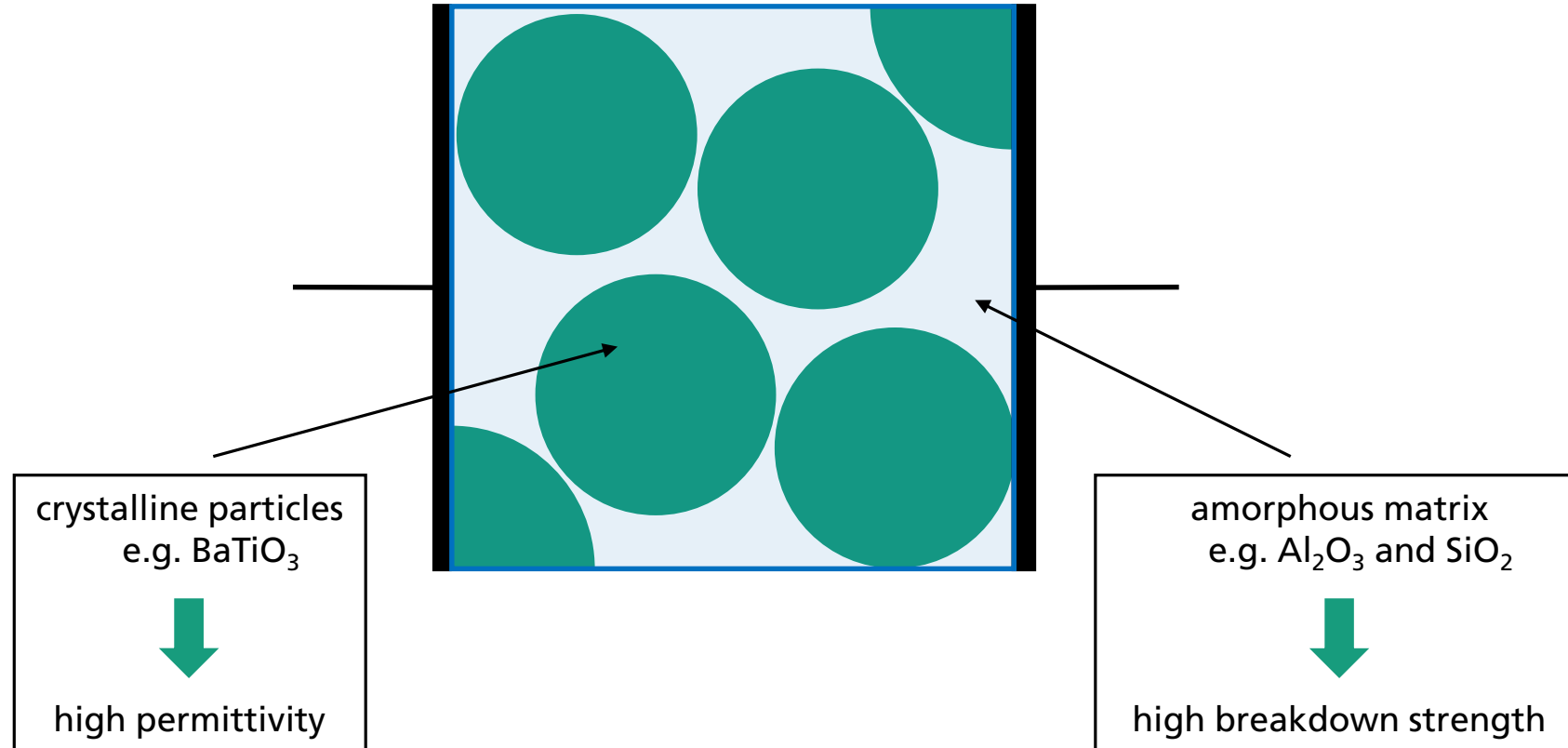


(1) <http://www.fotocommunity.de/pc/pc/display/29499476> by hansen-net

(2) <https://commons.wikimedia.org/wiki/File:HoheneckUmspannwerkTrafos.jpg>

(3) Luo, J.; Du, J.; Tang, Q.; Mao, C. *IEEE Trans. Electron Devices* **2008**, *55* (12), 3549–3554.

Motivation: Glass-Ceramic



Motivation: High Energy Density and High Capacity per Volume

Energy density as function of relative permittivity and dielectric breakdown strength

