Abstract—Current injection into solar cells may occur if cells in modules are connected in parallel without the protection of string diodes. This current injection can cause heating of the cell, which increases the recombination currents and, thus, the current injection. In this manner, a self-feeding process called thermal runaway is started. In concentrator photovoltaics modules, thermal runaway can cause substantial damage. In this paper, a model is introduced that calculates the conditions causing thermal runaway. This model is based on the two-diode model. As inputs for the model, three dependences were experimentally determined from dark IV measurements on the triple-junction cell: 1) series resistance on cell temperature, 2) saturation currents for each of the three junctions on cell temperature, and 3) cell temperature on the injected current. The model was tested by comparing the simulated and measured temperature increase in a triple-junction cell and their dependencies on the applied voltage. A reasonable agreement between the experiment and model was found whereby the voltage at which a thermal runaway occurred differs slightly by 0.04 V. The model was then applied to estimate the dependence of an expected temperature increase in a shaded solar cell on the number of cells connected in parallel and on the concentration factor of the sunlight.

Index Terms—Concentrator photovoltaics (CPV) modules, modeling, simulation, solar cell interconnection, SPICE, thermal runaway.

I. INTRODUCTION

PHOTOVOLTAIC modules consist of a certain number of solar cells. These solar cells are interconnected in series and/or in parallel to deliver the output power of the module. The choice of series and/or parallel connection depends on the electrical characteristics of the concentrator photovoltaics (CPV) module used. A series connection compensates for voltage differences between the solar cells but suffers power losses from current differences, whereas a parallel connection compensates for current differences between the solar cells but suffers power losses from voltage differences. The problem with a series connection in PV modules is that a reverse bias can be applied to a solar cell if the module is operating near the short-circuit current. This reverse bias causes heating in the cell from power dissipation. In the worst case, a hot spot [1] is caused and damages the cell. Power dissipation can be reduced through the usage of bypass diodes [2], [3]. The usage of bypass diodes causes no further power losses. The problem with a parallel connection is that a current can be injected into a solar cell if the module operates near the open-circuit voltage. This current injection heats the cell and, in the worst case, can cause thermal runaway, which possibly damages the module. Current injection can be avoided by the usage of string diodes or blocking diodes [4]. However, the usage of string diodes causes a significant power loss. Therefore, usage of a string diode should be avoided, and current injection should be accepted. However, current injection increases the temperature of the solar cell, and in turn, this temperature increase increases the current injection because of the increase of the recombination currents. In this manner, a self-feeding process called thermal runaway is started. However, current injection does not trigger thermal runaway under all conditions.

Thermal runaway occurs if the current injection is above a certain level. The question is what this level is. Current injection itself is determined by the number of cells connected in parallel and by the concentration factor of the sun light used in the PV modules. Therefore, the risk of causing thermal runaway in CPV modules is higher than it is in PV modules. Therefore, one possible method to prevent thermal runaway in CPV modules is to avoid parallel connections of solar cells. In contrast, parallel connections of solar cells can reduce power losses because of the differences in current generation in the individual solar cells [5]. For this reason, a model to determine the conditions under which thermal runaway will occur is important. This model must calculate the $I-V$ characteristics of interconnected solar cells and must additionally predict the resulting temperature rise from power dissipation. There are already publications that concern modeling of the $I-V$ characteristics of solar cells connected in series or in parallel and that consider the effects of current mismatch between solar cells. These publications also determine the power dissipation that results within the solar cells with lower currents. For instance, Bishop [4] introduced a model called PVNet, which numerically solves the one-diode equation to calculate various connection schemes. Bishop investigated the effects of bypass and string diodes on the $I-V$ characteristics of connected solar cells. Quaschning and Hanitsch [6] introduced a similar model by numerically solving the two-diode equation instead of the one-diode equation. However, the modeling approaches do not consider the resulting temperature increase from power dissipation in shaded solar cells. In this paper, a thermal runaway model to calculate the heating of shaded solar cells connected in parallel within CPV modules is...
introduced and is validated through experimental data. The initial results were previously shown in [7]. As an application of the thermal runaway model, we determine the maximum number of parallel connected solar cells and the maximum concentration factor of sunlight that can be used for a CPV module before thermal runaway is likely to occur in one completely shaded solar cell.

