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Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Smooth intensity maps and the Bortfeld-Boyer sequencer

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Abstract

It has been empirically verified that smoother intensity maps can be expected to produce shorter sequences when step-and-shoot collimation is the method of choice. This work studies the length of sequences obtained by the sequencing algorithm by Bortfeld and Boyer using a probabilistic approach.
The results of this work build a theoretical foundation for the up to now only empirically validated fact that if smoothness of intensity maps is considered during their calculation, the solutions can be expected to be more easily applied.

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1 Introduction

A treatment plan in intensity modulated radiotherapy treatment (IMRT) is applied by blocking parts of the beam surface for predetermined times. This is realized by multileaf collimators (MLCs). If the beam is switched off while the leaves configure a "shape" through which some radiation is emitted, then this is referred to as "step-and-shoot" delivery. The collection of all shapes and the time the beam is switched on for a shape (its "monitor unit") is referred to as the "sequence" of a given treatment plan.

It has been empirically verified that smoother maps can be expected to produce shorter sequences. However, the absolute statement that smoother maps always result in sequences with fewer shapes is incorrect. Counterexamples exist where a higher variation does not lead to an increase in the number of shapes. Therefore, any statement about the relationship between the smoothness of a map and the length of its sequence in static collimation can only be made in terms of their expected values when a probability distribution of the intensity maps is assumed. Thus, the appropriate mathematical equivalent statement to the observed phenomena can be given by

Proposition 1.1

If the total variation of an intensity map increases, the number of shapes required to sequence it can be expected to increase as well.

The total variation of an intensity map is taken as the inverse to the measure of smoothness and given by

$$TV := \sum_{i=1}^{m} \sum_{j=1}^{n} \left(x_{i,j} - x_{i,j-1}\right)^2 , \tag{1}$$

where m is the number of rows in the map, and n is the number of columns. $x_{i,j}$ denotes the intensity with which beamlet (i, j) contributes to the dose. In this work, we will study the number of shapes produced by the sequencing algorithm written by Bortfeld and Boyer [1] when a certain type of intensity map is sequenced. These special intensity maps are created using a specified probabilistic method. Then, a result similar to Proposition 1.1 for the case of sequences produced by the Bortfeld algorithm is proven. In particular, we will show that by varying a parameter of the probabilistic method used to produce the intensity maps both, the expected total variation and the expected number of shapes will increase. In the discussion we point to an obvious generalization of the probabilistic method to create the intensity maps which will not alter the results of the analysis. We then conclude that the stated Proposition is true for a rather large class of intensity maps.

This work is, to our knowledge, the first attempt to construct a theoretical foundation to the afore-mentioned empirical knowledge about the tendency of correlation between smoothness and sequences.

The implications are not limited to the mathematical insight: the message that smoothing maps is beneficial to the treatment is carried implicitly and explicitly in many publications [2, 3, 4, 5, 6, 7, 8, 9]. This work supplies some theoretical justification to these undertakings.

2 Random intensity maps

To construct a random intensity map, we let each beamlet intensity $x_{i,j}$ be a random variable with value depending on its left neighbor $x_{i,j-1}$ and an added stochastic term. To this end, let a random variable $\Delta_{i,j} \in \{-1, 0, 1\}$ be distributed as follows. p is a parameter between 0 and 1.

$$\Delta_{i,j} = \begin{cases} -1 & \text{with probability } \frac{1-p}{2} \\ 0 & \text{with probability } p \\ 1 & \text{with probability } \frac{1-p}{2} \end{cases} \tag{2}$$

We will then let each beamlet intensity be given by $x_{i,j} = x_{i,j-1} + c\,\Delta_{i,j}$, where c is a positive constant. $RandomIntMap$ defined in Algorithm 2.1 formalizes this procedure. To ensure non-negativity, $x_{i,0} := nc$ for every row i. Taking the expectation of the total variation TV (1), we have