$$\frac{\Pi_{max}^{el}}{V} = \frac{1}{2} \kappa \cdot E_d^2$$

$$\frac{C_{max} \cdot U^a}{V} = \kappa \cdot E_d^2$$

Π_{max}^{el} = maximal energy

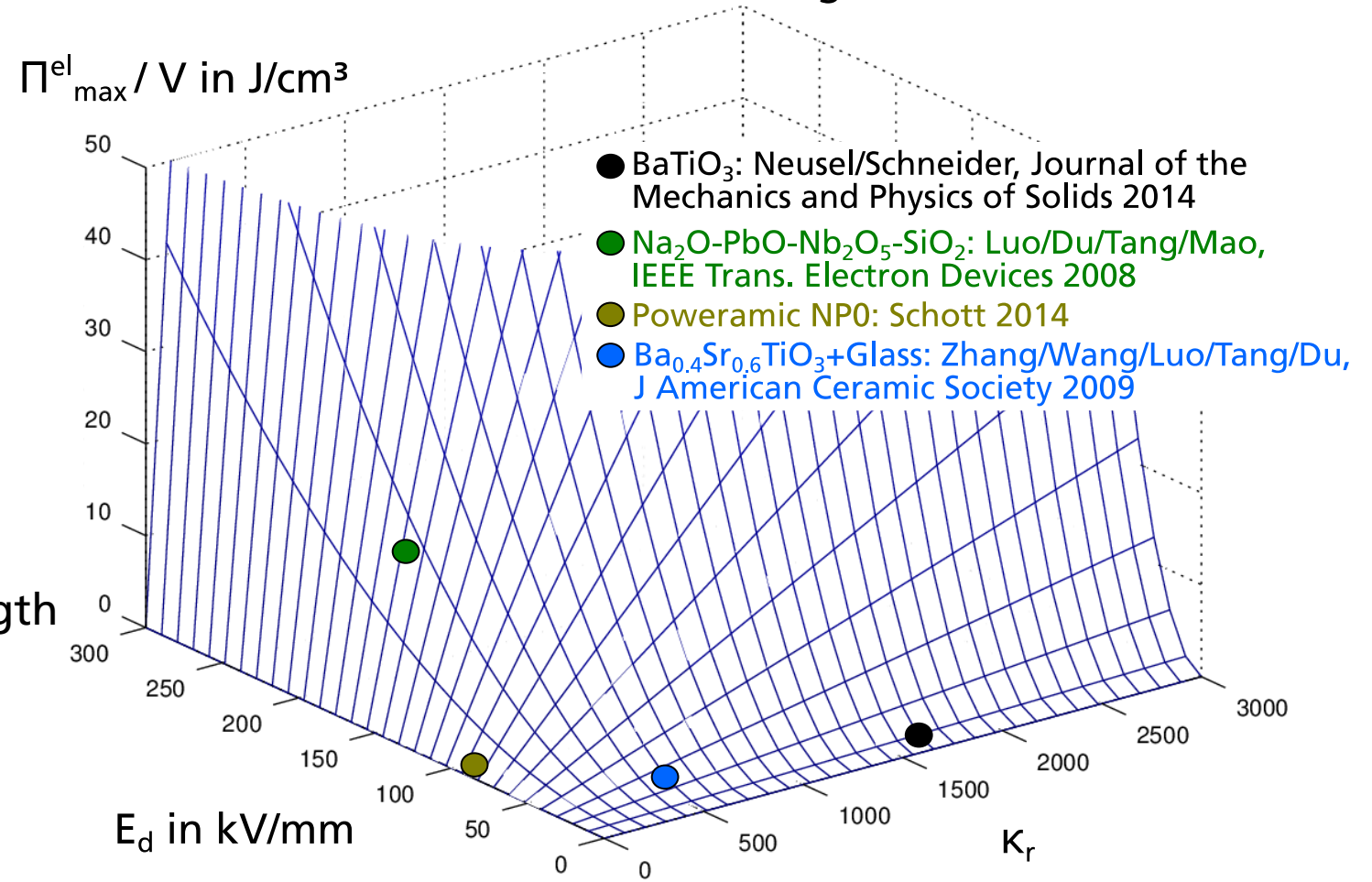
V = volume

κ = permittivity

E_d = dielectric breakdown strength

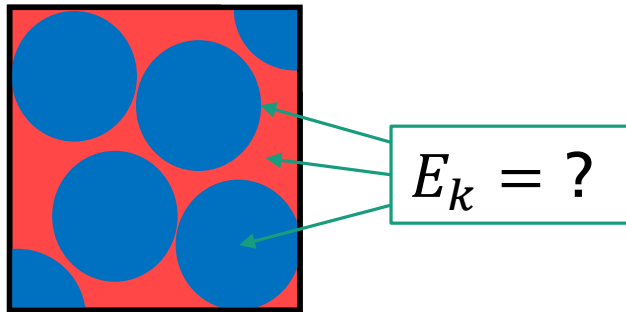
C_{max} = maximal capacity

U^a = applied voltage

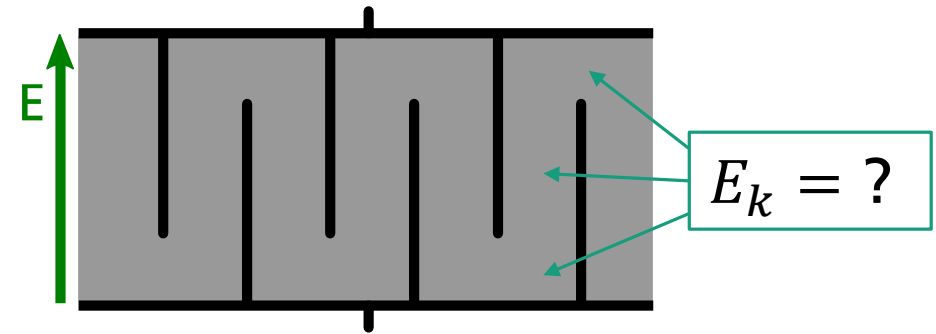


Motivation: Simulation of Glass-Ceramic

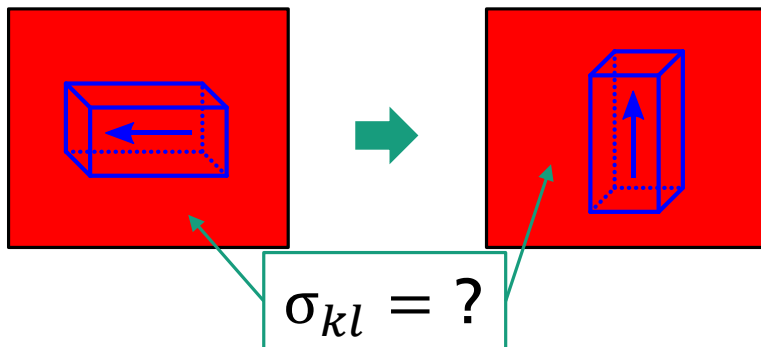
Distribution of electric field in microstructure



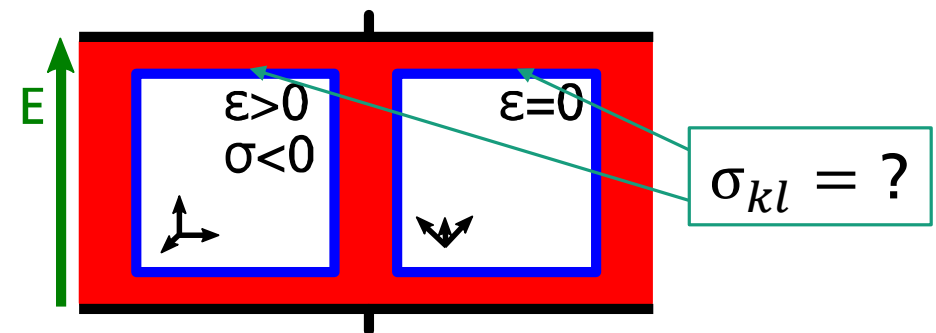
Inhomogeneous electric fields in component



Switching behavior: effect of matrix on particle



Mechanical tensions in microstructure caused by ferroelectric and piezoelectric grains



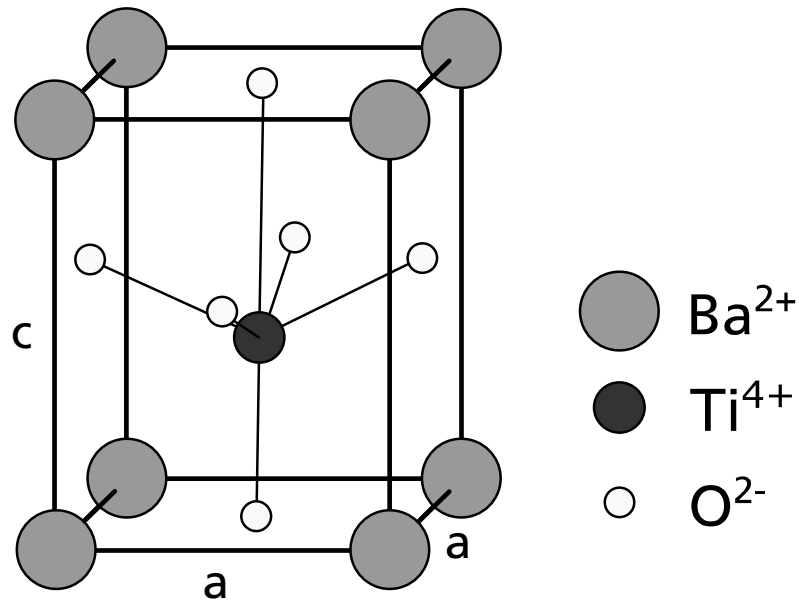
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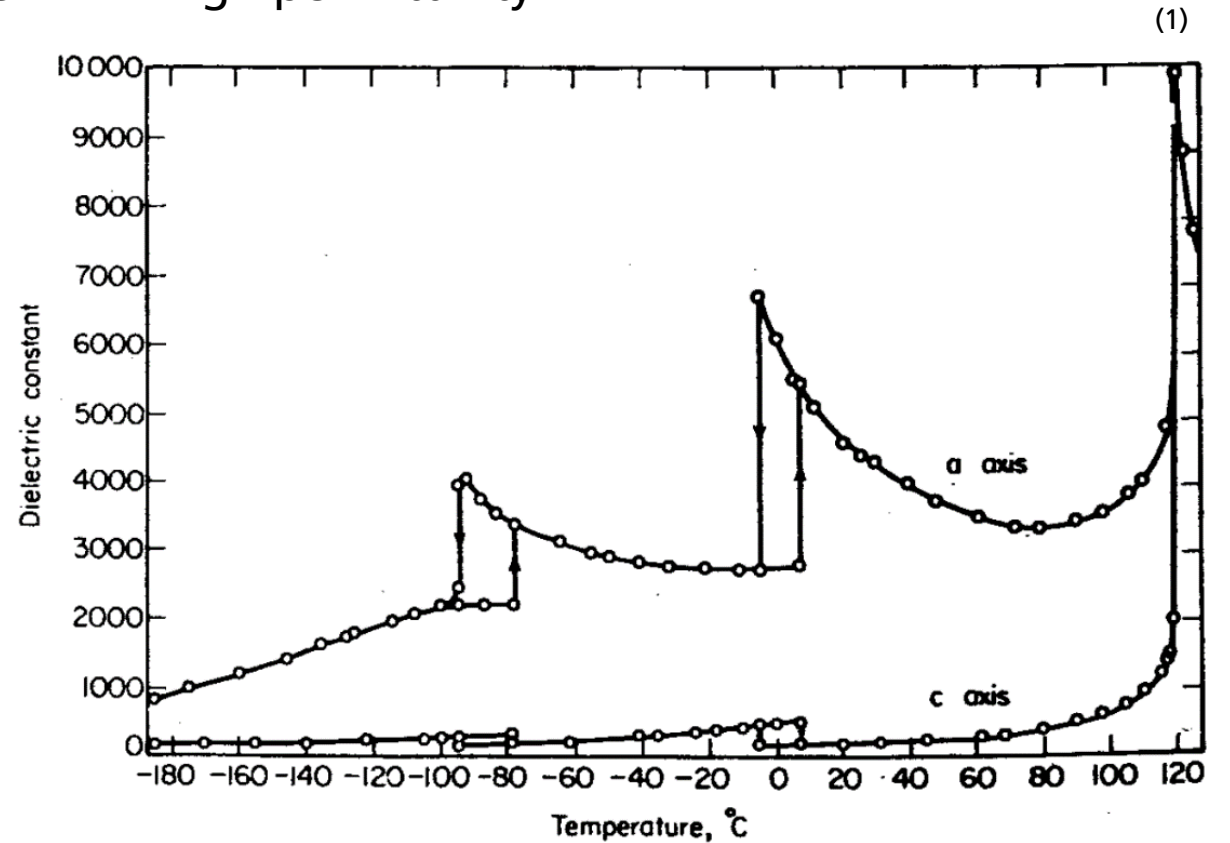
Basics: Barium Titanate

