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Implementation of a Monte Carlo-based Real Options Approach for the Evaluation of Residential Energy Technologies

Master Thesis

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DECLARATION OF INDEPENDENCE

I hereby declare that I have prepared the present work independently, that I have not presented it elsewhere for examination purposes, and that I have not used any other means than those indicated. All knowingly used text excerpts, quotations, or contents of other authors have been explicitly marked as such.

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Nomenclature

μ	Drift of the GBM
σ	Volatility
E_{exp}^{t}	Surplus Electricity feed into the Grid
E_{imp}^{t}	Imported Electricity
j	Investment Time Horizon
NPV_{AIV}	Additional Investment Value of the NPV
P_{e}^{t}	Electricity Price
P_0	Current Energy Price
p_e	State Variable of the Electricity Price
p_e^*	Threshold Value of the Electricity Price
p_g	State Variable of the Gas Price
p_g^*	Threshold Value of the Gas Price
P_{FIT}	Enumeration of the feed in surplus Electricity
Q_{imp}^{t}	Import Heat
t	Lifetime of the Technology
CAPEX	Initial Capital Expenditure
DCF	Discounted Cash Flow
DR	Discount Rate
GBM	Geometric Brownian Motion
MC	Maintenance Costs
NEC	Net Energy Cost
NPV	Net Present Value
OPEX	Operating and Maintenance Costs
RO	Real Option

1 Introduction

Miguel Arias Cañete, Commissioner for Climate Action and Energy described the urge of investments in renewable energies in the following way [30]: "Global investments hold the key to fighting climate change, with trillions already invested in solutions such as renewables and energy efficiency. The Paris Agreement is a massive investment opportunity[...]". The urgent need to transition to climate-neutral energy systems has made renewable energies a central focus of research and development efforts worldwide. Among their many advantages, these sources of energy are decentralized, giving cities and communities the ability to generate and supply themselves with a significant proportion of their energy needs. However: about 75 % of the world's energy consumption occurs in cities, resulting in approximately 70 % of global carbon dioxide emissions [9]. Buildings are a major contributor to these emissions, accounting for a significant share of global energy consumption and emissions. In 2021, buildings were responsible for 30 % of final energy consumption, with direct and indirect emissions contributing to a total of 27 % of global energy sector emissions [27]. Furthermore, it is projected that the global building stock will double by 2050 [30]. These points make it evident that there is a considerable amount of investments required to meet the energy transition goals.

The German building sector is heavily reliant of fossil fuel, accounting for over 80 % of its energy share [6]. However, the German government is now planning to install 500,000 heat pumps annually from 2024, in collaboration with relevant industry players [18]. In addition to the investment in renewable energies, a large part of the building stock is also being refurbished, with a refurbishment rate of currently 1 % which is to be increased up to 2 % [30]. Conversely, this means significantly more investment in residential energy concepts.

Usually, traditional approaches like the discount cash flow (DCF) method are being used when valuing investments into building utility concepts. Investing in renewable energies, and therefore residential energy technologies, is associated with many uncertainties, such as fluctuating energy prices and political framework conditions. The DCF method is not made for valuing investments that are connected to uncertainties, that is why there is a need for another method to value this kind of investments. Recent events like the war in Ukraine that led to supply shortages in gas supply or the Covid-19 pandemic have demonstrated the volatility of the energy market in response to external influences. Therefore, a method that considers uncertainty, evaluates planned investment accordingly and allows the investor options like waiting and abandoning the investment is crucial.

Real options (RO) theory provides an efficient way to take these uncertainties into account. Up to now, the RO analysis has found little application in the valuation of renewable energy projects because the method is often complex and hard to understand. Horn et al. [40] surveyed 1500 of the largest Scandinavian companies, discovering only 6 % of the respondents use RO, the most used technique, the NPV method, is used by 74 % of the companies. The lack of familiarity is the most named reason why companies are not using ROs, respondents that are familiar with ROs are mostly deterred by the complexity of this approach. The authors of Loncar et al. [54] claim decision makers will become more open to ROs if they acquire a deeper understanding, and possible investment strategies. The decision to base the algorithm used in this work on the approach developed by Locatelli et al. [53] is primarily because of its good usability and understandability.

