High resolution irradiance tailoring using multiple freeform surfaces

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Abstract: More and more lighting applications require the design of dedicated optics to achieve a given radiant intensity or irradiance distribution. Freeform optics has the advantage of providing such a functionality with a compact design. It was previously demonstrated in [Bäuerle et al., Opt. Exp. 20, 14477-14485 (2012)] that the up-front computation of the light path through the optical system (ray mapping) provides a satisfactory approximation to the problem, and allows the design of multiple freeform surfaces in transmission or in reflection. This article presents one natural extension of this work by introducing an efficient optimization procedure based on the physics of the system. The procedure allows the design of multiple freeform surfaces and can render high resolution irradiance patterns, as demonstrated by several examples, in particular by a lens made of two freeform surfaces projecting a high resolution logo (530 × 160 pixels).

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References and links

1. Fine irradiance tailoring remains a challenging task

The design of freeform optics is becoming the preferred route to achieve compact optical systems producing a prescribed irradiance or radiant intensity distribution. They are used in a wide range of applications ranging from general lighting (street lighting for example) to automotive lighting and also more specialized applications like laser beam shaping.

In a previous publication [1], the authors present a freeform design algorithm that can handle multiple optical surfaces whilst at the same time operating directly with a prescribed target irradiance pattern. In its practical implementation, this approach has some limitations with respect to the achievable resolution and the precision of the generated lighting patterns, in particular for more elaborate and highly asymmetrical geometries. A precise analysis of those limitations will be the subject of another article by the authors. Based on those findings, and still for a null-étendue source, we therefore propose an extended algorithm that enables improved homogeneity and higher achievable resolutions within the target irradiance patterns.

In the first part of this article, the principle of the ray mapping computation as described in [1] is briefly reviewed. The main contribution of this article is then presented in the subsequent section: the parametrization chosen for the surface representation allows to readily compute a localized light flux traversing the optical elements. A comparison of the flux hitting the target with the prescribed irradiance allows the optics to be directly optimized to fine-tune the results obtained from the mapping. Important numerical aspects have to be taken into account during calculations and will be highlighted.

2. Ray mapping computation to efficiently find a starting point

The overall design task amounts to finding optical surfaces that can be coupled to a zero-étendue source (for example a point source or a perfectly collimated source) so that after traversing the optical system, the light forms a prescribed irradiance distribution on a target plane. To this end, each source ray is uniquely associated with a point in a 2D plane \( \Omega_0 \) perpendicular to the optical axis via a suitable projection. The target irradiance is equally projected in a consistent way onto a target plane \( \Omega_1 \). The task of designing a (freeform) optical system can then be described as calculating a diffeomorphism ("ray mapping") so that the transformed irradiance distribution matches the target distribution:

\[
u: \Omega_0 \rightarrow \Omega_1, (x,y) \mapsto (t_x, t_y)\]

where \((t_x, t_y)\) represents the target point in \( \Omega_1 \) to be reached by a source ray identified by \((x,y)\) in \( \Omega_0 \). Figure 1 recalls this principle and shows how a point from the source plane \( \Omega_0 \) is uniquely mapped to a point on the target plane \( \Omega_1 \).

An initial mapping \( \tilde{u} \) between these two planes can easily be computed using successive linear integrations along the Cartesian coordinate axes [2]. In a second step, the mapping itself is optimized to make it irrotational. This ensures better adherence of the field of surface normal vectors \( \mathbf{N} \) to the integrability condition

\[
\mathbf{N} \cdot (\nabla \times \mathbf{N}) = 0
\]

required to build a smooth optical element [3, 4]. For the mapping optimization, Haker [5] proposed an evolution equation over a pseudo-time variable \( t \) converging towards a stationary irrotational solution \((t \rightarrow \infty)\):

\[
\frac{\partial u}{\partial t} = -\frac{1}{\mu_0} \text{Du} \cdot \nabla^\perp \left( \Delta^{-1} \text{div} u^\perp \right) \quad \text{with} \quad u|_{t=0} = \tilde{u}
\]
where $Du$ denotes the mapping’s Jacobian, $(x, y)^\perp = (−y, x)$ represents a rotation by 90 degrees in $\mathbb{R}^2$ and $\Delta^{-1} \text{div} u^\perp$ denotes the solution $f$ of Poisson’s equation $\Delta f = −\text{div} u^\perp$ with Dirichlet boundary conditions.