- Tetragonal unit cell below Curie-Temperature

Perovskite structure



- High permittivity



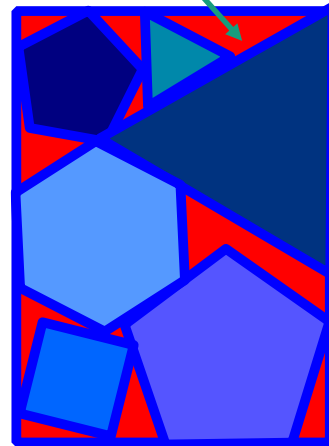
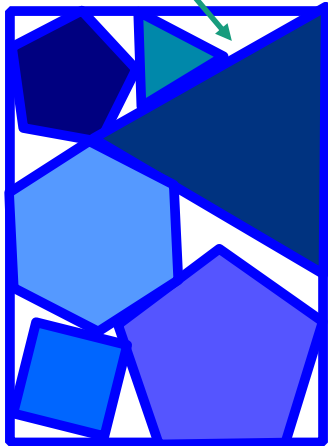
Basics: Advantages of Glass-Matrix

■ Dielectric properties:

Increase of breakdown strength

Air: 3 kV/mm ⁽³⁾

Glass: > 420 kV/mm ⁽²⁾



■ Mechanical properties:

- Decrease of micro cracks, voids and pinholes ⁽¹⁾
- Reduction of residual stresses ⁽¹⁾
- Increase of formability ⁽¹⁾

■ Nanosized grains of glass-ceramic:

- No switching
- No hysteresis
- Better mechanical strength

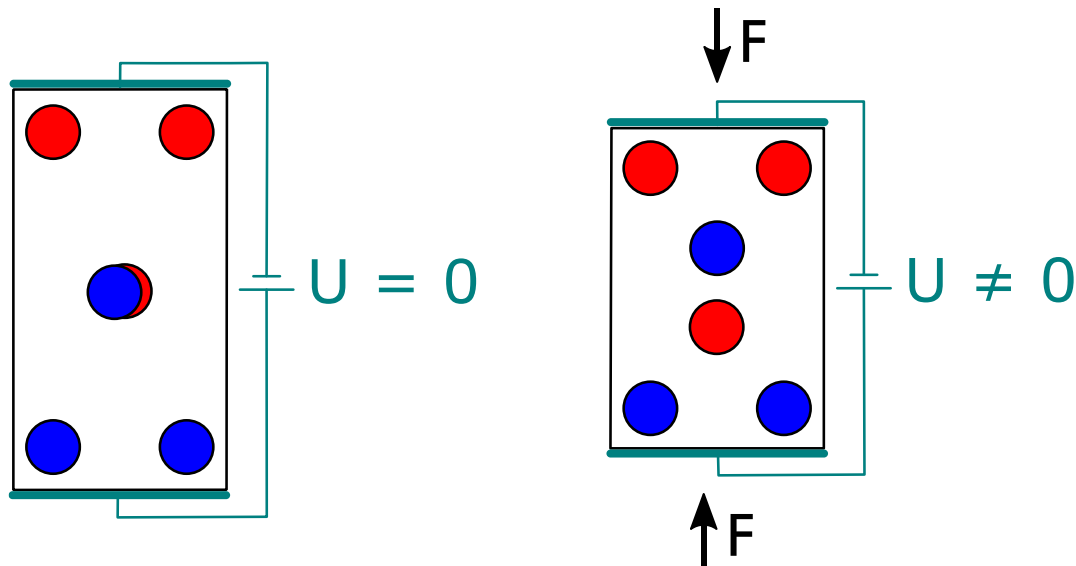
(1) Yao, K.; Zhu, W. *Thin Solid Films [Online]* 2002, 408 (1–2), 11–14.

(2) Smith, N. J.; Rangarajan, B.; Lanagan, M. T.; Pantano, C. G. *Materials Letters [Online]* 2009, 63 (15), 1245–1248.

(3) Küchler, A. *Hochspannungstechnik. Grundlagen - Technologie - Anwendungen*; VDI-Buch; Springer Berlin: Berlin, 2009.

