A geometrically and material nonlinear piezoelectric 3D-beam finite element formulation including warping effects

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The paper is concerned with a three-dimensional piezoelectric beam formulation and its finite element implementation. The developed model considers geometrically and materially nonlinear effects. An eccentric beam formulation is derived based on the Timoshenko kinematics. The kinematic assumptions are extended by three additional warping functions of the cross section. These functions follow from torsion and piezoelectrically induced shear deformations. The presented beam formulation incorporates large displacements and finite rotations and allows the investigation of stability problems. The finite element model has two nodes with nine mechanical and five electrical degrees of freedom. It provides an accurate approximation of the electric potential which is assumed to be linear in the direction of the beam axis and quadratic within the cross section. The mechanical degrees of freedom are three displacements, three rotations and three scaling factors for the warping functions. The latter are computed in a preprocess by solving a two-dimensional in-plane equilibrium condition with the finite element method. The gained warping patterns are considered within the integration through the cross section of the beam formulation. With respect to material nonlinearities, which arise in ferroelectric materials, the scalar Preisach model is embedded in the formulation. This model is a mathematical model for the general description of hysteresis phenomena. Its application to piezoelectric materials leads to a phenomenological model for ferroelectric hysteresis effects. Here, the polarization direction is assumed to be constant, which leads to unidirectional constitutive equations. Some examples demonstrate the capability of the proposed model.

Keywords:
three-dimensional beam; warping; piezoelectricity; shear actuator; geometrically nonlinear; ferroelectric hysteresis; Preisach model; finite element