Starting from a surface represented as a triangular mesh and using the vertex positions along the source rays as scalar variables, it is a key result of [1] that a multi-surface optical element can be constructed by performing a least-squares optimization. The corresponding merit function directly compares the actual position of rays traced through the optical system with their position prescribed by the irrotational mapping and thus significantly reduces the computation effort over optimization schemes that compare irradiance estimates derived from Monte-Carlo analyses (brute-force ray-tracing). It was demonstrated that this design procedure performs well in cases where homogeneous irradiance of a square area is required, and that the procedure is equally well suited for more complex cases found for example in the automotive industry. The design of a two-sided fog light lens demonstrates the latter [6].

3. High-resolution tailoring of the irradiance distribution

Previous work on high-resolution irradiance tailoring was most notably performed by Ries and Muschaweck [4], who outlined a single-step method that minimizes the deviation between the prescribed and the realized irradiance pattern on the target. For a given surface shape, the irradiance realized on the target is derived from the curvature tensor of the outgoing wave field, which in turn is computed using the curvature tensors of the incoming wave-front and that of the optical surface. The computation of the curvature tensor of the optical surface involves the second derivatives of its parametrization and the resulting equations are highly non-linear. Convergence to the global minimum requires the initial solution to be sufficiently close to the global minimum. One way to achieve this is using a multi-grid technique and a good first guess [7].

The main advantage of this method is that the optical surface is readily represented by a smooth function, as the field of surface normals automatically conforms to the integrability condition Eq. (2). However, a disadvantage is the high computational cost which is partly due...
to the sensitivity of the target irradiance to the surface’s second derivatives. In addition, this procedure has, to the best of the authors’ knowledge, yet only been published and proven for a single optical surface, which is a major difference to the work presented in [1] and here.

Taking a different approach, we now propose an extended algorithm that allows to significantly improve the initial design resulting from the optimized mapping derived in the previous section and that also allows to design multiple optical surfaces. Assuming as in the previous section that the optical surface is represented as a three-dimensional triangular mesh, each node of the mesh constituting the first surface has its position determined by the direction of a source ray (a unit vector), multiplied by a scalar value. Those directions are held constant during the construction of the surfaces (only the scalar factors are varied).

We observe that, on the one hand, the flux emitted by the source $\Phi_S$ can be written as follows:

- for a point source:
  $$\Phi_S = \iiint d^2\Phi_S = \iiint I(\theta, \phi) d^2\Omega$$  \hspace{1cm} (4)

  where $d^2\Phi_S$ is the elementary light flux, $d^2\Omega$ the elementary solid angle surrounding the direction of emission $(\theta, \phi)$, and $I(\theta, \phi)$ is the radiant intensity (in W/sr) of the point source. The elementary solid angle $d^2\Omega$ is approximated by the solid angle encompassed by the three source rays around a triangular face (see Fig. 2). This solid angle remains constant throughout the construction.

- for a collimated source:
  $$\Phi_S = \iiint d^2\Phi_S = \iiint B_S(x, y) dx dy$$  \hspace{1cm} (5)

  where $B_S(x, y)$ is the source (radiant) irradiance (in W/m²). In this case, the source rays are all directed towards the same direction (e.g. along the $z$-axis) and the cross section of a triangular tube remains constant between the source and the first optical surface.

In both cases, the flux from the source within a triangular tube remains constant along its path through the optical system, as illustrated by Fig. 2.

On the other hand, the flux reaching the target $\Phi_T$ can be written as a function of the irradiance (in W/m²) on the target. A triangular flux tube is projected as a triangular area on the target.
target as illustrated in Fig. 2. The flux in this tube can be computed as:

\[ \Phi_T(j) = \int \int_{A_j} B_T(x,y) \, dx \, dy \]

where \( A_j \) is the area of the target triangle corresponding to the \( j \)-th face of the surface, and \( B_T(x,y) \) the desired irradiance at the target point \((x,y)\). This integral is approximated with the simplest quadrature:

\[ \Phi_T(j) = B_T(x_j,y_j)A_j \]

(6)

with \((x_j,y_j)\) being the center of the triangular face on the target as illustrated in Fig. 2.