Algorithm 2.1 Method to create random intensity maps

Procedure: $RandomIntMap$

Input: Constant c, a stream of random variables $\Delta_{1,1}, \ldots, \Delta_{m,n}$ identically and independently distributed with probability mass function (2)

Output: Intensity map $\mathbf{x}$

```
for row i = 1 to m do
    Let $x_{i,0} := nc$
    for column j = 1 to n do
        Let $x_{i,j} := x_{i,j-1} + c\,\Delta_{i,j}$
    next j
next i
```

$$E\,(TV) = E\left(\sum_{i=1}^{m}\sum_{j=1}^{n}(x_{i,j} - x_{i,j-1})^2\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} E\,(x_{i,j} - x_{i,j-1})^2 .$$

The expected value of the squared level jump is given by

$$E\,(x_{i,j} - x_{i,j-1})^2 = E\,(x_{i,j-1} + c\,\Delta_{i,j} - x_{i,j-1})^2 = E\,(c\,\Delta_{i,j})^2 = c^2\,E\left(\Delta_{i,j}^2\right),$$

and the expected value of $\Delta_{i,j}^2$ is given by

$$E\left(\Delta_{i,j}^2\right) = (-1)^2\,\frac{1-p}{2} + (0)^2\,p + (1)^2\,\frac{1-p}{2} = 1 - p.$$

The expected total variation is then given by

$$E\left(TV\right) = mnc^2\left(1-p\right),$$

and is linear in the probability that the level jump is not zero. This parameter $q := 1-p$ can be thought of as our “control” for the smoothness of the maps we produce.

3 Number of shapes in Bortfeld sequences

The sequencing algorithm given by Bortfeld and Boyer [1] was the first that resulted in sequences with provably optimal beam-on time (total number of monitor units). The number of shapes resulting from an application of this sequencer can approximately be given by

$$NS(\text{Bortfeld}) \approx \max_{i=1,\dots,m} SPG_i = \max_{i=1,\dots,m} \sum_{j=1}^{n} \max\left(0, x_{i,j} - x_{i,j-1}\right),$$

where SPG_i stands for the “sum of positive gradients” in row i. This is the number of iterations the algorithm will perform, and in each iteration a shape with monitor unit 1 is created. The actual number of shapes will be slightly less since different iterations may produce the same shapes. However, this is very unlikely if the intensities are continuos variables and we neglect this fact for the remainder of this work.

Next we determine the expected value of maximum sum of positive gradients in terms of $q = 1-p$. Denote by $SPG_{(m)}$ the maximum sum of positive gradients over m rows of the intensity map:

$$SPG_{(m)} := \max_{i=1,\dots,m} SPG_i.$$

Further denote by L the maximum intensity value in $\mathbf{x}$: $L := \max_{i,j} x_{i,j}$. Note that the maximum sum of positive gradients in one row is bounded by $\hat{n} := \frac{nL}{2}$. Then

$$\begin{aligned} E\left(NS(\text{Bortfeld})\right) \approx E\left(SPG_{(m)}\right) &= \sum_{t=0}^{\hat{n}} t \Pr\left(SPG_{(m)} = t\right) \\ &= \sum_{t=1}^{\hat{n}} t \left(\Pr\left(SPG_{(m)} \le t\right) - \Pr\left(SPG_{(m)} \le t-1\right)\right) \\ &= \hat{n} \Pr\left(SPG_{(m)} \le \hat{n}\right) - \sum_{t=0}^{\hat{n}-1} \Pr\left(SPG_{(m)} \le t\right) \\ &= \hat{n} - \sum_{t=0}^{\hat{n}-1} \left(\Pr\left(SPG_i \le t\right)\right)^m, \end{aligned}$$

where SPG_i is treated as a random variable.
Now we are interested in the rate of change of $E\left(SPG_{(m)}\right)$ with respect to $q := 1-p$. If it can be shown to be positive for all $0 \leq q \leq 1$, then a positive relationship between the variation and the length of Bortfeld sequences would be established.
Let us first study the distribution of SPG_i. That is, we would like to calculate

$$\Pr\left(SPG_i \leq t\right) = \Pr\left(\sum_{j=1}^{n} \max(0, x_{i,j} - x_{i,j-1}) \leq t\right).$$