Basics: Piezoelectric Effect

- Applying a force F on a piezoelectric material results in an electric tension U



- $\tilde{\varepsilon}_{ij} = \tilde{S}_{ijkl}^E \tilde{\sigma}_{kl} + \tilde{d}_{kij} \tilde{E}_k + \tilde{\varepsilon}_{ij}^r$
- $\tilde{D}_i = \tilde{d}_{ikl} \tilde{\sigma}_{kl} + \tilde{\kappa}_{ik}^\sigma \tilde{E}_k + \tilde{P}_i^r$
 - Macroscopic physical variables
 - $\tilde{\sigma}_{kl}$ (mechanical tension), \tilde{E}_k (electric field), \tilde{D}_i (dielectric displacement)
 - Macroscopic material properties
 - $\tilde{\varepsilon}_{ij}^r$ (remanent strain), \tilde{P}_i^r (remanent polarization)
 - \tilde{S}_{ijkl}^E (compliance), \tilde{d}_{kij} (piezoelectric coupling tensor), $\tilde{\kappa}_{ik}^\sigma$ (relative permittivity)

CONTENT

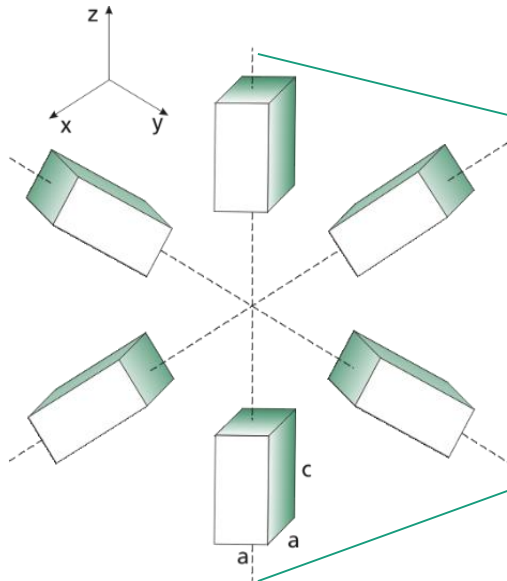
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Simulation: Multi-Scale Model

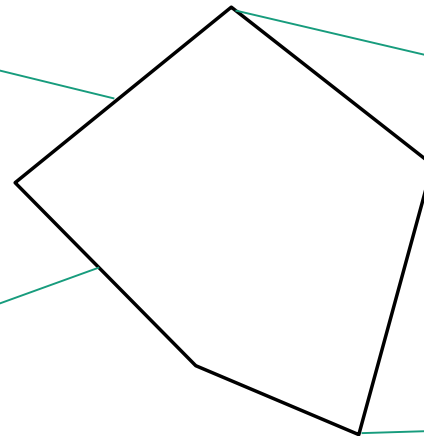
■ Micro scale
↓
Unit cell

■ Orientation

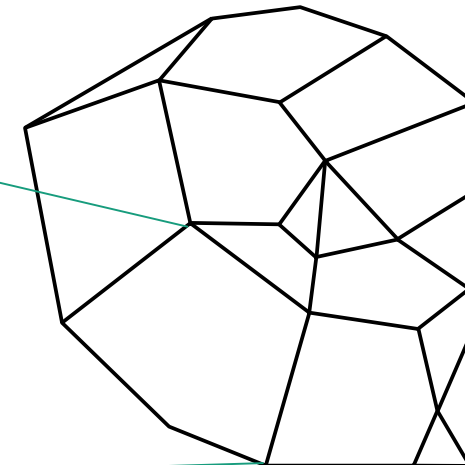
■ Properties



■ Meso scale
↓
Grain



■ Macro scale
↓
Polycrystal



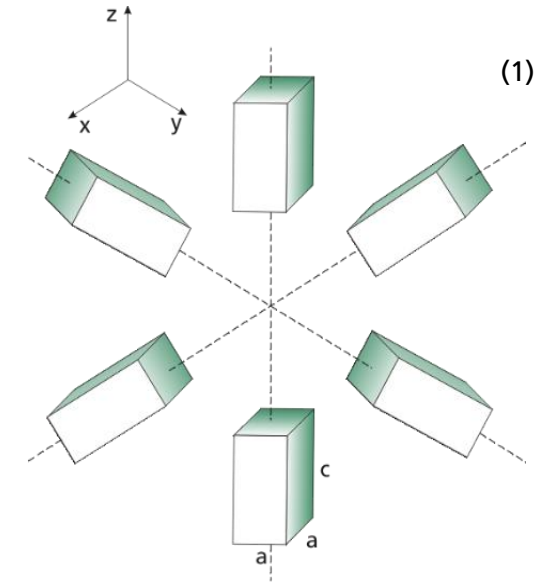
Simulation: Multi-Scale Model

Micro Scale - Unit Cell

- 6 possible poling directions
 - corresponding to volume fractions v^α
 - $\sum_{\alpha=1}^6 v^\alpha = 1$
- Microscopic material properties

$$\hat{\varepsilon}^{S,+z} = \begin{pmatrix} -0.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \varepsilon^S, \quad \hat{p}^{S,+z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot p^S,$$

$$\hat{\kappa}^{\sigma,+z} = \begin{pmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{11} & 0 \\ 0 & 0 & \kappa_{33} \end{pmatrix}, \quad \hat{d}^{+z,T} = \begin{pmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{S}^{E,+z} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{pmatrix}$$



Simulation: Multi-Scale Model

Meso Scale - Grain

- Mesoscopic physical variables

$$\sigma_{kl}, E_k$$

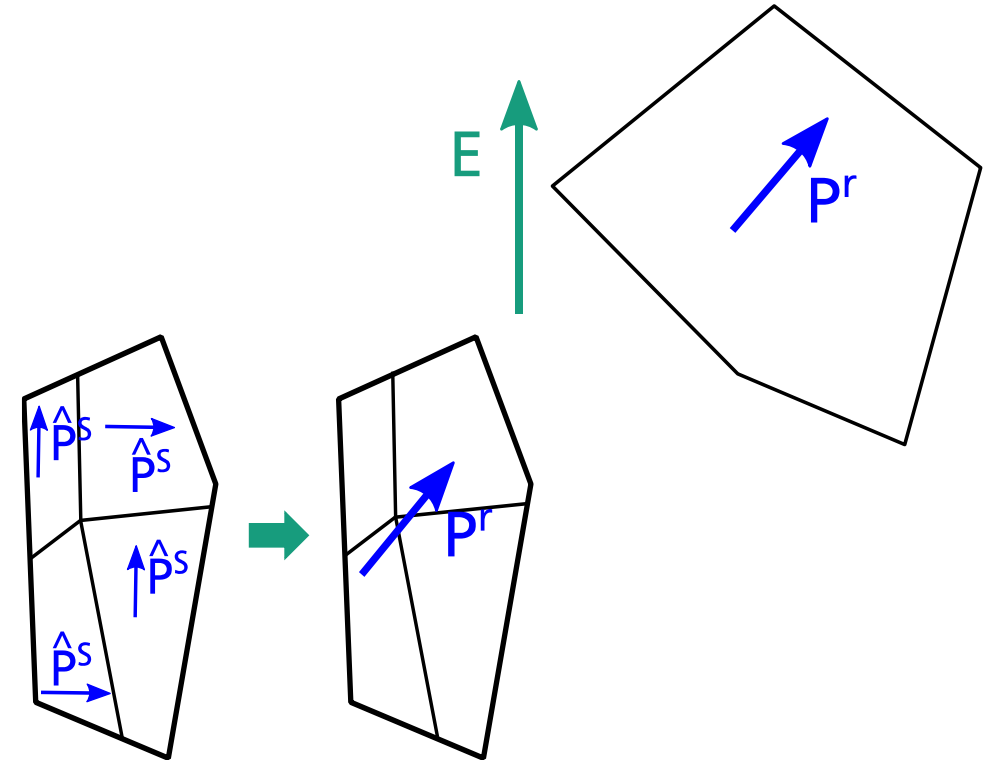
- Mesoscopic material properties

Homogenization

$$P_i^r = \sum_{\alpha=1}^6 \hat{P}_i^{S,\alpha} \cdot v^\alpha$$

(analog for $\varepsilon_{ij}^r, \kappa_{ik}^\sigma, S_{ijkl}^E, d_{kij}$)