3.1. Objective function components

Similar to the purely mapping-based approach in [1], a least-squares optimization procedure is proposed. Taking into account the description of the light flux above, the merit function being optimized can now be detailed as a collection of three components:

- **light flux difference**: this component aims to equalize the light flux in Eqs. (4) (or (5)) and (6). It ensures that the desired irradiance \( B_T(x,y) \) is achieved on the target;

- **smoothing component**: in order to avoid local minima corresponding to triangular mesh components with abnormally high or low radiant intensity values (distorted triangles), a smoothing component is added. It enforces that face rays intersect the target near the center of the respective triangular area on the target. A face ray is closely correlated to the normal vector of the corresponding triangular face on the optical surface. Thus this component gives an advantage to nearly co-planar neighboring triangles on the optical surface;

- **distance to target’s boundary**: the source and the target total light flux are both normalized to unity, so that all the source rays going through the system must eventually reach the target. This is explicitly enforced by penalizing edge rays which deviate too far from the target boundary.

3.2. Numerical considerations

A consistent and homogeneous scaling of the three merit function components allows a better numerical stability, the ability to easily scale up or down the resolution of the surface (number of triangles), or the possibility to change the geometry of the problem without re-adjusting from scratch the convergence criteria of the optimizer.

All three components of the objective function above are hence chosen to be homogeneous to unity and to remain of the same order of magnitude, regardless of the geometry or resolution of the problem:

- the flux difference for a face is multiplied by the number of faces. Since the source flux is already normalized to unity, this ensures the scalability with regards to the number of triangles in the mesh. The effective light flux going through a face hence always has the order of magnitude unity;

- the smoothing component is scaled by the typical size of a triangle area on the target which varies as the square root of the total number of triangles;

- the distance to the target boundary is scaled by the target size.
The optimization procedure also involves the computation of a numerical gradient in the form of a standard forward difference using a step size $h$. It can be shown [8] that on a finite-precision machine with machine precision $\varepsilon_m$ such a computation leads to the following final relative error:

$$\tilde{f}'(x) = f'(x) \left(1 + \varepsilon_m \frac{2f(x)}{h} + \frac{f''(x)}{2f'(x)} h \right)$$

with $\tilde{f}'(x)$ the numerical approximation of the derivative and $\varepsilon_m$ the machine round-off precision. The last two terms are competing one with another and dictate an optimum value for $h$.

Noting $\varepsilon_m = 10^m$, $h = 10^n$, $2f/(hf') = 10^u$ and $f''/(2f') = 10^v$ the optimum value of $n$ satisfies $v + n = -n + u + m$, hence:

$$n = (u - v + m)/2$$

Typically, with the scaling considerations mentioned above $(u, v, m) = (-1, -1, -15)$ giving a step size $h \approx 10^{-7}$ and an overall relative computation precision around $\varepsilon = 10^{u+n} \approx 10^{-8}$. However, the computation above discards the error induced by the evaluation of $f$ itself (propagation error); a slightly bigger step size (around $h \approx 10^{-6}$) gives in practice better results and has been retained in the final implementation.

Finally, the computation time using this new objective function is about a factor 4 longer than the mapping-only approach, but this remains within minutes on a standard PC for typical applications (general lighting, automotive, etc.).

4. Sample applications

The procedure presented above allows the design of various optical configurations with a wide range of applications. We present two complex cases. For each design below the improvement between the initial mapping approach and the flux optimized approach is presented, and the examples are given in order of increasing complexity.

4.1. Double sided freeform lens

First, a refractive case with two freeform surfaces is presented. The target plane is positioned perpendicular to the light axis and is centered on it at a distance of 100 mm. The prescribed irradiance is the letter “B”.

![Fig. 3. Double sided freeform lens projecting the letter “B” (lengths in mm, irradiance in a.u.)](image)

(a) with mapping optimization only; (b) with mapping and flux optimization; (c) illustration of the geometry.

The flux optimization provides a sharper cut-off at the boundaries of the pattern as can be seen in the two central holes, and on the upper and lower tips of the left part of the distribution.
The two freeform surfaces forming the lens were designed using 4,000 triangles for each optical element. The final surfaces consist of two NURBS fitted to the two triangular meshes. The design assumes that the lens is made of glass ($n = 1.5$) and that a light cone with a half-angle of 45 degrees is captured from the point source.

4.2. Logo lens

To demonstrate the abilities of the presented algorithm a much more elaborate example is presented. The optics has been designed for a perfectly collimated source. The logo is projected on a surface of $55 \times 17.5 \text{ mm}^2$ at a distance of $100 \text{ mm}$ from the source, and has a resolution of $530 \times 160$ pixels. We present the results with one and two freeform surfaces, respectively. The optics has a size close to the target one: $50 \times 15 \text{ mm}^2$.