This master thesis aims to implement a Monte Carlo-based RO analysis to evaluate technology options for the transformation paths of residential energy systems. Despite the named reasons, so far, the literature lacks examples of applying RO theory for use cases in the residential energy sector. In this thesis, the method developed by Locatelli et al. [53] will be utilized, which was particularly developed for use cases in the energy sector. The approach they developed is the basis of this work, the concept of "exercise thresholds" is introduced and used in this work to receive the relevant investment parameters. The aim of this work is to apply the method and answer the following questions:

- How should the applicability of the Monte Carlo algorithm according to Locatelli be evaluated for the assessment of investments in technology options for residential energy systems?
- What influence does the consideration of waiting options have on the evaluation of residential technology options?

In the context of this work, a prototype is developed to answer these questions.

At the beginning, different methods for investment valuation are presented. Advantages and disadvantages are worked out, and it is seen to what extent these methods are suitable for the evaluation of uncertainties and flexibilities. Subsequently, the method of the RO is explained by taking relevant literature in the energy sector and in particular for residential energy technologies into account. Then an overview of the current transformation paths for the building sector is presented.

The next chapter presents the basic framework of the model and discusses the methods used to determine the key parameters for building the RO model.

In the results section, it is presented how the developed model was parameterized and the results for each technology. In addition, a sensitivity analysis is performed.

Afterwards, the mentioned research questions are answered, resulting questions and research areas for the improvement of the model are presented.

2 Real Options Theory

2.1 Approaches to Investment Valuation

There are multiple ways to determine the value of investments, some of them are shown in Figure 2.1. Depending on the selected method for calculating the investment value, the uncertainty or flexibility of the investment is considered, and it is possible to combine these methods to end up with the desired results.