II. DESCRIPTION OF THE THERMAL RUNAWAY MODEL

A. Modeling Approach for the Triple-Junction Solar Cell

A commonly used approach to model the \( I-V \) characteristics of multijunction solar cells uses the two-diode model for each subcell in an electrical circuit. Fig. 1 shows such an electrical circuit for a triple-junction solar cell. Each of the three subcells is represented by two diodes and a current source connected in parallel. All series resistances are combined to give one lumped series resistance. The parallel connection of several solar cells used in CPV modules is realized by connection of several of the elements shown in Fig. 1 at node 1 and node 2 [5], [8]. The current–voltage characteristics of a triple-junction solar cell, and Silvestre et al. [8] used a similar SPICE-based model to simulate the behavior of PV modules.

The realization of the electrical circuit shown in Fig. 1 with LTSpice requires input parameters of the series resistance \( R_S \), the dark recombination current densities \( J_{01} \) and \( J_{02} \), and the short-circuit current densities \( J_{SC} \) of all three subcells. \( J_{01} \), \( J_{02} \), and \( J_{SC} \) depend on the temperature, the band gap, and the quality of the materials of the subcell. \( J_{SC} \) also depends on the spectral irradiance illuminating the solar cell. The temperature dependence of \( J_{01} \) and \( J_{02} \) on the energy gap and on the temperature \( T \) were considered in the thermal runaway model through (1)–(3). Equation (1) describes the absolute temperature dependence of the energy gap, \( E_{\text{gap}}(T) \), as introduced by Varshni in [12]. In this paper, the parameters \( E_0 \), \( \alpha \), and \( \beta \) used for the Ga\(_{0.50}\)In\(_{0.50}\)P, Ga\(_{0.99}\)In\(_{0.01}\)As, and Ge subcells were taken from Levinshtein et al. [13], [14].

\[
E_{\text{gap}}(T) = E_0 - \alpha T^2/(T + \beta). \tag{1}
\]

Subsequently, the dark recombination current densities \( J_{01} \) and \( J_{02} \) were calculated with (2) and (3), which were published by Reinhardt et al. [15]. In these equations, \( k \) is the Boltzmann constant, and \( E_{\text{Gap}}(T) \) is the temperature-dependent energy gap described by (1)

\[
J_{01}(T) = k_{01} \cdot T^{3.0} \exp \left[ - \frac{E_{\text{gap}}(T) - \Delta E_{\text{gap}}}{kT} \right] \tag{2}
\]

\[
J_{02}(T) = k_{02} \cdot T^{2.5} \exp \left[ - \frac{E_{\text{gap}}(T) - \Delta E_{\text{gap}}}{2kT} \right]. \tag{3}
\]

The parameters \( k_{01} \), \( k_{02} \), and \( \Delta E_{\text{gap}} \) required for (2) and (3) are obtained through a fit of the dark \( J-V \) curves measured at 298 K to the dark \( J-V \) curve calculated with LTSpice for the electrical circuit shown in Fig. 1. This procedure is described in more detail in [5], and the resulting parameters and the calculated dark recombination currents of the three subcells at 298 K are listed in Table I.

Furthermore, the temperature dependence of the series resistance, \( R_S \), is extracted from the dark \( J-V \) curves measured at different temperatures. Fig. 2 displays the derived dark series resistance of the measured triple-junction solar cell at different temperatures. The dark series resistance decreases as temperature increases. For a triple-junction solar cell, Sakurada published a similar dependence of the series resistance on temperature [11]. The experimental data for \( R_S(T) \) were well fitted by an exponential equation shown in Fig. 2. This parameterization for \( R_S(T) \) was used as an input to the model. Please note that a series resistance determined under dark conditions

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**TABLE I**

\( k_{01} \), \( k_{02} \), and \( \Delta E_{\text{gap}} \) derived at 298 K for the top, middle, and bottom subcells and the dark recombination current densities \( J_{01} \) and \( J_{02} \) at 298 K