v^α = volume fraction of domains polarized in directions +x, +y, +z, -x, -y, -z



Simulation: Multi-Scale Model

Macro Scale - Polycrystalline Compound

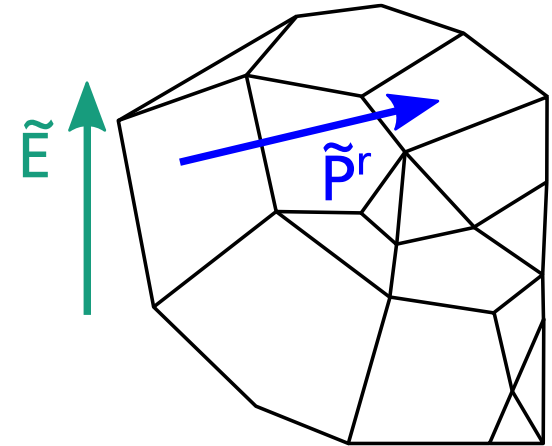
- Macroscopic physical variables

$$\tilde{\sigma}_{kl}, \tilde{E}_k$$

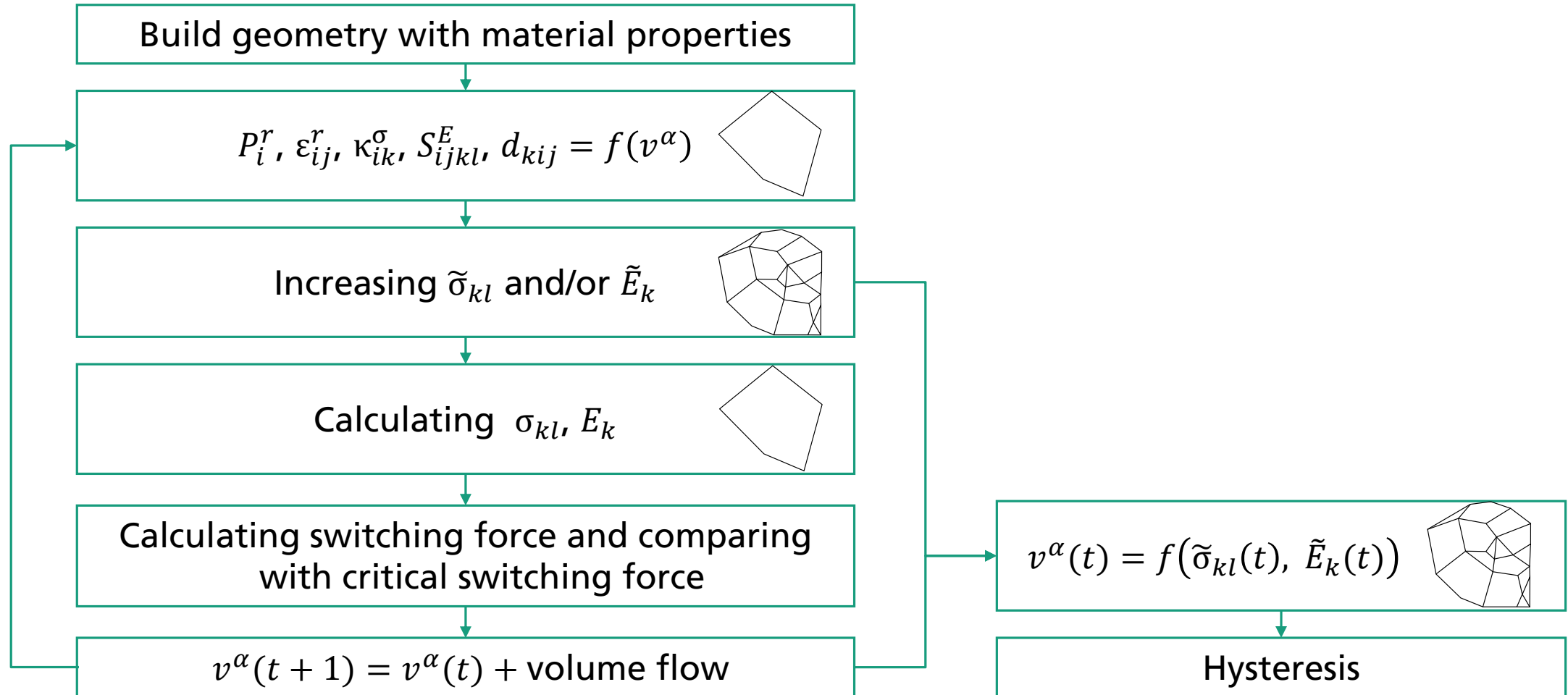
- Macroscopic material properties

Homogenization

- Equal sized grains: $\tilde{P}_i^r = \sum_i P_i^r$ (analog for $\tilde{\varepsilon}_i^r, \tilde{S}_{ijkl}^E, \tilde{d}_{kij}, \tilde{\kappa}_{ik}^\sigma$)
- Variable sized grains: FEM



Simulation: Process

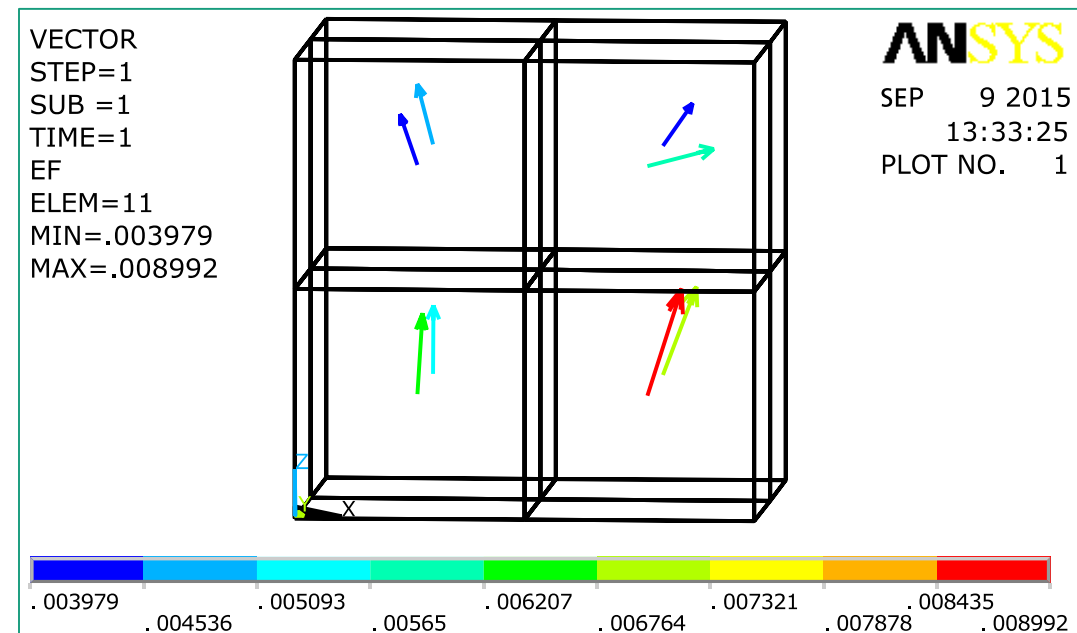
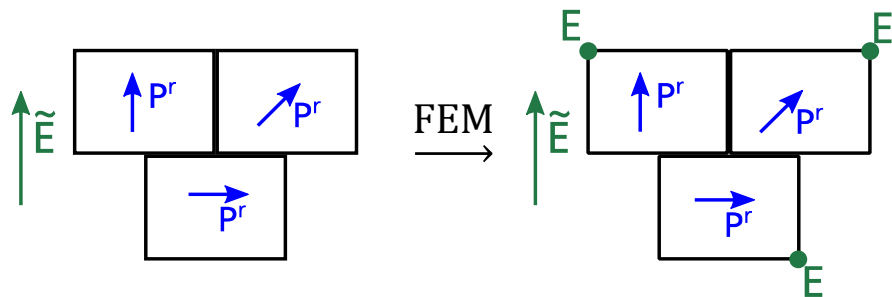