Expressing SPG_i a little differently, we obtain

$$\begin{aligned} SPG_i &= \sum_{j=1}^{n} \chi\left(x_{i,j} > x_{i,j-1}\right)\left(x_{i,j} - x_{i,j-1}\right) \\ &= \sum_{j=1}^{n} \chi\left(x_{i,j} > x_{i,j-1}\right) c\, \Delta_{ij}, \end{aligned}$$

where $\chi\left(A\right)$ represents the characteristic function of event A. Thus, we obtain

$$\Pr\left(SPG_i \leq t\right) = \Pr\left(\sum_{j=1}^{n} \Delta_{ij}\, \chi\left(x_{i,j} > x_{i,j-1}\right) \leq \left\lfloor \frac{t}{c} \right\rfloor\right) =: \Pr\left(P_i \leq \hat{t}\right),$$

where P_i is the number of positive level jumps in row i, and $\hat{t} := \left\lfloor \frac{t}{c} \right\rfloor$.
Note that by (2), the random figure P_i is a binomial random variable with distribution parameters $\left(n, \frac{1-p}{2}\right)$. Thus,

$$\Pr\left(SPG_i \leq t\right) = \Pr\left(P_i \leq \hat{t}\right) = \sum_{u=0}^{\hat{t}} \binom{n}{u} \left(\frac{1-p}{2}\right)^{u} \left(\frac{1+p}{2}\right)^{n-u}$$

Differentiating now $E\left(SPG_{(m)}\right)$ with respect to q gives

$$c(q) := \frac{\partial E\left(SPG_{(m)}\right)}{\partial q} = -m \sum_{t=0}^{\hat{n}-1} \Big(\Pr\left(SPG_i \leq t\right)\Big)^{m-1} d(t,q),$$

where

$$\begin{aligned} d(t,q) &= \sum_{u=0}^{\hat{t}} \binom{n}{u} \left(\frac{u}{2}\left(\frac{q}{2}\right)^{u-1}\left(\frac{2-q}{2}\right)^{n-u} - \frac{n-u}{2}\left(\frac{q}{2}\right)^{u}\left(\frac{2-q}{2}\right)^{n-u-1}\right) \\ &= \sum_{u=0}^{\hat{t}} \binom{n}{u} \left(\frac{q}{2}\right)^{u}\left(\frac{2-q}{2}\right)^{n-u}\left(\frac{u}{q} - \frac{n-u}{2-q}\right) \\ &= \frac{1}{q(2-q)} \sum_{u=0}^{\hat{t}} \left(2u - nq\right) \Pr\left(P_i = u\right). \end{aligned}$$

Let now

$$v := \frac{q}{2-q}, \quad 0 < v < 1. \tag{3}$$

Then

$$c(q) = -\frac{m\,(1+v)^2}{2v} \sum_{t=0}^{\hat{n}-1} \left[\left(\sum_{u=0}^{\hat{t}} \binom{n}{u} v^u \left(\frac{1}{1+v}\right)^n \right)^{m-1} \cdot \left(\sum_{u=0}^{\hat{t}} \binom{n}{u} u v^u \left(\frac{1}{1+v}\right)^n - \frac{nv}{1+v} \sum_{u=0}^{\hat{t}} \binom{n}{u} v^u \left(\frac{1}{1+v}\right)^n \right) \right]. \tag{4}$$

