

# Systematic Tuning of Fixed-Structure Speed and Active Tower Damping Controllers using $H^\infty$ -Norm Criteria in the Frequency Domain

Martin Shan, Boris Fischer, Peter Loepelmann

Fraunhofer IWES Kassel,  
Department Control Engineering, Email: [martin.shan@iwes.fraunhofer.de](mailto:martin.shan@iwes.fraunhofer.de)

## Abstract

This paper considers how structured  $H^\infty$ -control design based on a given wind spectrum can be used as a systematic, transparent and efficient way for tuning the parameters of fixed-structure controllers for rotor speed control and active tower damping for a horizontal-axis wind turbine using collective pitch actuation.

*Keywords:* Pitch Control, Active Tower Damping,  $H^\infty$

## 1 Introduction

If collective pitch is used for both rotor speed control and active tower damping of a wind turbine, control design effectively becomes a multivariable problem. Speed control and axial tower motion are highly coupled, as changes in pitch angle always influence both the aerodynamic thrust and the aerodynamic torque acting on the rotor. Modern control design methods, in principle, can optimize both control loops in parallel once the overall optimization criterion is defined, e.g. [1]. This, however, requires proper weighting of different control objectives as a starting point, which in many cases is not obvious. As a consequence, the problem of controller parameter tuning is often shifted towards tuning of weighting matrices or weighting functions.

Furthermore, the resulting MIMO-controllers are not very comprehensible and, depending on the control design model, may be of high order. In practice, this can cause problems for gain scheduling or pitch actuator saturation / controller switching between part- and full-load operation.

It is common practice in industry to design separate control loops for speed control and tower damping. The controller structure typically consists of simple PID schemes or filters that are designed in an iterative process. In many cases, an existing speed controller is augmented with an additional control loop for active tower damping [2]. Clearly, this approach will not result in an optimal controller parame-

terization regarding both speed control and tower fatigue objectives.

The approach taken in this paper is to apply a pragmatic multivariable control design to a controller with predefined, i.e. fixed structure. The advantages of a simple and comprehensible controller structure should be combined with those of a systematic multivariable control design.

The results and conclusions presented in this paper have been derived for the well-known 5 MW NREL reference wind turbine [3]. However, similar results have been observed also for models of different multi-MW wind turbines.

## 2 Formulation of Control Objectives in the Frequency Domain

The assumed control objective in this study is:

**For a given wind spectrum,**

- **minimize the fatigue damage related to the fore-aft tower base bending moment**
- **while keeping the rotor speed deviations below a defined threshold**
- **and keeping pitch speed and pitch acceleration below given limits**

Since information on the wind field is typically given in the frequency domain, an  $H^\infty$ -norm based approach is chosen. The original control objectives, which are actually time domain criteria, are translated into weighting functions. While some authors propose “black-box” numerical optimization of weighting function parameters, see e.g. Ozdemir [4], in this paper the dependency between weighting function parameters and the original control objectives should be made transparent. For this purpose, tower bending fatigue and maximum speed deviations must be related to the frequency domain properties of the wind turbine, i.e. the shape of the closed-loop transfer functions.

As described in [5], fatigue due to tower fore-aft bending can be estimated directly based on

the power spectral density (PSD) of the bending moment signal, using the Dirlik-method [6].

Also, the maximum speed deviations can be estimated based on the PSD of the generator speed signal using the Rice-method for estimating the probability distribution of maximum amplitudes of a normally distributed signal [7]. In this paper, the value of 2 times the amplitude corresponding to 95% in the cumulated probability distribution was used as (conservative) estimation for the maximum rotor speed deviation from mean in a 600 s time series.

The evaluation of controllers with regard to the control objective formulated above can thus conveniently be carried out on the basis of PSDs. As long as the assumption that the turbine behaves linearly is valid, these PSDs can be calculated directly from the transfer function and the assumed wind spectrum without any time domain simulations, see e.g. [2]. Thus, optimizations are computationally very efficient. Note that the linear system assumption will be violated for large deviations from the operating point and especially if the transition between full-load and part-load is considered. However, for optimizing the controller for individual operating points in the full-load region of the wind turbine, the approach is considered to be very useful.

### 3 Wind Turbine Model for Control Design

In this paper, a linearized model for only one operating point at 18 m/s mean wind speed is considered for control design. As we consider only full-load operation, the generator torque was assumed to be constant and pitch speed reference was used as single control input.

In order to reduce complexity and calculation time for the full-order  $H^\infty$ -control design, the linear control design model was derived from a 40<sup>th</sup> order reference wind turbine model by model reduction using modal condensation. For full-order and structured  $H^\infty$ -design, an 18<sup>th</sup>-order wind turbine model was found to be sufficient, which describes the relevant system dynamics up to approximately 4 Hz, including

- 1<sup>st</sup> and 2<sup>nd</sup> tower bending modes fore-aft
- 1<sup>st</sup> flapwise blade bending collective and whirling modes
- 2<sup>nd</sup> flapwise blade bending collective mode
- 1<sup>st</sup> drivetrain mode coupled with edge-wise collective blade bending

If the model is further reduced, the interaction between tower and blade bending modes is not properly described.

To be more accurate, however, the resulting transfer functions, step responses and frequency domain performance indicators are computed based on the more detailed linear model of order 40.

### 4 $H^\infty$ -Reference Control Design

$H^\infty$ -control design in this paper is considered in the interpretation of shaping closed loop transfer functions. In the first step, full-order  $H^\infty$ -control design is carried out as a reference, using the *hinfsyn*-function in MATLAB [8].

By means of suitable weighting functions, it is possible to define an upper bound for the magnitude of transfer functions from wind speed to generator speed, denoted as  $N_{Om}(j\omega)$ , and from wind speed to tower top acceleration  $N_{aT}(j\omega)$ . The schematic of the control design model combined with weighting functions is shown in Figure 1. The applied types of weighting functions are shown in Figure 9 as inverse bode magnitude plots and will be discussed in the following.