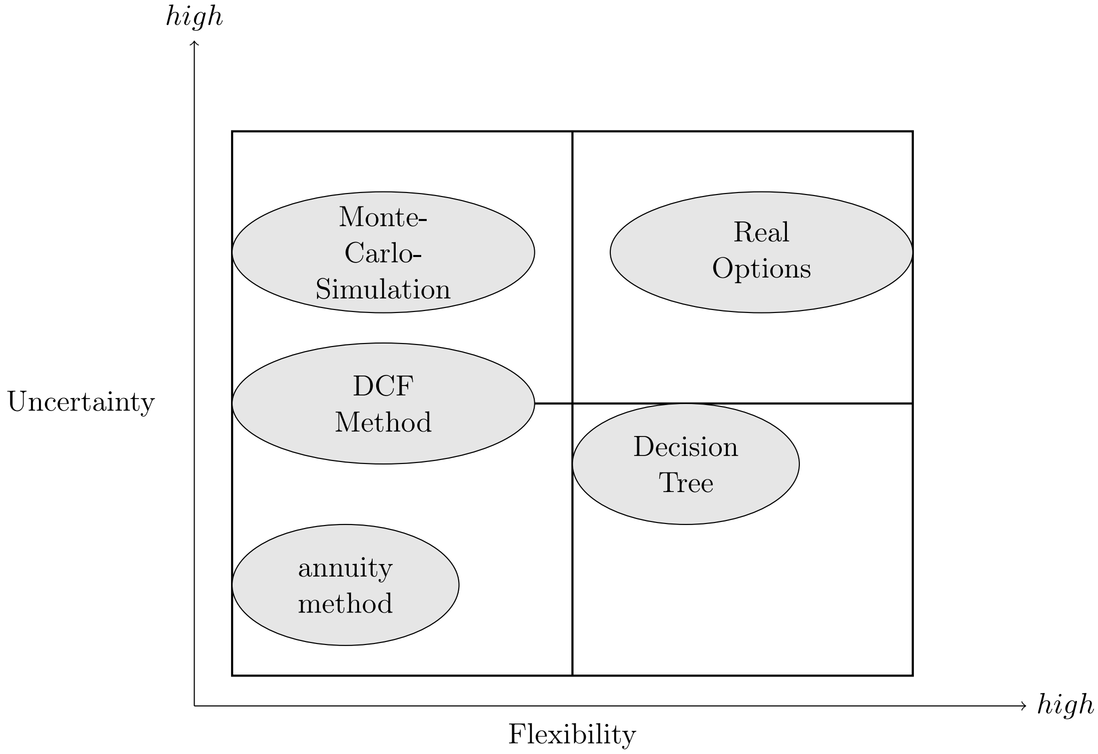


Figure 2.1: Methods for the assessment of investments based on [39].

The most commonly used method to assess projects, including Renewable Energy projects, is the DCF method [83]. When applying this method, the cash flows are discounted to the current value, and the Net Present Value (NPV) is the sum of DCF over the investment life cycle [61]. This method is, due to its simplicity, easy to implement. However, the DCF method has three major drawbacks that have favored the development of the RO approach. The drawbacks are the following [53]:

1. It is not easy to estimate the future cash flow of a project. The price of the output, production rate and the investment cost are assumed to be constant over time, which is not the case in reality.

2. The choice of the discount rate reflects the risks of the cash flows, which is unavoidably subject to estimation errors.

3. The assumed passivity of the management, such as the option to delay, expand or to abandon the project if the outcome is extremely negative.

The consequence of this drawbacks in particular for energy projects leads to the interest in more flexible tools to value investment projects [80, 81, 43], so the uncertainty of investment can be handled more accurate [49, 37].

One alternative approach is the decision tree method, where parameters that generate uncertainties of investments are considered with the help of a finite number of environmental states and their probabilities of occurrence in a dynamic model [36]. Investment decisions can then be made (sequentially) as a function of these environmental states and the predefined uncertainties can be taken into account [57].

In finance, the annuity method, also known as the equivalent annual cost method, is used to determine the cost per year of owning and operating an asset over its lifetime. It involves calculating the equivalent annual cost by dividing the negative NPV of a project by the "present value of the annuity factor." This approach allows for a more straightforward assessment of the project's expenses on an annual basis, making it easier to compare different investment options.

Another approach is to use the Monte Carlo method in project evaluation, which enables the creation of a distribution of potential values by considering various sources of uncertainty. This technique is particularly suitable for assessing projects with high levels of uncertainty, allowing decision-makers to gain a more comprehensive understanding of risks and potential outcomes. By running multiple simulations, the Monte Carlo method generates a range of potential project values, along with their associated probabilities. This statistical approach is particularly valuable for projects with high levels of uncertainty, as it provides decision-makers with a more comprehensive understanding of the project's risks and rewards [8]. The RO approach, which will be used in this thesis, will be explained in detail in the next section.

2.2 Real Options Theory

The problems with using DCF methods were discussed in the previous section. In order to overcome these limitations, the concept of "options" can be employed, which provides the right to react to new market situations and introduces additional flexibility to investment decisions. To better understand the theory of the RO approach, it is crucial to clarify the difference between *financial options* and *real options.*

Financial options include a large and diverse group of assets of individual stocks, stock indices, government bonds, currencies, precious metals, and futures contracts. *Real options* include capital budgeting, investment decisions, and business transactions. [20]

Table 2.1 shows the analogy between financial and ROs, highlighting their respective characteristics.

Table 2.1: Analogy between financial and real options [71].

Financial Options	Real Options
Stock Price	Present value of cash flows from the Project
Exercise Price	New investment required
Time to expiration	Length of time until decision must be made
Risk-free rate of return	Time value of money
Volatility	Risk of expected returns

The RO approach takes three characteristics into account to determine the optimal decisions of investors [29]. Firstly, investments are partially or fully irreversible, that means once an investment decision is made, it is difficult or connected with high costs to reverse it (irreversibility). Making irreversible investment decisions when facing uncertainty is associated with a high risk. Secondly, the value of an investment can be increased by delaying the investment decision and gathering more information (flexibility). By postponing the investment, the investor is allowed to reduce the risk of the investment by making more informed investment decisions once additional information is available. Lastly, investments are inherently subject to uncertainties, that originate from various factors, such as technological advancements, market dynamics, policy changes, and societal influences [48] (uncertainty). These uncertainties introduce potential risks for investment decisions and have a substantial impact on investment outcomes.