Taking the terms $(1+v)^{-1}$ out of the summations, we get

$$c(q) = -\frac{m}{2v\,(1+v)^{mn-2}} \sum_{t=0}^{\hat{n}-1} \left[\left(\sum_{u=0}^{\hat{t}} \binom{n}{u} v^u \right)^{m-1} \cdot \left(\sum_{u=0}^{\hat{t}} \binom{n}{u} u v^u - \frac{nv}{1+v} \sum_{u=0}^{\hat{t}} \binom{n}{u} v^u \right) \right]. \tag{5}$$

Simplifying once by taking out the v in the denominator of the first fraction, we obtain the following difference in the last bracket

$$\left(\sum_{u=0}^{\hat{t}} \binom{n}{u} u v^{u-1} - \frac{n}{1+v} \sum_{u=0}^{\hat{t}} \binom{n}{u} v^u \right) \tag{6}$$

Now note that

$$\sum_{u=0}^{\hat{t}} \binom{n}{u} v^u = (1+v)^n - \sum_{u=\hat{t}+1}^{n} \binom{n}{u} v^u,$$

and

$$\sum_{u=0}^{\hat{t}} \binom{n}{u} u v^{u-1} = \frac{\partial}{\partial v} \sum_{u=0}^{\hat{t}} \binom{n}{u} v^u.$$

This implies

$$c(q) = -\frac{m}{2\,(1+v)^{mn-2}} \sum_{t=0}^{\hat{n}-1} \left[\left(\sum_{u=0}^{\hat{t}} \binom{n}{u} v^u \right)^{m-1} \cdot \left(n(1+v)^{n-1} - \sum_{u=\hat{t}+1}^{n} \binom{n}{u} u v^{u-1} - n(1+v)^{n-1} + \sum_{u=\hat{t}+1}^{n} \binom{n}{u} v^u \right) \right]. \tag{7}$$

And this finally simplifies to

$$c(q) = -\frac{m}{2\,(1+v)^{mn-2}} \sum_{t=0}^{\hat{n}-1} \left[\left(\sum_{u=0}^{\hat{t}} \binom{n}{u} v^u \right)^{m-1} \sum_{u=\hat{t}+1}^{n} \binom{n}{u} v^{u-1}(v-u) \right]. \tag{8}$$

Now let us look at the terms involved:

$$\left(\sum_{u=0}^{\hat{t}} \binom{n}{u} v^u\right)^{m-1} > 0 \tag{9}$$

$$\sum_{u=\hat{t}+1}^{n} \binom{n}{u} v^{u-1}(v-u) < 0 \tag{10}$$

$$\text{thus} \quad \sum_{t=0}^{\hat{n}-1} \left[\left(\sum_{u=0}^{\hat{t}} \binom{n}{u} v^u\right)^{m-1} \sum_{u=\hat{t}+1}^{n} \binom{n}{u} v^{u-1}(v-u) \right] < 0 \tag{11}$$

for all possible choices of t and q. And, because $-\frac{m}{2(1+v)^{mn-2}} < 0$, we arrive at the conclusion that $c(q) > 0$.

In other words, the Bortfeld sequencing algorithm produces sequences in probably increasing lengths as the intensity maps from the random method become less smooth.

4 Discussion

The main result from this work is that for the type of intensity map created by $RandomIntMap$, the Bortfeld sequences increase with total variation. Notice that the crucial point in the argumentation was the fact that $RandomIntMap$ produces level jumps that are binomially distributed - just like a series of coin flips to decide whether the jump goes up or not. That is, the magnitude of the jumps, c remains in the calculations, but has less impact on the validity of the result.

Imagine a modified randomized method that creates level jumps according to (2) but distributes the magnitudes of the jumps randomly. It is easy to see, that if the jumps are not too small, the variation (1) of the intensity maps increases. It is also relatively easy to imagine that SPG will increase as a result of sufficient variation introduced by differing level jump magnitudes c_{ij}. In other words, the main result that sequences will increase with increasing variation will hold even for the generalized random intensity map creation.

This argument shows that Propsition 1.1 holds for a large class of intensity maps.

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