#### Weighting function for generator speed

The first idea was to put a threshold  $k_{Wom}$  on the transfer function  $N_{Om}(j\omega)G_{Wind}(j\omega)$ , where  $G_{Wind}(j\omega)$  is a transfer function describing the wind turbulence spectrum. The type of the transfer function was chosen according to [9] and the parameter adjusted to fit the magnitude to the square root of the considered PSD of wind speed variations. The corresponding weighting function would thus be  $k_{Wom} * G_{Wind}(j\omega)$ . However, while for this choice there is a tight correlation between weighting function gain factor  $k_{Wom}$  and the resulting bandwidth of speed deviations, it was found that this specific weighting function did not result in minimum tower bending fatigue.

Instead, the observation was made that weighting functions crossing the open loop transfer function  $G_{Om}$  with zero slope, in general, result in smaller tower bending fatigue. For this reason, an inverse weighting function as shown in Figure 9 (a) was chosen, having proportional behavior around the desired controller bandwidth and derivative behavior at low frequencies, resulting in zero steady-state speed deviation of the closed-loop system. The first corner frequency is denoted as  $f_{c,Wom}$ . At a frequency of 0.8 Hz, the slope again changes to 20 dB/decade to avoid interaction with higher frequency modes of the wind turbine.

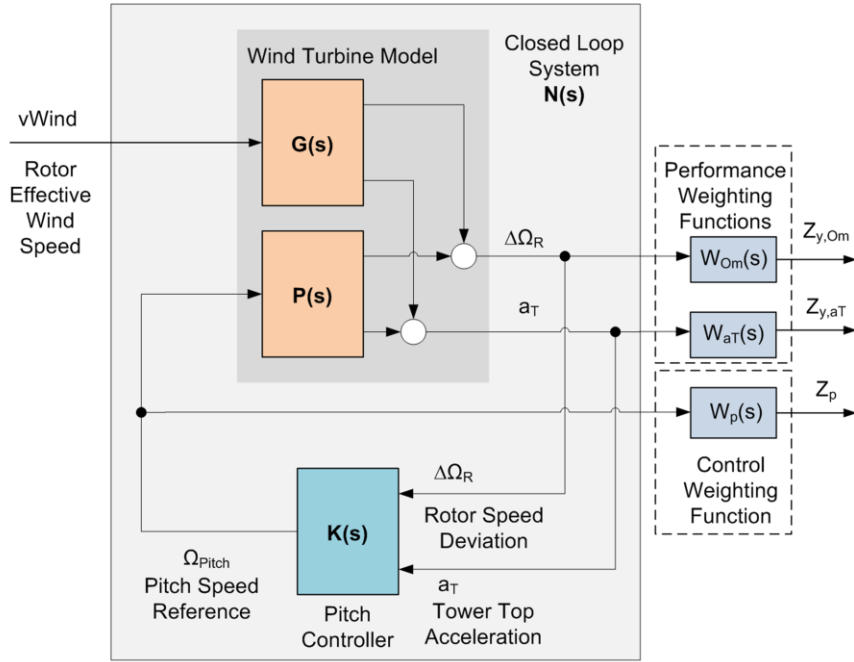


Figure 1: Augmented Model Scheme considered for control design

The free parameters of the inverse weighting function are thus corner frequency  $f_{c,W_{Om}}$  and the gain in the horizontal part  $k_{W_{Om}}$ .

Considering Figure 8, if tower bending fatigue is compared for different controllers with equal maximum generator speed deviation (Rice-estimate), it turns out that controllers with small values of  $f_{c,W_{Om}}$  achieve lower values of tower bending fatigue damage.

As a consequence, a pure D-controller acting on pitch rate reference (or a pure P-controller acting on the pitch angle reference) seems to be most suitable if steady-state rotor speed deviations can be tolerated. If zero steady-state error is required, a desired time compensation constant for the steady state error can be specified. In the following it will be assumed that  $f_{c,W_{Om}}$  is set to a defined value corresponding to a comparably large time constant of 10s.

### Weighting function for tower top acceleration

For tower top acceleration, a simple constant  $k_{W_{aT}}$  is used as inverse weighting function, see Figure 9 (b). The aim is to attenuate the peak in the transfer function from wind speed to tower top acceleration, which corresponds to an active damping of the tower fore-aft-motion.

### Weighting function for pitch speed

The aim of this weighting function is to represent the actuator limits in terms of pitch speed and pitch acceleration. Furthermore, it should provide sufficient roll-off to the controller for

higher frequencies in order to avoid interaction with high-order structural modes and noise, and to increase robustness against model uncertainties. Additionally, it was observed that proper bandwidth limitation is effective to avoid the calculation of unstable controllers by the *hinf* function.

The inverse of the chosen weighting function is shown in Figure 9 (c). In the low frequency region, the requirement to limit the pitch speed results in

$$W_p(j\omega) = G_{Wind}(j\omega) / \Omega_{Pitch,max} \quad (1)$$

Here the maximum pitch speed  $\Omega_{Pitch,max}$  was assumed to be 5 deg/s.

For higher frequencies, two zeros are placed in order to limit the bandwidth of the pitch controller. The bandwidth of the inverse transfer function was chosen at approximately 2 Hz to roll off above the first flapwise blade bending mode.