RO theory considers the factors of irreversibility, flexibility, and uncertainty, enabling decision-makers to incorporate them into their investment analysis. By considering these elements, RO theory provides a comprehensive framework to assess and manage risks associated with investment decisions.

The most common RO valuation techniques will be explored, which serve as essential tools for integrating the dimensions of time, uncertainty, and flexibility into investment decision-making. These techniques provide practical approaches to assess and evaluate the value of ROs within investment scenarios. These techniques have gained attention in various research areas and offer effective ways to assess the value of ROs. There are three commonly used methods to solve this problem [68]: (1) Black-Scholes equation [17] (stochastic differential equations), (2) dynamic programming [25] (binomial models), and (3) stochastic simulation models (Monte Carlo). Black and Scholes model [17] was the first successful attempt to quantify the value of options in financial markets. They developed a partial derivative equation that can price the value of an option, based on the assumption that stock prices follow a Brownian Motion. Cox et al. [25] assume that the stock price follows a binomial process over a series of discrete time steps. At each time step, the price can either move up or down. The Monte-Carlo simulation has three significant advantages over the Black-Scholes and binomial tree techniques [68]: It is possible to model multiple underlying assets, to introduce cash flows from different sources and to input multiple sources of uncertainty with different types of stochastic behaviour. One major drawback of the Monte-Carlo simulation is that it reveals uncertain paths forwards, but optimal timing for investments must be obtained in backwards fashion [10], like it is done in dynamic programming. The approach tackling this issue in the Least Squares Monte-Carlo algorithm developed by Longstaff and Schwartz [55]. This approach can reduce computation time compared to a traditional Monte-Carlo simulation and also obtains the optimal exercise strategy for each path.

ROs can be categorized into three main types, as proposed by Copeland and Keenan

[23, 24]. These categories can be classified into growth options, deferral/learning options, and abandonment options:

1. *Investment and growth options:* These include (1) **scale-up** options, early entrants can take advantage of cost-effective follow-on investments later on when the market grows; (2) **switch-up** options in which a rapid commitment to the first generation of a product or technology gives a company a preferred position in moving to the next generation of the product or technology; and (3) **scope-up** options. Investing in proprietary assets in one industry allows a company to enter another industry at a low cost.

2. *Deferral and learning options*: Also called **study/start** options, delay the investment until more information or skill is acquired.

3. *Disinvestment and shrinkage options:* These include (1) **scale-down** options, where the investor shuts down or shrinks a project part until new information are changing the expected payoffs; (2) **switch-down** options to more cost effective and flexible assets as new information is obtained. In the case of investing into technologies to supply buildings with energy, that would be a switch in the used technology; and (3) **scope-down** options where I limit the scope of (or abandon) operations when there is no further potential in a business opportunity.

2.3 Real Options in the Energy Sector

The characteristics of irreversibility, uncertainty, and flexibility can all be found in the energy sector. At this point it might be important to mention that the term flexibility, usually refers to the flexible operation of energy assets when used in the energy sector, in the case of this work it refers to the flexibility of a decision maker during the planning phase of the system.

There are a variety of uncertainties that can arise when investing in the energy sector, that significantly influence the value of the project. In the energy sector in particular, these uncertainties include volatile market prices for gas and electricity, the political framework, which include different financial promotion of the respective technologies and changes in Feed-in-Tarif laws, as example for Germany [52, 12]. The constantly evolving and changing conditions are leading to uncertainties, when performing in an investment in the energy sector and make it a difficult challenge to value investment in the energy sector. Such uncertainties raise questions about the long-term viability of technologies and which investments will ultimately yield the highest returns.