<table>
<thead>
<tr>
<th>Subcell</th>
<th>( k_{01} ) [A/cm²]</th>
<th>( k_{02} ) [A/cm²]</th>
<th>( \Delta E_{\text{gap}} ) [eV]</th>
<th>( J_{01} ) [A/cm²]</th>
<th>( J_{02} ) [A/cm²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>top</td>
<td>3.2E+00</td>
<td>1.4E-04</td>
<td>-0.006</td>
<td>8.0E-24</td>
<td>6.1E-14</td>
</tr>
<tr>
<td>mid</td>
<td>7.0E-02</td>
<td>7.0E-06</td>
<td>0</td>
<td>1.9E-18</td>
<td>1.1E-11</td>
</tr>
<tr>
<td>bot</td>
<td>1.6E-06</td>
<td>4.0E-06</td>
<td>0</td>
<td>2.7E-10</td>
<td>1.5E-05</td>
</tr>
</tbody>
</table>

---

Fig. 1. Electrical circuit of a triple-junction solar cell using the 2-diode model for each subcell. The current–voltage characteristics of this network are solved with LTSpice [9]. The parallel connection of the solar cells is realized by connecting several of these elements at node 1 and node 2.

Fig. 2. Temperature dependence of the series resistance for a triple-junction solar cell; this dependence was extracted from fits of the two-diode equation to dark \( J-V \) curves for different solar cell temperatures. This dependence is included in the thermal runaway model with the exponential fit shown in the inset of the graph.
I–V \text{ curve is shown in Fig. 3 for curve.} This function is determined empirically, I–V \text{ curve of solar cells connected in parallel.} This dependence is determined \text{ in Fig. 3, the \textit{I–V} curve of the two cells connected in parallel.} One of the cells is shaded. Dashed line: \textit{I–V} curve for two solar cells connected in parallel; one of the solar cells is shaded and is illuminated with a 1000 \text{ W/m}^2, AM1.5 \text{ d spectrum. Both solar cells operate at the same temperature of 298 K. In another curve in Fig. 3, the \textit{I–V} curve of the two cells connected in parallel is shown if the temperature of the shaded cell is increased to 323 K and the temperature of the illuminated cell is maintained at 298 K. Furthermore, in Fig. 3, the \textit{I–V} curves of the shaded cell at cell temperatures of 298 K and at 323 K are shown by a dotted line and by a short dashed line, respectively. The crosses on the \textit{I–V} curves for the shaded solar cell mark the working point for the shaded cell if the system of the two cells connected in parallel is under open-circuit conditions. The increased temperature of the shaded solar cell results in a higher injected current.

is smaller compared with the series resistance determined under light conditions. For this paper, the series resistance under dark conditions is more accurate, because the series resistance is used for the calculation of the \textit{I–V} characteristic of a completely shaded cell.

B. Modeling Approach for the Thermal Runaway Effect

The \textit{I–V} curve exemplarily for two solar cells connected in parallel is calculated with the modeling approach that is described in Section II-A and is shown in Fig. 3. Of course any other parallel and/or series connection of solar cells can be calculated with this modeling approach. Here, it was assumed that one of the two solar cells is shaded, whereas the other cell is illuminated with a 1000 \text{ W/m}^2, AM1.5 \text{ d spectrum. Both solar cells operate at the same temperature of 298 K. In another curve in Fig. 3, the \textit{I–V} curve of the two cells connected in parallel is shown if the temperature of the shaded cell is increased to 323 K and the temperature of the illuminated cell is maintained at 298 K. Furthermore, in Fig. 3, the \textit{I–V} curves of the shaded cell at cell temperatures of 298 K and at 323 K are shown by a dotted line and by a short dashed line, respectively. The crosses on the \textit{I–V} curves for the shaded solar cell mark the working point for the shaded cell if the system of the two cells connected in parallel is under open-circuit conditions. These working points indicate that the shaded cell has a negative current. A negative current means that a current is injected into the shaded solar cell by the illuminated solar cell. The quantity of this injected current depends on the operating voltage of the two solar cells connected in parallel. In any case, the injected current causes the temperature of the shaded solar cell to increase. The quantity of the injected current and, thus, the temperature increase are maximized if the system of the two connected solar cells is operating under open-circuit conditions. The rising temperature of the shaded cell causes an increase in the dark recombination current, and this increase changes the \textit{I–V} curve of the shaded cell. An example of this change in the \textit{I–V} curve is shown in Fig. 3 for a temperature increase from 298 to 323 K. Increasing the dark recombination currents lowers the operating voltage and consequently leads to increased current injection into the shaded cell and thus a further temperature increase. In this manner, a self-feeding process called thermal runaway can be initiated where current injection causes a higher solar cell temperature; in turn, this higher temperature causes current injection and so on. In this paper, this self-feeding process is modeled in an iterative manner following steps 1 to 6.