### Influence of weighting function parameters on tower bending fatigue and maximum speed deviation

In the following, the weighting function for pitch speed is considered as fixed, while the gains  $k_{W_{Om}}$  and  $k_{W_{aT}}$  of the weighting functions for generator speed and tower acceleration are considered as free parameters for control design. One of the advantages of the control design using parametric weighting functions is the interpretation of these free parameters in terms of upper limits on transfer functions. To

illustrate this, a number of  $H^\infty$ -controller calculations have been carried out on a grid in the 2-dimensional parameter space  $[k_{WOM}, k_{WaT}]$ . For the resulting controllers, the dependency of maximum speed deviations and tower bending fatigue damage on the maximum magnitude values of  $N_{Om}(j\omega)$  and  $N_{aT}(j\omega)$  have been investigated.

For that purpose, the parameters  $k_{WOM}$  and  $k_{WaT}$  have been reduced stepwise beginning from starting values  $k_{WOM,0}$ ,  $k_{WaT,0}$ . These starting values can be interpreted as absolute upper bounds on the magnitude of the transfer functions  $N_{Om}(j\omega)$ ,  $N_{aT}(j\omega)$ . For  $k_{WaT,0}$  the natural choice is to set the starting value to the maximum of the open-loop transfer function from wind speed to tower top acceleration  $G_{aT}(j\omega)$ . This means, the controller should not reduce the damping in comparison to the open loop. Regarding the speed control loop, an upper bound for  $|N_{Om}|$  can be easily found from the admissible maximum speed deviation, as shown below.

For every  $H^\infty$ -controller computation, the function *hinfsyn* returns a performance value  $\gamma$  which is smaller than 1 if all specifications in terms of weighting functions have been met. For  $\gamma > 1$  this is not the case and some closed loop transfer functions exceed the weighting functions. The lower limit of the parameter space is thus given by the combinations  $[k_{WOM}, k_{WaT}]$  that result in  $\gamma = 1$ , forming the border to the parameter region that is not feasible in terms of actuator limits or robustness requirements.

The dependency of the Rice estimate for maximum generator speed deviation on the maximum magnitude of the transfer function  $N_{Om}(j\omega)$  is shown in Figure 2. Here, the relation is quite clear: a reduction in  $\max|N_{Om}|$  - as expected - will result in a proportional reduction of the maximum speed deviation. There is almost no dependency on  $\max|N_{aT}|$ . If the slope of the linear dependency is known, the maximum value of  $k_{WOM,0}$  as upper bound on  $k_{WOM}$  can thus be directly derived from the maximum admissible speed deviation.

Considering the dependency of tower bending fatigue damage  $D_{MYT}$  on the individual maxima of  $|N_{Om}(j\omega)|$ ,  $|N_{aT}(j\omega)|$  the relation is shown by the color map in Figure 3. The red dots denote the calculated controllers. For constant  $\max|N_{Om}|$ , a decrease in  $\max|N_{aT}|$  will result in reduced fatigue damage. On the other hand, for constant  $\max|N_{aT}|$ , also a decreasing  $\max|N_{Om}|$  will result in reduced fatigue damage. In the considered case, the minimum is located on the lower border of the plane which is deter-

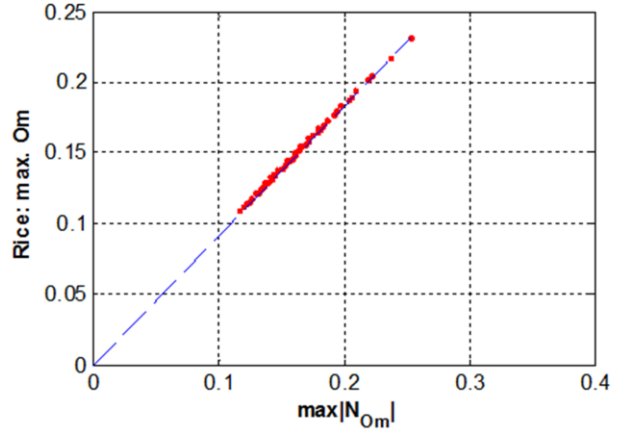


Figure 2: Dependency of Rice estimate for maximum speed deviation on the maximum of the transfer function  $N_{Om}(j\omega)$

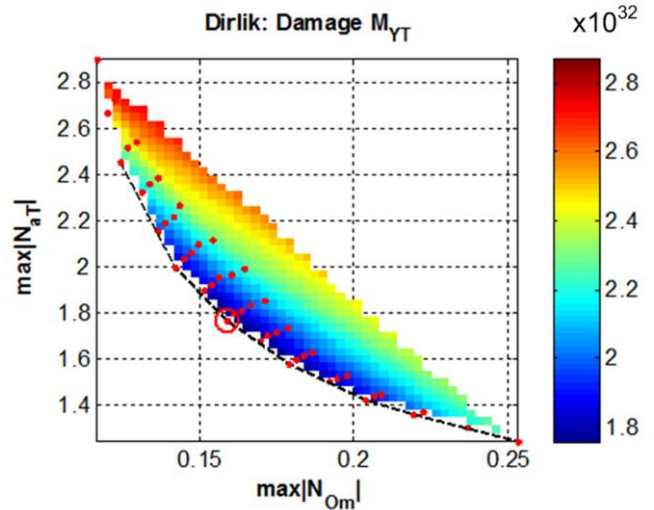


Figure 3: Dependency of tower bending fatigue damage on the maxima of transfer functions  $N_{Om}(j\omega)$  and  $N_{aT}(j\omega)$ , red dots: calculated controllers, black dashed line: border  $\gamma=1$  in the  $[\max|N_{Om}|, \max|N_{aT}|]$  plane, red circle: controller with minimum  $D_{MYT}$

mined by the condition  $\gamma = 1$ , and is thus mainly influenced by the pitch speed weighting function. Since  $\max|N_{aT}|$  and  $\max|N_{Om}|$  are not independent of each other, the optimum trade-off has to be found. Especially, for the NREL wind turbine, it is not true in any case that a more aggressive active tower damping (reduced  $\max|N_{aT}|$ ) will result in lower tower bending fatigue as it might mean an increase in  $\max|N_{Om}|$ . Also relaxing the speed controller will not in any case result in lower fatigue loads.