In addition to uncertainty, the flexibility of the investment also plays a crucial role in decision-making in the energy sector. The flexibility of using investment options allows the investor to choose when to invest, providing the opportunity to wait and make a more informed decision. Moreover, flexible decision-making allows investors to choose from a variety of technologies available in the market. This further enhances the value of using ROs, as each technology is influenced by different uncertainties [71]. The ability to consider these uncertainties and select the most suitable technology becomes a significant advantage in the investment evaluation process.

In the energy sector, especially residential energy in the case of this work, the utilization of ROs can be a powerful tool to value the flexibility of investment decisions and to take uncertainties into account. To gain insights into potential applications within the energy sector, first, general applications in this field will be examined. Subsequently, the focus will shift to residential energy technologies and the selected methodology employed in this thesis.

Relevant Literature Real Options in the Energy Sector The literature review first examines the sources of uncertainty and the evaluation techniques commonly associated with renewable energy projects. The review papers Alonso-Travesset [10], Kozlova [46] and Ceseña [21] investigate sources and different types of uncertainties when applying RO methods for the valuation of renewable energy projects. The most commonly uncertainties analyzed in these papers are the electricity price, capital expenditure and operational expenditure, governmental subsidies, CO2 Price, Fuel price. Electricity prices had the biggest share of interest in the reviewed papers with 70 % in Alonso-Travesset and 48 % in Kozlova. Kozlova also names the commonly used approaches for modeling and valuing RO, such as partial differential equations expressed with the Black-Scholes model, Binomial trees, simulations in particular the Monte Carlo Simulation, Fuzzy Theory and Dynamic Programming.

Other papers that present relevant use cases are the following: Nadajarah and Secomandi [63] give an overview of the current state of applications of RO theory in the energy sector. They investigate the RO elements - option type, valuation methodology, model formulation, price risk dynamics and optimization scheme. They also give an outlook of possible future research opportunities, which are - the transition to a clean energy landscape, rich models and advanced methods, and the analysis, support, and improvement of practice.

Bøckman et al. [19] propose a RO model to assess small hydropower projects that are subject to uncertain electricity prices. They found a certain price limit for initiating the project. Three projects were assessed using this method, with two being good investments and one not being optimal due to the value of the RO being higher than the NPV.

In Muche et al. [62], the valuation of a pump storage using a RO approach that considers future price based unit commitment planning is discussed. The developed power price model is used to capture the properties of power prices, the simulation is carried out to determine the optimal unit commitment and annual total contribution margin for each simulated price path. The RO based valuation is compared with the traditional NPV approach, results show that the NPV approach could result in smaller investment values and therefore face wrong investment decisions.

Mutale et al. [59] discuss the importance of renewable energy generation and the need for economic drivers for investment in the sector. ROs theory is explored as a possible economic driver, but further development is required. An advanced ROs methodology is introduced and applied to a hydro power case study, showing higher expected profits compared to other planning methodologies.

Moriaty et al. [45] propose a model for optimal and dynamic control and long-term valuation of CHP-thermal storage in the presence of uncertain market prices. The proposed approach has been tested on a UK district energy system with CHP, back-up boiler, and thermal storage, and the findings indicate that the operating cost reductions offered by heat storage are robust. They state that thermal storage can be a key component to enable flexibility in district energy systems and is improving their business case.

Xiu et al. [86] determine the investment value of different storage technologies (li-ion batteries, vanadium redox flow batteries, and sodium sulfur batteries) using the binary tree analysis method for RO pricing. The study shows that the investment economy of li-ion battery energy storage system is better than the other two systems under the condition of storage system cost. The Li-ion battery energy storage system has investment income if considering its direct benefits, coal saving benefits, and environmental benefits from the view of social benefits. The authors suggest that policymakers can consider the significant role of energy storage systems on power system stability and introduce subsidy policies on storage projects based on coal saving benefits and environmental benefits of the energy storage system.

Applications from Literature in the Residential Energy Sector To date, there have been almost no cases in the literature where ROs have been used to evaluate investments in energy-generating technologies in buildings and districts [82].