![Fraunhofer logo images](image)

Fig. 4. Ray-tracing of high-resolution logo generated with 16 million rays (lengths in mm, irradiance in a.u.); (a) prescribed irradiance distribution (passed to the algorithm in black and white); (b) one freeform surface with mapping optimization only; (c) one freeform surface with mapping and flux optimization; (d) two freeform surfaces with mapping and flux optimization.

The ray tracings presented in Fig. 4 were performed with 16 million rays and a detector grid having the same resolution as the prescribed irradiance. Once again, the improvement over the mapping optimization is clearly visible: the contours of the letters making the logo are much sharper and better defined, particularly in the bottom right part (see analysis focused on the “ILT” part below). The overall result looks equally satisfactory with one or two freeform surfaces.

Similar to what has been done previously by Bortz and Shatz [9], the following fractional RMS measure was computed to assess the quality of the design:

$$\text{RMS} = \sqrt{\sum \left( \frac{I_c - I_p}{I_{ref}} \right)^2}$$

where $I_c$ is the computed irradiance as given by the ray tracing, $I_p$ is the prescribed irradiance and $I_{ref}$ is a reference irradiance. Bortz and Shatz restricted their computation to an area where the irradiance was sufficiently high and hence took $I_{ref} = I_p$. In our case, given the contrast at hand (about 4:1) and the fact that the analysis should not be restrained to the letter cores (areas of high irradiance), we computed the RMS with both $I_{ref} = I_{\text{avg}}$, the (arithmetical) average of the light irradiance, and $I_{ref} = I_{\text{max}}$, the peak irradiance found at the core of a letter, for example.
For statistical relevance, the analysis was also restrained to a sub-part of the logo where a fine level of detail is found (see Fig. 5). The sub-area was traced with 2 million rays and covers $66 \times 39$ detector pixels. With this setup, the ray tracing of a homogeneous pattern would result in a statistical relevance of 3-4%.

![Fig. 5. Zoom on the lower right part of the logo for RMS analysis. Ray-tracing with 2 million rays on a grid of $66 \times 39$ points (lengths in mm, irradiance in a.u.). (a) one freeform surface with mapping optimization only; (b) one freeform surface with mapping and flux optimization; (c) two freeform surfaces with mapping and flux optimization.](image)

Table 1 presents the resulting fractional RMS computed with the two different irradiance references (average and maximum). The improvement brought on by the flux optimization is clearly visible for both references. The last two lines show that the ray-tracing falls slightly outside the statistical boundary given above. The sharp edges of the letters forming the logo are challenging to project and prove to be the main deteriorating factor. Comparing part b) and c) of Fig. 5, we can also identify a slight loss of irradiance homogeneity within the letters, confirming that the single surface design is less complicated. However, in both cases, the shape of the pattern has clear and well defined contours.

Finally, the overall optical efficiency in this case is close to its maximum theoretical value, since all the rays of the collimated source are captured by the optical system.

<table>
<thead>
<tr>
<th></th>
<th>$RMS_{\text{avg}}$</th>
<th>$RMS_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>One surface (mapping optimization only)</td>
<td>33.7%</td>
<td>16.0%</td>
</tr>
<tr>
<td>One surface (mapping and flux optimization)</td>
<td>10.1%</td>
<td>4.82%</td>
</tr>
<tr>
<td>Two surfaces (mapping and flux optimization)</td>
<td>10.2%</td>
<td>4.84%</td>
</tr>
</tbody>
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5. Conclusion and future work

This article presents an extension of the work outlined in [1]. An efficient optimization procedure based on the physics of the system was introduced to overcome the limitations posed by the sole mapping optimization. The procedure allows the design of multiple freeform surfaces and can render high resolution irradiance patterns, as demonstrated by the projection of
a simple pattern, and by the design of a lens made of two freeform surfaces projecting a high resolution logo (530 × 160 pixels). For this last example, the quality of the irradiance given by the ray-tracing was analyzed through the computation of the fractional RMS which confirms the notable improvement over the mapping optimization.

The next challenge is to gain a better understanding of the effects of the source extent, which poses fundamental limits on the achievable irradiance resolution in real-world applications.

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