For finding the optimum set of weighting function parameters, the most transparent way, as described above, is to apply  $H^\infty$ -control design for all parameter points  $[k_{WOM}, k_{WaT}]$  on a sufficiently dense grid in the feasible region.

The simple algorithm is as follows:

- (1) Start with  $k_{WOM} = k_{WOM,0}$ ,  $k_{WaT} = k_{WaT,0}$ .
- (2) Reduce  $k_{WaT}$  in sufficiently small steps, until  $\gamma > 1$ . Calculate the  $H^\infty$ -controller and evaluate  $D_{MYT}$  for each point.
- (3) Reduce  $k_{WOM}$  by one step. If  $\gamma > 1$ , stop.
- (4) Repeat steps (2) and (3) until stop.
- (5) Select the controller for the grid point with minimum  $D_{MYT}$ .

The whole procedure can be easily automated. The gridding approach is feasible as each controller calculation and evaluation takes only a few seconds. For the grid, in the present case, a logarithmic step size for  $\Delta k_{WOM}$ ,  $\Delta k_{WaT}$  of 1dB was found to be reasonable.

Even faster solution is possible by applying a numerical search algorithm, e.g. Nelder-Mead-Simplex. The regions  $k_{WOM} > k_{WOM,0}$ ,  $k_{WaT} > k_{WaT,0}$  and  $\gamma > 1$  can be excluded from the search area by suitable penalty offsets.

## 5 Fixed-Structure Control Design

In the second step, structured  $H^\infty$ -control design was carried out. The idea was to use the optimum  $H^\infty$ -controller described in the previous section as a reference and find a fixed-structure controller that is sufficiently close.

For that purpose, the *hinfstruct* function in Matlab was used [8], which applies non-smooth optimization to find the free parameters of a prescribed controller structure. Especially, the same weighting functions for specifying the control design objectives can be used for the  $H^\infty$ -design. Refer to [10] for more detailed information on the method.

As supported by experience, it was found that the speed control objectives can be achieved by a simple PD-controller, where an additional 2<sup>nd</sup>-order low-pass filter was applied for roll-off in the high frequency region.

For the tower damping controller, it was not possible to identify a transparent transfer function structure, e.g., a bandpass filter. Instead, a state space model with free parameters was assumed. The order of this model was increased, until the *hinfstruct* algorithm provided sufficient agreement with the  $H^\infty$ -reference controller. It was found that a 5<sup>th</sup>-order state space model is sufficient to meet the design objectives, however, the pitch speed weighting function had to be relaxed somewhat by shifting the roll-off to higher frequencies.

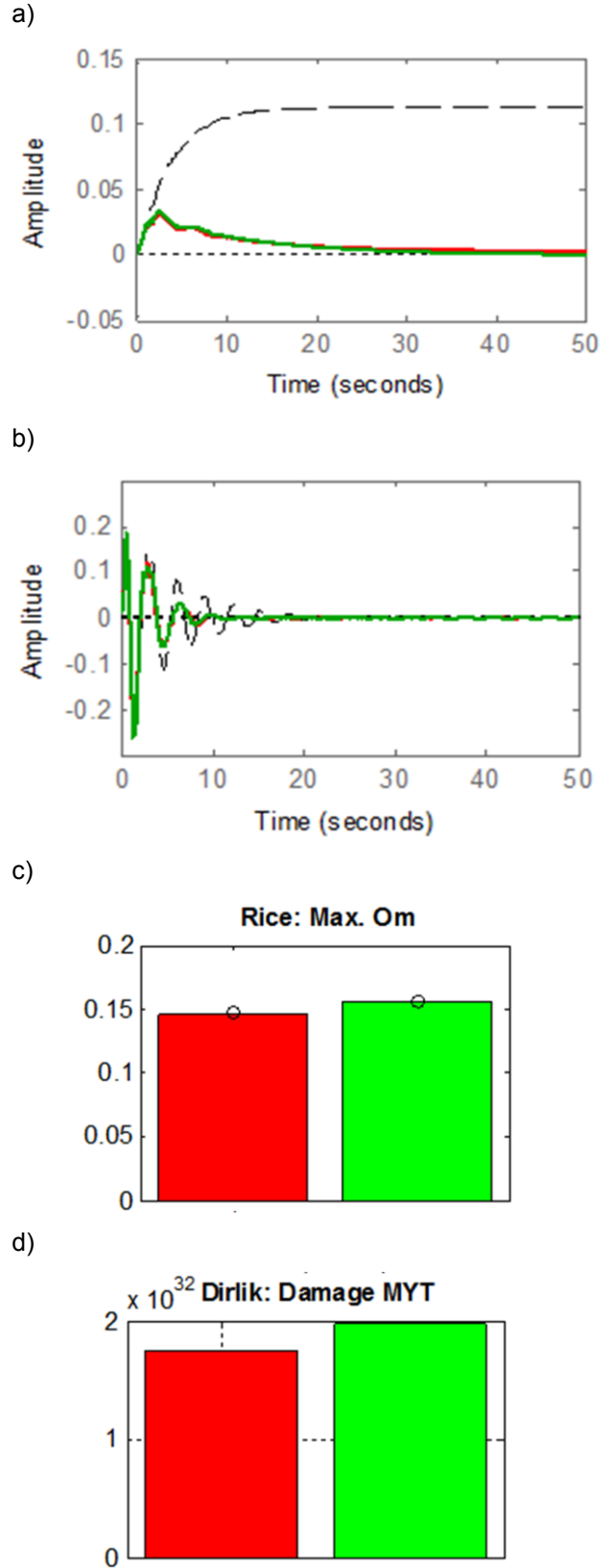


Figure 4: Performance comparison of  $H^\infty$  (red) and fixed-structure controller (green): (a) step response wind speed to rotor speed; (b) step response wind speed to tower top acceleration fore-aft; (c) max. speed deviation (Rice), (d) tower base fore-aft bending fatigue damage (Dirlik)

The comparison with the  $H^\infty$ -controller is shown in Figure 4 in terms of performance and in Figure 9 regarding the resulting transfer functions. For the fixed-structure controller, it is interesting to see that although the step response and the result of fatigue estimation are very similar, the transfer function of the tower damping part of the calculated *hinfstruct*-controller is quite different from that of the  $H^\infty$ -controller, compare Figure 10.