1) Calculate the \textit{I–V} curve of solar cells connected in parallel. One of the cells is shaded.
2) Determine the working voltage of the calculated \textit{I–V} curve.
3) Calculate the current that is injected into the shaded solar cell at this working voltage.
4) Determine the temperature of the shaded solar cell caused by this injected current.
5) Adjust the dark recombination current densities of the shaded solar cell according to the increased temperature.
6) Repeat steps 1 to 5 until the temperature of the shaded solar cell remains constant.

All steps except step 4 can be realized with the electrical circuit solved by LTSpice and described in Section II-A. Step 4 requires a function $T(I_{\text{Injected}})$ that gives the temperature dependence of the shaded solar cell on the current injected $I_{\text{Injected}}$ into this cell. This function is determined empirically, as described in the next section.

III. EXPERIMENTAL SETUP AND TEST OF THE MODEL

The thermal runaway model that is described in the previous section requires the temperature $T$ of the solar cell as a function of the injected current $I_{\text{Injected}}$. This dependence is determined experimentally with a triple-junction solar cell that is mounted onto a copper heatsink. The copper heatsink, which is electroplated with gold, is glued to a glass plate similar to that used in FLATCON-type CPV modules [16], [17]. The back of the glass plate is positioned with minimal thermal contact to the metal to ensure that the heat is mainly dissipated by air. In this manner, similar conditions as in CPV modules are achieved. Furthermore, this solar cell is connected to a power supply, which injects a specific current into the solar cell. The energy introduced by this current in the solar cell is converted into radiation and heat. The temperature resulting from the produced heat is approximated with a PT100 sensor that is mounted onto the 30 mm\textsuperscript{2} heatsink at a distance of approximately 10 mm from the 5 mm\textsuperscript{2} solar cell. Fig. 4 shows the correlation determined between the injected current density and the resulting temperature of the solar cell. From the temperature in the laboratory, the solar cell temperature rises up to approximately 425 K for an injected current density of approximately 55 A/cm\textsuperscript{2}. This current density corresponds to 3 A current flowing through the 5 mm\textsuperscript{2} solar cell and corresponds to the current generated in the solar cell by approximately 3500 suns (1 sun, 1000 W/m\textsuperscript{2}, AM1.5d ASTM G-173-03). The power supply was limited to 3 A; the solar cell temperature is expected to increase further with higher current injection. Fig. 4 presents a linear fit to the measurement data. This linear fit is used as an input in the thermal runaway
data and used as an input in the thermal runaway model. The temperature of the laboratory was used for each voltage. As the initial temperature for the iterative calculation for calculations of thermal runaway in a triple-junction solar cell. The initial temperature was again connected to the solar cell mounted onto a heatsink, which is glued to a glass plate. The solar cell temperature is determined by the thermal runaway model and a comparison of this calculated correlation with the measured data. Below a fixed voltage of approximately 2.5 V, the temperature of the solar cell is determined by the room temperature of the laboratory. From 2.5 to 3.1 V, the solar cell temperature increases slightly and shows good agreement between the measurement and model. At an applied voltage of 3.1 V, thermal runaway is triggered within the modeled solar cell and causes an abrupt, steep temperature increase. However, the measured data indicate that the thermal runaway is triggered at a fixed voltage of 3.14 V. Therefore, the thermal runaway model underestimates the fixed voltage that triggers thermal runaway by approximately 0.04 V. This result means that the thermal runaway model has satisfactory agreement with the measurements for calculations of thermal runaway in a triple-junction solar cell. As the initial temperature for the iterative calculation of the solar cell temperature with the developed model, the temperature of the laboratory was used for each voltage.