Simulation: Material Description

Mesoscopic Loads

Mesoscopic loads calculated out of macroscopic loads and mesoscopic material properties by FEM

$$\tilde{\sigma}_{kl}, \tilde{E}_k, P_i^r, \varepsilon_{ij}^r, \kappa_{ik}^\sigma, S_{ijkl}^E, d_{kij} \xrightarrow{\text{FEM}} \sigma_{kl}, E_k$$



Simulation: Material Description

Volume Flows – Switching Criterion

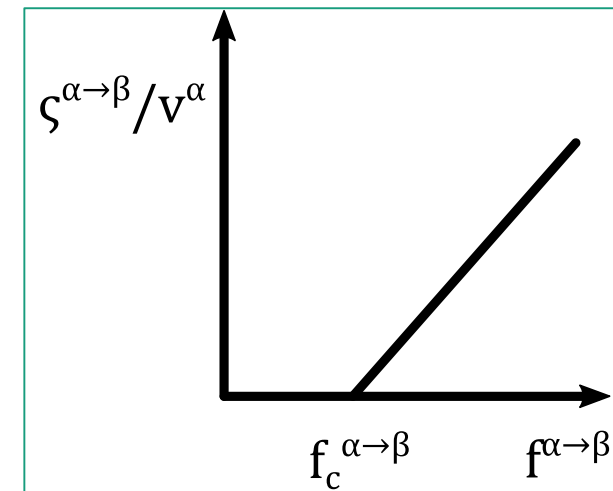
- $f^{\alpha \rightarrow \beta} = \sigma_{ij} \Delta \hat{\varepsilon}_{ij}^{S, \alpha \rightarrow \beta} + E_i \Delta \hat{P}_i^{S, \alpha \rightarrow \beta}$ ^(1, 2) driving force for switching system $\alpha \rightarrow \beta$

- Switching rule: $\zeta^{\alpha \rightarrow \beta} = \begin{cases} k_s^{\alpha \rightarrow \beta} \left(\frac{f^{\alpha \rightarrow \beta}}{f_c^{\alpha \rightarrow \beta}} - 1 \right) v^\alpha & \text{if } f^{\alpha \rightarrow \beta} \geq f_c^{\alpha \rightarrow \beta} \text{ }^{(3)} \\ 0 & \text{else} \end{cases}$

$\zeta^{\alpha \rightarrow \beta}$ = volume flow from domain orientation α to domain orientation β

$k_s^{\alpha \rightarrow \beta}$ = creep constant / inverse viscosity

$f_c^{\alpha \rightarrow \beta}$ = critical driving force



(1) Landis, C. M. *Current Opinion in Solid State and Materials Science* **2004**, 8 (1), 59–69.

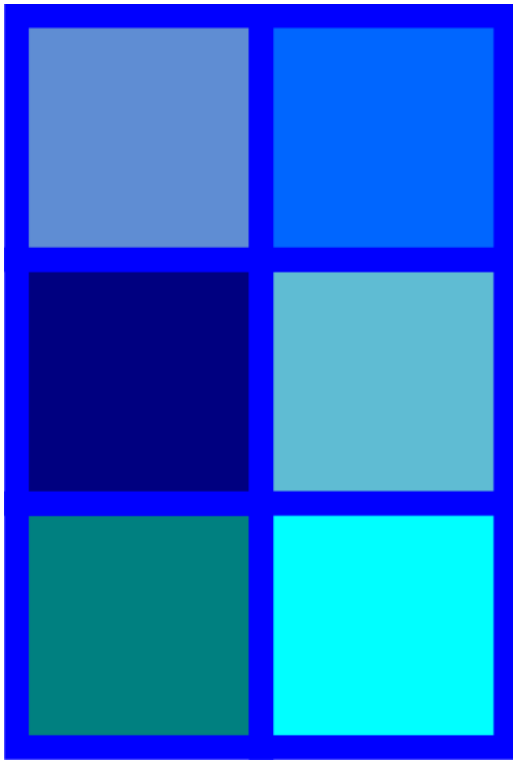
(2) Kessler, H.; Balke, H. *Journal of the Mechanics and Physics of Solids [Online]* **2001**, 49 (5), 953–978.

(3) Neumeister, P. *Mikromechanische Modellierung morphotroper PZT-Keramiken*. 2011