## 6 Tower Damping Controller without Tower Acceleration Measurement

The  $H^\infty$ -design methodology was also used to design a controller for speed control and active tower damping based on solely the generator speed input. This is possible, in theory, since the  $H^\infty$ -controller includes a full model of the plant and the tower acceleration is observable in the generator speed.

However, if similar performance specifications were used, as have been for the controllers in the previous sections, the *hinfsyn*-algorithm calculated controllers with unstable poles. These controllers are clearly not practicable for use in real wind turbines. With relaxed speed controller specifications, still it was possible to find stable controllers that provide active tower damping. These controllers, however, proved to be unstable in nonlinear simulations. A possible explanation was found by considering the robust stability for higher frequency unstructured uncertainty, as will be discussed in Section 8.

All controllers considered in this paper are then listed in Table 1.

## 7 Verification with nonlinear Simulations

For verification of the control design results, nonlinear simulations have been carried out with the IWES in-house wind turbine simulation tool WTsim [11]. This simulation tool is implemented in MATLAB/Simulink. It includes a structural dynamics description comparable to the full order linear model, which is scheduled with operational point, as well as a nonlinear aerodynamic model based on a state-of-the-art BEM implementation.

In order to make the time domain simulations fully comparable with the linear model predictions of the control design, a homogeneous wind field was applied. Furthermore, the tower shadow was disabled. A turbulence intensity of 10% at 18 m/s mean wind speed was assumed, leading to considerable deviations from

the steady-state operational point of the wind turbine model. Only a single turbulence seed of 600s duration was simulated, leaving some room for statistical uncertainty in the time domain results.

The comparison of the results for fatigue damage related to the tower base fore-aft bending moment  $M_{YT}$  and rotor speed deviations are shown in Figure 5 and Figure 6. It can be concluded that there is good agreement with the linear model predictions, even though only one turbulence seed was simulated. The decrease in tower bending fatigue by means of active tower damping as well as the deviations in rotor speed are well predicted by the linear control design procedure, as compared to the nonlinear simulation results.

Table 1: Overview of considered controllers

Controller	Order	Description
$K_{ref}$	2	Reference Speed controller with same speed controller settings as $K_{struct}$
$K_{struct}$	7	Fixed structure controller, PD speed controller and 5 <sup>th</sup> order tower damping controller
$K_{hinf}$	22	$H^\infty$ -controller with generator speed and tower acceleration input
$K_{hinf,Om}$	23	$H^\infty$ -controller with only generator speed input

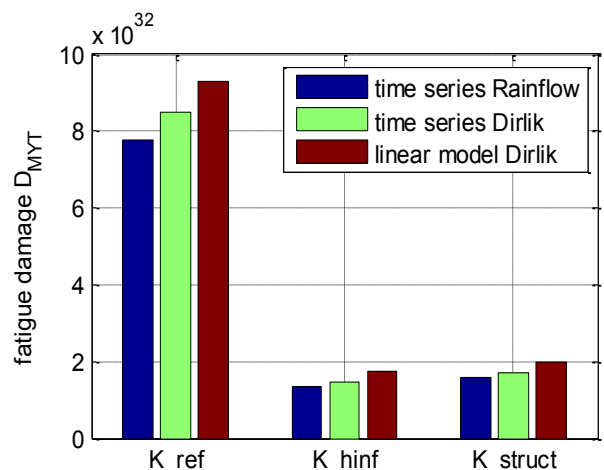


Figure 5: Comparison of tower base bending fatigue  $D_{MYT}$  for nonlinear simulation (time series) and linear model prediction.

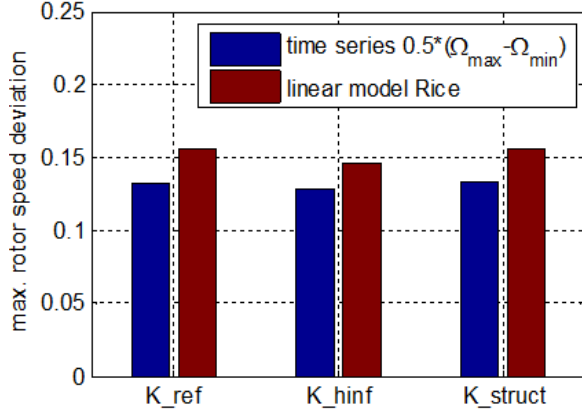


Figure 6: Comparison of maximum rotor speed deviation for nonlinear simulation (time series) and linear model prediction.

## 8 Robust Stability Analysis

In order to evaluate the robust stability of the four different controllers discussed above, two uncertainty descriptions have been considered:

### 1. Multiplicative output uncertainty margin MMO:

Here, the perturbed plant  $P_p$  is described by

$$P_p = (I + \text{diag}(\Delta_{Om}, \Delta_{aT}))P_0 \quad (2)$$

where  $\Delta_{Om}$ ,  $\Delta_{aT}$  are complex perturbations at the generator speed and tower acceleration outputs of the nominal turbine model  $P_0$ . This is a structured uncertainty which would cover for instance uncertainties in the damping of the tower and blade bending modes. Equivalently to the classical gain and phase margin for SISO systems, multivariable output margins MMO can be calculated that provide the value of gain and phase variations which can be tolerated independently on both outputs [12].