IV. Calculation of Temperature Increase and Thermal Runaway

In the previous section, we showed that the application of a voltage to a solar cell can lead to a steep increase in the temperature of the entire solar cell because the injection of a current starts a self-feeding process. If this steep increase occurs in a solar cell within a CPV module, damage can occur. For instance, the glass back plate of the module could break from the stress induced by the huge temperature differences. For this reason, it is important to understand under which conditions a strong temperature increase or even a thermal runaway can occur within a CPV module. First, two conditions must be fulfilled so that current injection into a solar cell within a CPV module can occur.

1) The solar cells must be connected in parallel to strings, and the strings must not be protected by a string diode.
2) The string of solar cells connected in parallel must operate at a voltage greater than the open-circuit voltage of the solar cell that generates the lowest current.

The fulfillment of condition 1) depends on the design of the CPV module. Condition 2) is fulfilled if one solar cell generates less current or if the temperature of one solar cell is greater than the temperatures of the other solar cells. If these two conditions are fulfilled, a current is injected into the solar cell and leads to an increase in the temperature of the solar cell. The value of this temperature increase and the starting of a thermal runaway depends on the amount of current that is injected into the solar cell. The maximum current that is injected into a solar cell is determined by the number of solar cells connected in parallel to the cell and by the concentration factor of the sunlight. The fulfillment of condition 1) depends on the design of the CPV module. Condition 2) is fulfilled if one solar cell generates less current or if the temperature of one solar cell is greater than the temperatures of the other solar cells. If these two conditions are fulfilled, a current is injected into the solar cell and leads to an increase in the temperature of the solar cell. The value of this temperature increase and the starting of a thermal runaway depends on the amount of current that is injected into the solar cell. The maximum current that is injected into a solar cell is determined by the number of solar cells connected in parallel to the cell and by the concentration factor of the sunlight. The fulfillment of condition 1) depends on the design of the CPV module. Condition 2) is fulfilled if one solar cell generates less current or if the temperature of one solar cell is greater than the temperatures of the other solar cells. If these two conditions are fulfilled, a current is injected into the solar cell and leads to an increase in the temperature of the solar cell. The value of this temperature increase and the starting of a thermal runaway depends on the amount of current that is injected into the solar cell. The maximum current that is injected into a solar cell is determined by the number of solar cells connected in parallel to the cell and by the concentration factor of the sunlight. The fulfillment of condition 1) depends on the design of the CPV module. Condition 2) is fulfilled if one solar cell generates less current or if the temperature of one solar cell is greater than the temperatures of the other solar cells. If these two conditions are fulfilled, a current is injected into the solar cell and leads to an increase in the temperature of the solar cell. The value of this temperature increase and the starting of a thermal runaway depends on the amount of current that is injected into the solar cell. The maximum current that is injected into a solar cell is determined by the number of solar cells connected in parallel to the cell and by the concentration factor of the sunlight. The fulfillment of condition 1) depends on the design of the CPV module. Condition 2) is fulfilled if one solar cell generates less current or if the temperature of one solar cell is greater than the temperatures of the other solar cells. If these two conditions are fulfilled, a current is injected into the solar cell and leads to an increase in the temperature of the solar cell. The value of this temperature increase and the starting of a thermal runaway depends on the amount of current that is injected into the solar cell. The maximum current that is injected into a solar cell is determined by the number of solar cells connected in parallel to the cell and by the concentration factor of the sunlight. The fulfillment of condition 1) depends on the design of the CPV module. Condition 2) is fulfilled if one solar cell generates less current or if the temperature of one solar cell is greater than the temperatures of the other solar cells. If these two conditions are fulfilled, a current is injected into the solar cell and leads to an increase in the temperature of the solar cell. The value of this temperature increase and the starting of a thermal runaway depends on the amount of current that is injected into the solar cell. The maximum current that is injected into a solar cell is determined by the number of solar cells connected in parallel to the cell and by the concentration factor of the sunlight. The fulfillment of condition 1) depends on the design of the CPV module. Condition 2) is fulfilled if one solar cell generates less current or if the temperature of one solar cell is greater than the temperatures of the other solar cells. If these two conditions are fulfilled, a current is injected into the solar cell and leads to an increase in the temperature of the solar cell. The value of this temperature increase and the starting of a thermal runaway depends on the amount of current that is injected into the solar cell. The maximum current that is injected into a solar cell is determined by the number of solar cells connected in parallel to the cell and by the concentration factor of the sunlight.
in parallel operates at open-circuit conditions. Initially, all the solar cells have the same temperature. Temperature increases above 125 K were calculated by extrapolation. As shown in Fig. 6, the calculated increase in temperature because of the injected current is limited to 10 K at concentration factors less than 300, regardless of how many solar cells are connected in parallel. If a concentration factor of 500 is used, the number of solar cells connected in parallel should be limited to six to prevent temperature increases greater than 25 K. If ten solar cells are connected in parallel, the concentration factor should not be greater than 400. Note that these statements are calculated for a specific type of the solar cell and heat sink. For other heat sink technologies and solar cell sizes, the model must be adapted. However, the calculation revealed that the concentration factor is more critical to get high temperature increases and triggering a thermal runaway than the number of connected solar cells. This result occurs because the concentration factor increases the open-circuit voltage of the illuminated CPV cells, whereas the number of connected solar cells increases the maximum injected current only. For the demonstration of the relevance of the used heat sink technology, the same calculations as shown in Fig. 6 are done with a modified dependence of current injection and solar cell temperature. As a modification, the linear fit shown in Fig. 4 is used with a 50% steeper slope. The reason for this modified dependence of current injection and solar cell temperature could be a smaller heatsink, for instance. The results of these modified calculations are shown in Fig. 7. For the modified dependence of current injection, the steep increase in temperature occurs for lower concentrations and lower number of solar cells in parallel. The same calculations were performed with a 50% flatter slope of the dependence shown in Fig. 4. These calculations revealed that for a 50% flatter slope, no increase in temperature above 15 K occurs up to concentrations of 1000 and up to 20 of solar cells connected in parallel.