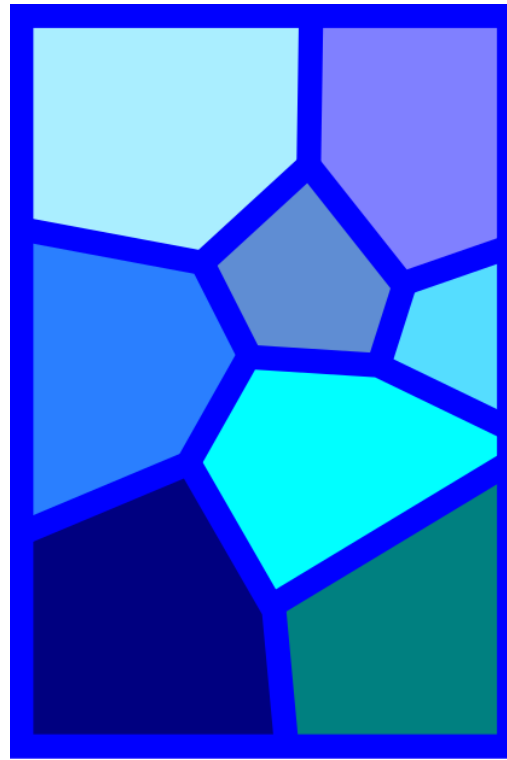


Simulation: Geometric Representation

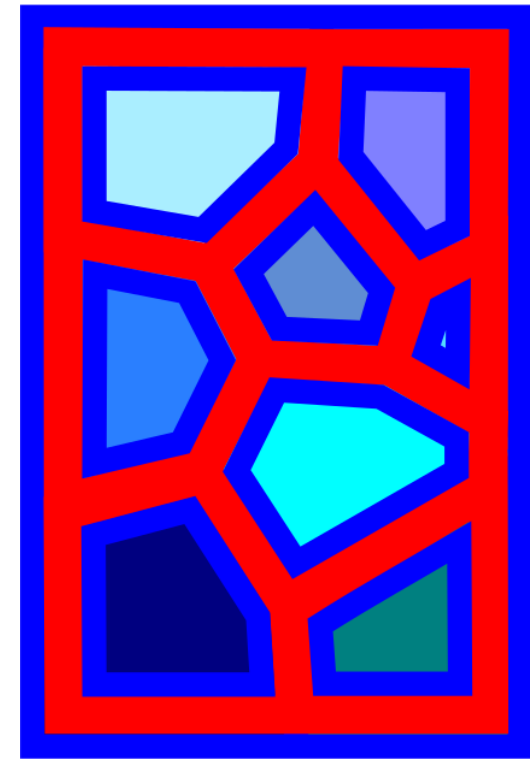
■ BaTiO₃ polycrystal cubed



■ BaTiO₃ polycrystal voronoi



■ Glass-ceramic voronoi

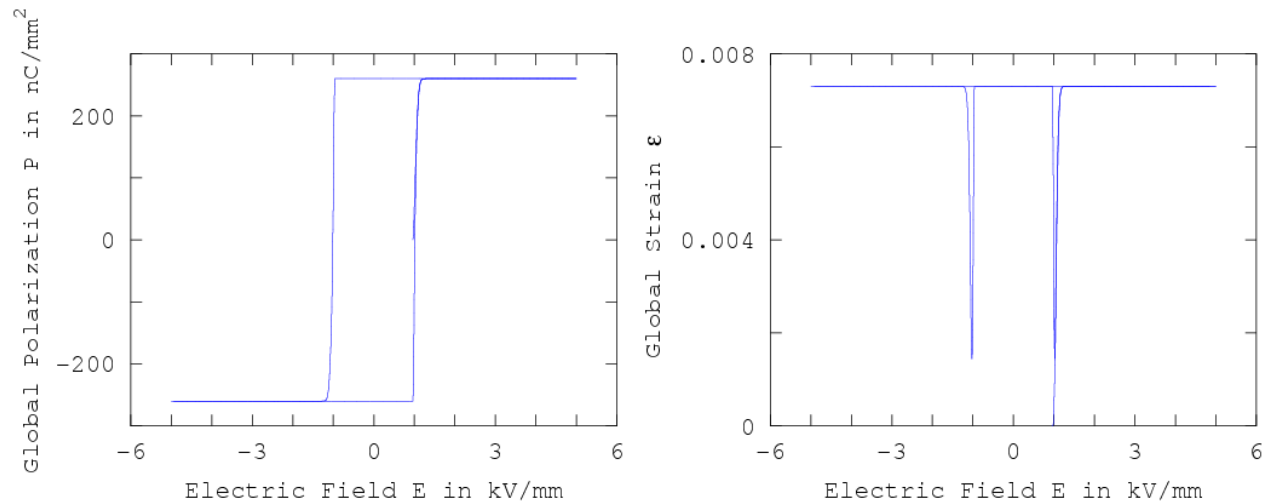


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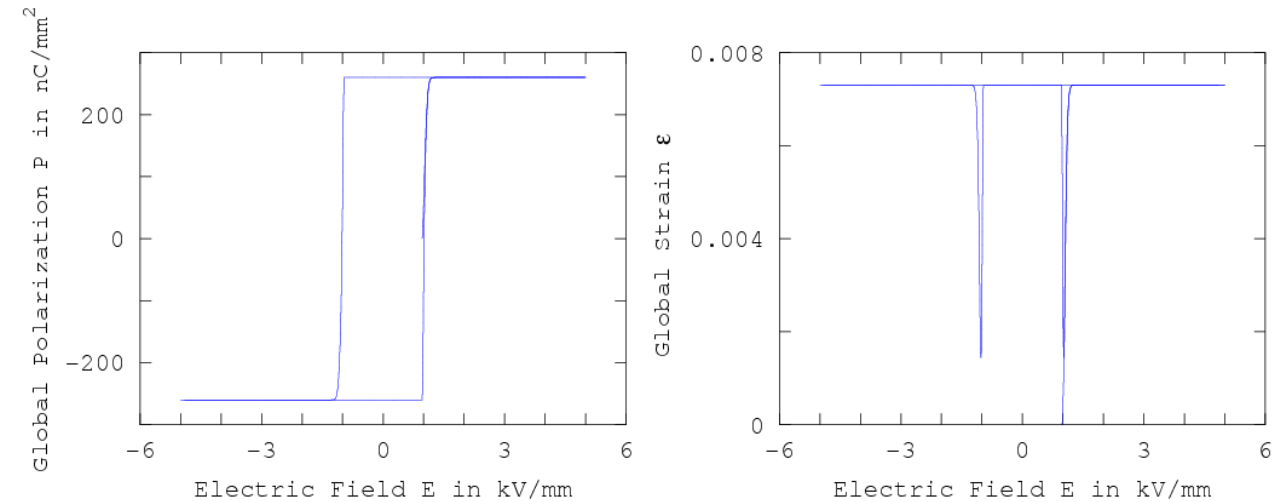
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Results Validation

- Single crystal one element
<100>-orientation



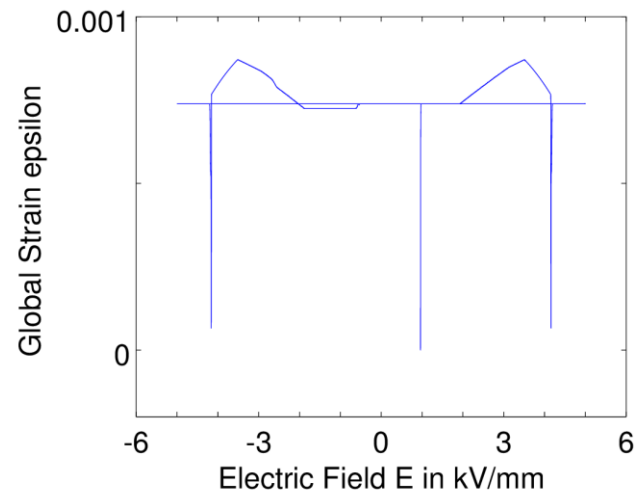
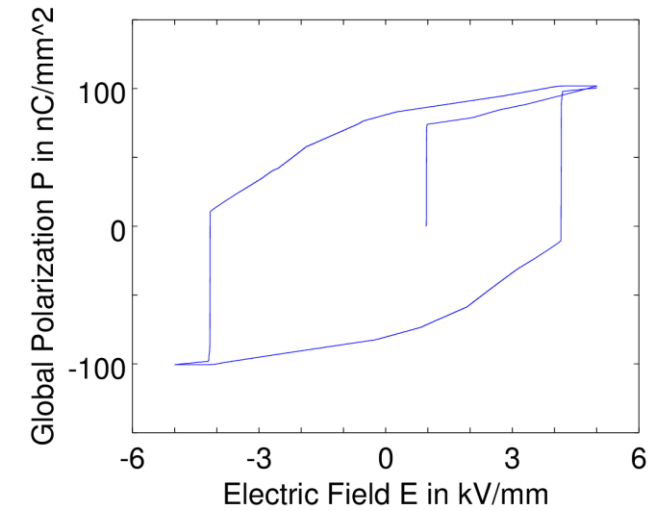
- Single crystal 8 elements
All elements <100>-orientation



Results

- Polycrystal 8 elements
Arbitrarily oriented elements

- Polycrystal 27 elements
Arbitrarily oriented elements



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Fabrication of Glass-Ceramic: Composition and Procedure