### 2. Normalized coprime factor uncertainty NCFM:

This is a fairly general description for unstructured uncertainty, which has proven useful in practical applications [13]. The key statement is that the maximum singular value of the expression

$$M = [K^T \ I]^T (I + P_0 K)^{-1} M_L^{-1} \quad (3)$$

is a measure for the unstructured combined input / output uncertainties  $\Delta_M$ ,  $\Delta_N$  in the perturbed plant

$$P_p = (M_L + \Delta_M)^{-1} (N_L + \Delta_N) \quad (4)$$

that can be tolerated without losing closed-loop stability, where  $P_0 = M_L^{-1} N_L$  is a left coprime factorization of the nominal plant  $P_0$ .

## Overview of robustness measures

For the current control problem, we consider the MMO as a useful measure of uncertainty for frequencies up to the first flapwise blade-bending eigen-frequency, whereas the NCFM is additionally evaluated as a measure for tolerated unstructured uncertainty. NCFM is considered only for frequencies larger than the first tower eigen-frequency.

Table 2 shows an overview of MMO multivariable output gain and phase margins. As can be seen, all considered controllers show very similar results regarding the MMO. This is somewhat unexpected, especially for  $K_{ref}$  which includes no active tower damping. It was observed, however, that the MMO is mainly influenced by the speed control loop and the minimum value occurs just below the first tower bending eigen frequency.

On the other hand, the robustness regarding unstructured uncertainty as described by the NCF is quite different, as can be seen in Table 3 and Figure 7. Especially the  $H^\infty$ -controller without acceleration measurement provides poor robustness to unstructured plant variation in the frequency region close to the first flapwise blade bending mode.

Future research is required for improving the understanding of the stability measures and considering also robust performance aspects.

Table 2: Comparison on multivariable output gain and phase margins for the considered controllers

	MMO Gain tol [dB]	MMO Phase tol [°]	MMO Freq [Hz]
$K_{ref}$	7.1	42	0.23
$K_{struct}$	7.0	41	0.22
$K_{hinf}$	6.7	40	0.25
$K_{hinf,Om}$	6.6	40	0.20

Table 3: Comparison of minimum NCF gain (for frequencies > 0.3 Hz) and the corresponding frequencies for the considered controllers

	NCF Margin	NCF Freq [Hz]
$K_{ref}$	0.074	1.68
$K_{struct}$	0.059	9.09
$K_{hinf}$	0.0047	1.24
$K_{hinf,Om}$	0.0014	1.02

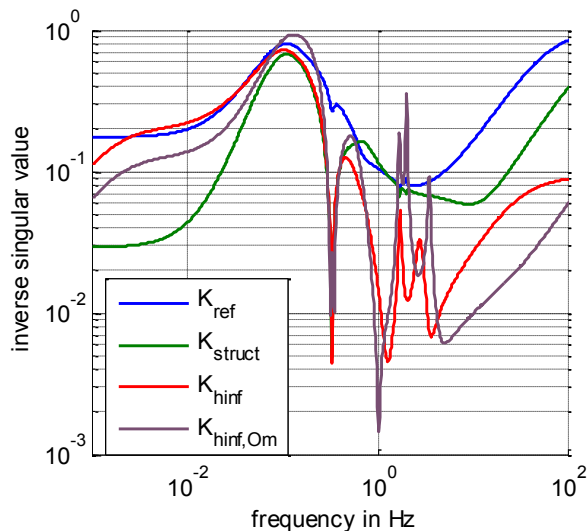


Figure 7: Inverse max. singular value of  $M$  in Eq. (3) for the different controllers as a measure for the NCF margin at different frequencies

## 9 Summary and Conclusions

A pragmatic multivariable approach for parallel design of rotor speed control and active tower damping is proposed. The control design is carried out in the frequency domain and provides a high level of transparency to the control engineer. In detail, the following conclusions can be drawn:

- $H^\infty$ -criteria can be conveniently used to formulate relevant pitch control objectives in the frequency domain. If speed controllers with sufficiently large compensation time constants for the steady state error are applied, the maximum magnitude of the transfer function from wind speed to generator speed is directly related to the maximum values of speed deviations for a given turbulence spectrum.
- Unfortunately, such a direct link between weighting function and time domain control objective was not found for the tower bending fatigue damage. Here, the minimum results from a trade-off between tight speed control and active tower damping, which can be found, e.g., by applying a numerical search algorithm on weighting function gain parameters. In comparison to direct controller parameter optimization, however, the  $H^\infty$ -approach allows to restrict the search area by considering upper bounds on the closed-loop transfer functions.
- If both generator speed and tower top acceleration feedback are used, structured control design can achieve simi-

lar results as full order  $H^\infty$ -controllers. The prescribed controller structure, however, is more transparent and easier to handle in a practical arrangement including gain scheduling and handling of actuator saturations. It has also been shown to provide more robustness to unstructured uncertainty in the higher frequency region.

- For the speed controller, D-control (acting on pitch speed reference) seems to be most effective to minimize tower bending fatigue while restricting speed deviations to a prescribed threshold.
- For the full order  $H^\infty$ -control design, in principle, both speed control and active tower damping would be possible using only generator speed feedback. For robustness reasons, however, omission of the tower top acceleration feedback is not recommended.

## 10 Future Work

In this work, only one full-load operational point of the turbine was considered. If required, however, the approach can be easily extended to a number of operating points. Especially for the fixed order control design, gain-scheduling between the different controllers is then quite straightforward because of the low order and the transparency of the prescribed controller structure.