Solar cells connected in parallel within CPV modules without the protection of string diodes can suffer current injection. This current injection can cause a heating of the cell which results in an increase of its recombination currents. In this manner, a self-feeding process called thermal runaway can be started. A thermal runaway can cause substantial damage in CPV modules.

In this paper, we introduced a model to calculate the thermal runaway that can occur within CPV modules. This thermal runaway model uses an electrical circuit of a triple-junction solar cell that is based on the two-diode model for all three subcells. The current–voltage characteristics of the electrical circuit are calculated with LTSpice [9]. The thermal runaway model requires as input the correlation between the injected current and the resulting temperature increase within the solar cell. In this paper, this correlation is obtained experimentally with a triple-junction solar cell.

As a test of the model, a fixed voltage is applied to a triple-junction solar cell by a power supply. At a fixed applied voltage of 3.14 V, a thermal runaway occurs in the solar cell. The thermal runaway model calculates that thermal runaway for an applied fixed voltage of 3.10 V.

As an application of the thermal runaway model, we calculated the dependence of the temperature increase within a completely shaded solar cell on the number of parallel connected solar cells and on the concentration factor of sunlight incident on the solar cells. This calculation shows that above a certain number of parallel connected solar cells and above a certain concentration factor, the shaded solar cell suffers a steep increase in temperature of over 100 K. For the investigated solar cell and heat sink technology, the temperature increase is limited to 20 K for a concentration factor below 350 and for up to 20 solar cells connected in parallel in a string. For a concentration factor of 500 and five solar cells connected in parallel per string, the calculated temperature increase is limited to 30 K. For a concentration factor of 750, fewer than four solar cells are recommended.
and for a concentration factor of 1000 suns, fewer than three solar cells are recommended in parallel per string to prevent strong temperature increases and thermal runaways within the CPV module for the used solar cell and heat sink technology.

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