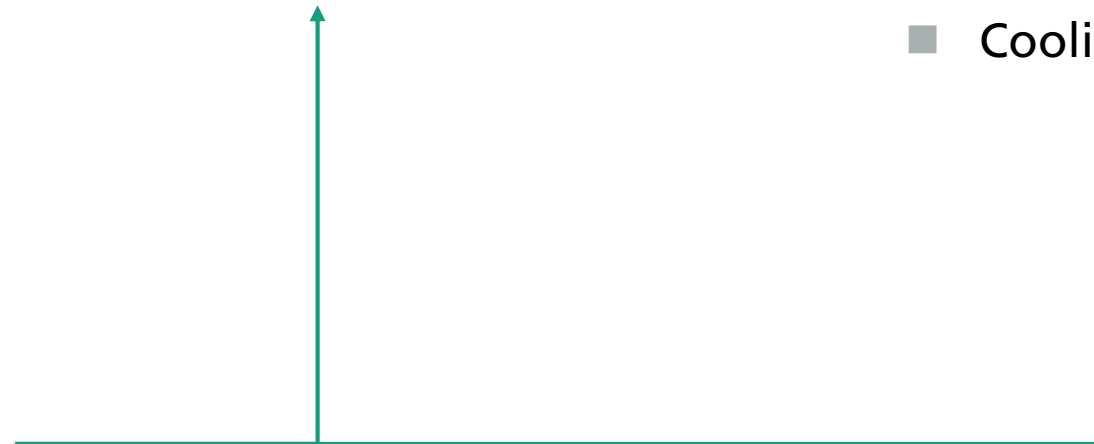
■ Glass: Composition and Melting⁽¹⁾

Component	Mol%
Al ₂ O ₃	10
BaO	38.2
SiO ₂	20
TiO ₂	31.8

- Melting for 2 h at 1550 °C
- Casting onto brass plate
- Cooling from 650 °C (1 K/min)

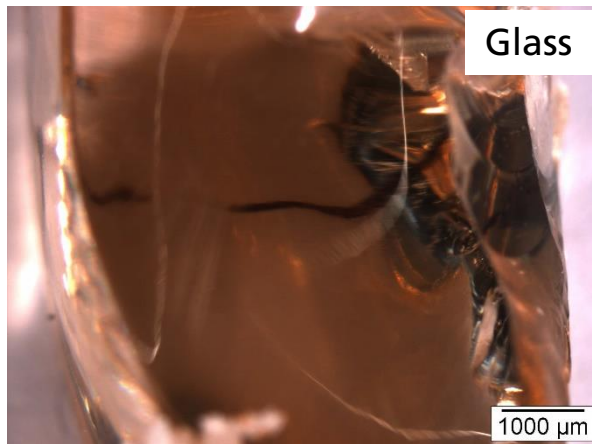
- ## ■ Glass-Ceramic Crystallized:⁽²⁾
- Heating to 950 °C (1K/min)
 - Holding for 15 h in air
 - Cooling (5 K/min)

- ## ■ Glass-Ceramic Sintered:⁽³⁾
- Grinding to powder
 - Heating to 870 °C (3 K/min)
 - Holding for 30 min in air
 - Cooling (3 K/min)

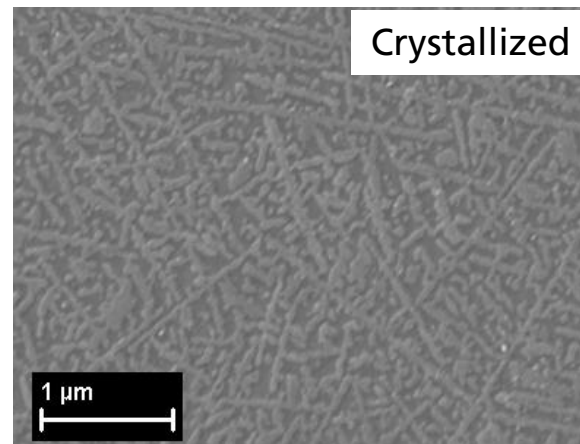


Fabrication of Glass-Ceramic: Dielectric Properties and Microstructure

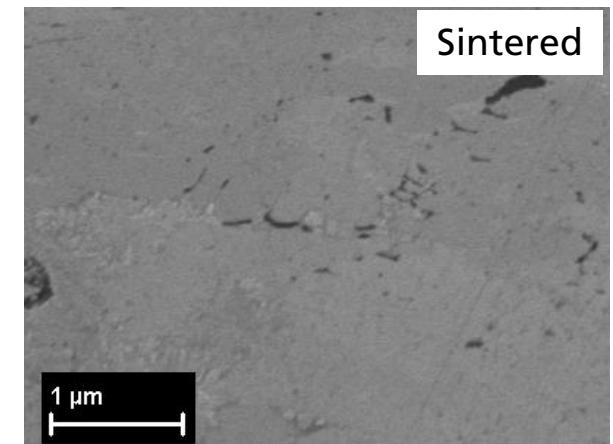
	Capacity	Loss Tangent	Relative Permittivity	Dielectric Breakdown Strength	Coercivity	Polarization
	[nF] (1kHz)	[1e-4]	[-]	[kV/mm]	[kV/mm]	[$\mu\text{C}/\text{cm}^2$]
Crystallized	0.011	930	588	6.44	0.6152	1.0681
Sintered	0.001	9921	53	9.57	0.2610	0.1341



(1)



(2)



(3)

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Summary

■ Glass-Ceramic Material

Dielectric material for capacitor application

- High dielectric breakdown strength
- High energy density

■ Simulation

Energy based switching criterion

- Validation of code on single crystal
- First calculations polycrystal with 8 and 27 cubed elements

■ Fabrication

- Crystallized and Sintered Samples
- Good relative permittivity, poor dielectric breakdown strength

Acknowledgement



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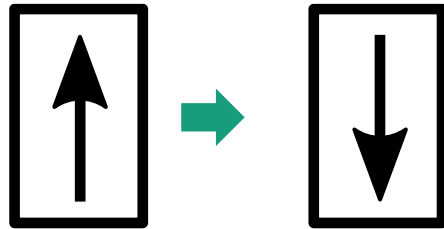
„Werkstoffmechanik der Funktionskeramiken:
Von der Mikrostruktur zum Bauteilverhalten“

Thank you for your attention!

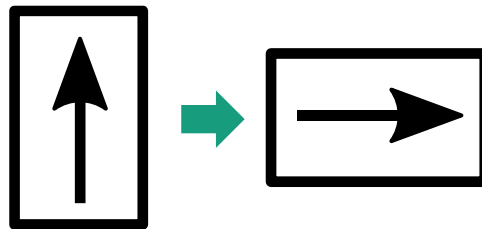
Basics: Ferroelectric Domains

- Regions with unit cells polarized in the same direction
- Domains separated by domain walls
- Movement of domain walls

■ 180° switching

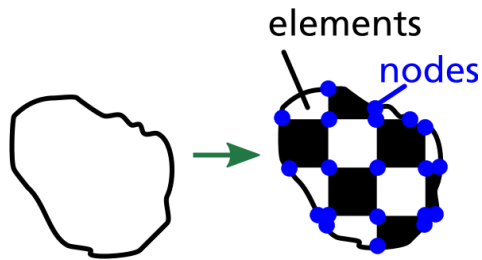


■ 90° switching



Basics: FEM (Finite Element Method)

- Numerical method for calculating physical fields
- Procedure:
 - Segmentation of desired volume into small elements



- Approach for each element
- Linear combination of element approaches
- Insertion into differential equation
- Solving equation system by aid of boundary, transition and initial conditions

Simulation: Material Description

Volume Flows – Switching Criterion I

- $f^{\alpha \rightarrow \beta} = \sigma_{ij} \Delta \hat{\varepsilon}_{ij}^{S, \alpha \rightarrow \beta} + E_i \Delta \hat{P}_i^{S, \alpha \rightarrow \beta}$ ^(1, 2) driving force for switching system $\alpha \rightarrow \beta$

- $f^{\alpha \rightarrow \beta} = E_i \left(\hat{P}_i^{S, \beta} - \hat{P}_i^{S, \alpha} \right)$

- $f^{\alpha \rightarrow \beta} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} E \cdot \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \right) P^S$

- $f^{\alpha \rightarrow \beta} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} E \cdot \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \right) P^S$