One of the open points is the inclusion of unsymmetrical effects like spatial turbulence and 3p harmonic excitation into the fatigue load prediction in the frequency domain. These effects have been omitted in this study but will lead to deviations.

## Acknowledgement

The presented research was partly carried out in the joint research project „ELBA – control systems for the reduction of extreme loads at large-scale wind turbines” (0325731A) funded by the German Federal Ministry for Economic Affairs and Energy.

Gefördert durch:



aufgrund eines Beschlusses  
des Deutschen Bundestages

## References

- [1] M. Geyler and P. Caselitz: *Robust multivariable pitch control design for load reduction on large wind turbines*. Journal of Solar Energy Engineering, 130(3):031014, 2008.

[2] W. E. Leithead, S. Dominguez: *Analysis of tower/blade interaction in the cancellation of the tower fore-aft mode via control*. Proc. EWEK 2004, London, UK.

[3] J. Jonkman, S. Butterfield, W. Musial, and G. Scott: *Definition of a 5 MW Reference Wind Turbine for Offshore System Development*, National Renewable Energy Laboratory, Technical Report NREL/TP-500-38060, February 2009

[4] A. A. Ozdemir: *Preview Control for Wind Turbines.*, dissertation submitted to the faculty of the graduate school of the university of Minnesota, August 2013

[5] P. Ragan, L. Manuel: *Comparing estimates of wind turbine fatigue loads using time-domain and spectral methods*. Wind Engineering, Vol. 31(2), pp. 83–99, 2007.

[6] T. Dirlik: *Application of computers in fatigue analysis*, PhD-Thesis submitted to the department of engineering, University of Warwick Coventry, England, 1985.

[7] S. O. Rice, *Mathematical Analysis of Random Noise: Part III*, Bell Systems Technical Journal, vol. 1945, no. 24, pp. 46–156. 1945

[8] MATLAB, robust control system toolbox, <http://de.mathworks.com/products/robust/>

[9] *Von Karman Wind Turbulence Model*, MATLAB documentation, aerospace blockset: <http://de.mathworks.com/help/aeroblks/vonkarmanwindturbulencemodelcontinuous.html>

[10] P. Gahinet and P. Apkarian: *Decentralized and Fixed-Structure  $H^\infty$  Control in MATLAB*, 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC), 2011

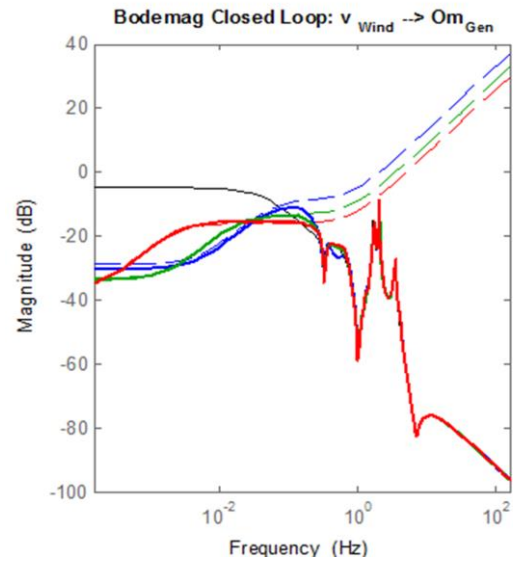
[11] B. Jasiewicz, M. Geyler: *Wind turbine modeling and identification for control system applications*, in: Scientific proceedings of the EWEA 2011, Bruxelles, Belgium, pp. 280-283.

[12] Blight, J.D., R.L. Dailey, and D. Gangsass, *Practical control law design for aircraft using multi-variable techniques*, International Journal of Control, Vol. 59, No. 1, 1994, pp. 93-137

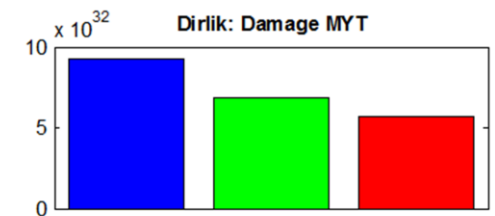
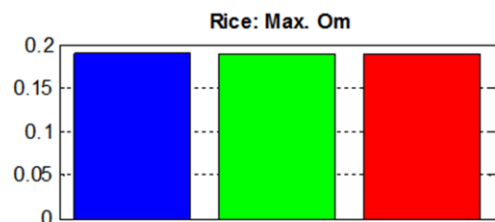
[13] D. McFarlane, K.Glover, *Robust Controller Design using Normalized Coprime Factor Plant Descriptions*, Vol 138 of Lecture Notes in Control and Information Sciences, Springer-Verlag, Berlin, 1990.

Figure 8: Comparison of three different  $H^\infty$ -controllers with the same maximum speed deviation: (a) Bode Magnitude Plot from wind speed to rotor speed, black line: open loop; (b) estimates of maximum rotor speed deviation (Rice) and tower bottom bending fatigue damage (Dirlik); (c) step responses from wind speed to rotor speed; black dashed line: open loop

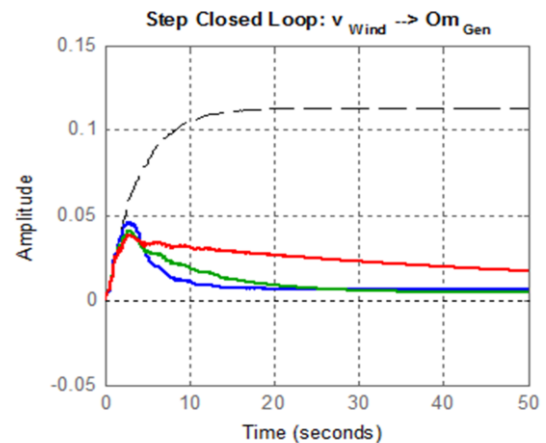
a)



b)



c)



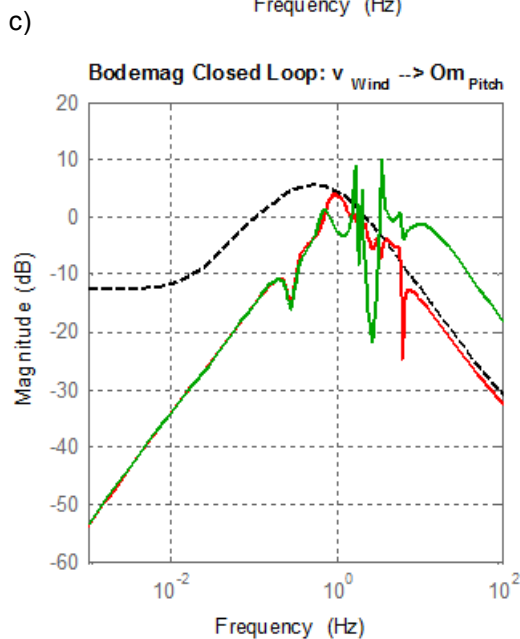
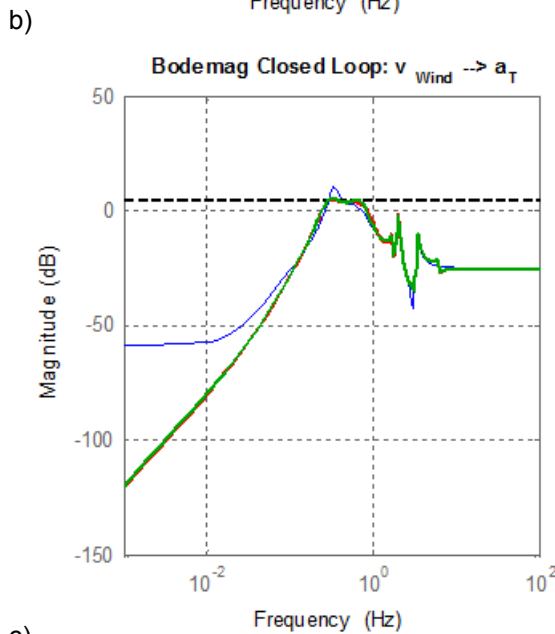
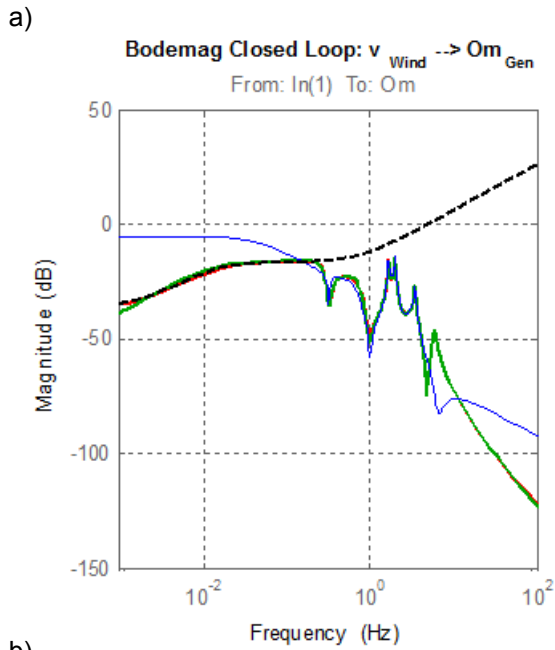


Figure 9:(left) Bode Magnitude Plots of open loop and closed loop, comparison of  $H^\infty$ - and fixed-structure controller:

(a) from wind speed to generator speed;

(b) from wind speed to tower top acceleration fore-aft;

(c) from wind speed to pitch speed

blue: open loop, black dashed: inverse weighting functions, red: closed loop  $H^\infty$ -controller, green: closed loop fixed-structure controller

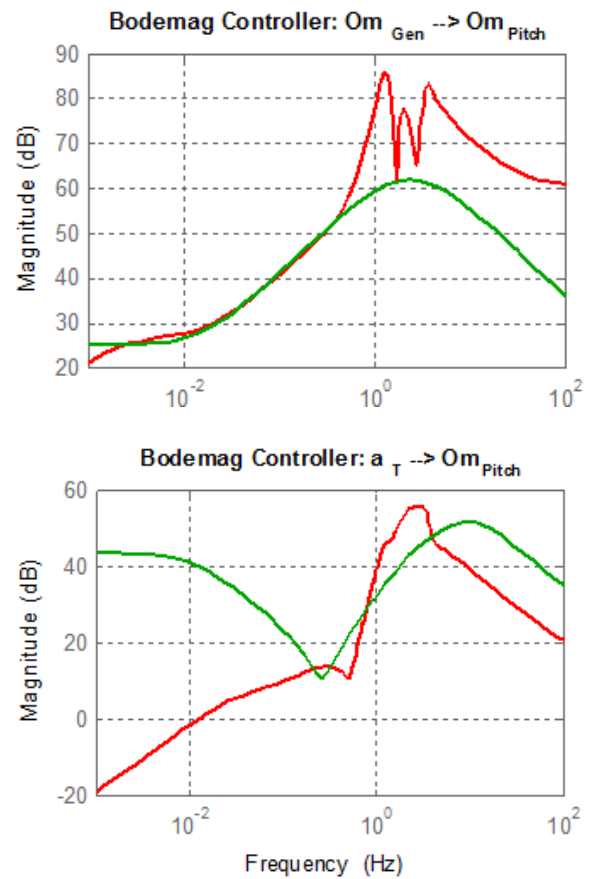


Figure 10: Bode Magnitude Plots of  $H^\infty$  (red) and fixed-structure controller (green); speed controller (top) and active tower damping controller (bottom).