In Andreolli et al. [11], a theoretical framework is proposed using a ROs approach to model households decisions to invest in domestic photovoltaic plants coupled with battery storage. The findings suggest that the option of storing energy via batteries increases investment value and managerial flexibility, encouraging households to invest in larger photovoltaic systems.

Another recently published study [66] shows the challenges of high initial costs, uncertain future expenses, and inflation rates. The study proposes using RO valuation the Least Square Monte Carlo Simulation to evaluate residential photovoltaic investment decisions in Turkey. Comparing the traditional NPV method with the RO approach, it is demonstrated that while NPV deems the investment infeasible, the RO approach with the Least Squares Monte Carlo method supports it by considering future uncertainties and providing cost-effective options, thus highlighting potential government incentives for promoting solar energy generation in Turkey.

Martinez et al. [58] are proposing a ROs methodology for the assessment of domestic photovoltaic systems, which takes into account the expected evolution of photovoltaic technologies and the option to delay investment. The results of the case studies suggest that an RO's convenience to defer investments is mainly influenced by the time value of money, the type of loan, and the forecasts. Through the utilization of ROs to defer investments, domestic photovoltaic projects tend to experience an increase in economic value across the majority of studied scenarios in this paper.

In Tian et al. [79], a RO approach to assess the investment value of photovoltaic power generation in a carbon market-linked context is developed, considering uncertainties and market co-movements. The study finds a negative investment value in the case of Dunhuang, recommending either postponing or abandoning the investment unless conditions improve, while also suggesting potential enhancements through technical improvements and increased subsidy payments.

Kim et al. [44] are proposing a framework for determining the appropriate level of government subsidy for private entities to install clean energy systems using a ROs analysis. The framework considers factors such as greenhouse gas emission trading, life cycle costs, socioeconomic and political factors. A case study was conducted in Seoul, Korea, to verify the proposed method, which involved the installation of a photovoltaic system in an average-sized household. The findings suggest that the proposed framework could help policymakers choose the proper level of government subsidy that will effectively encourage private entities to install clean energy systems.

These case studies demonstrate the practical application and benefits of using ROs in energy investment decision-making in the residential energy sector. For approaches in the residential energy sector, there is a huge focus on photovoltaic systems but no holistic analysis of building or household related technologies, especially considering the heating sector. Furthermore, is has been showcased how ROs can provide valuable insights into optimal system sizing, the role of storage options, consideration of future uncertainties, the role of policy assessments and the evaluation of investment viability in dynamic market environments.

Used Method for the Evaluation Locatelli et al. [53] developed a simple to implement method to value investments in the energy sector. The method is based on simulating several scenarios and creating so-called "exercise thresholds". An exercise threshold gives the investor the right to make a decision. An exercise threshold is therefore a rule to decide whether to exercise or not a certain option on the basis of the values of one or more state variables. Using state variables and exercise thresholds, the probability distribution of the NPV can be calculated. By using this method, the value of the ROs and relevant indications, such as when and in what type of technology to invest, can be determined. This enables decision-makers to assess the value of ROs and make informed choices about when and in what type of technology to invest, most importantly, having a tool to value the risk of waiting to initiate an investment. The method is still relatively new but has already been applied in Biggins et al. [16] to assess whether it is worth waiting to invest in a polymer electrolyte membrane electrolyser for hydrogen production in a wind farm and how much value this investment adds to the wind farm. The findings indicate that contemplating the option to wait not only enhances the expected mean value of the electrolyser investment but also reduces the likelihood of encountering scenarios where the returns are negative.

2.4 Transformation Paths of Buildings

The Federal Ministry for Economic Affairs and Climate Action of Germany developed four different long-term scenarios and strategies for the expansion of renewable energies in Germany until 2045 [74]. In order to represent the spectrum of possible transformation paths for the energy system as comprehensively as possible, various target scenarios are being modeled. Four main scenarios with different focuses are developed: "T45-Electricity", "T45-Hydrogen", "T-45-PowerToGas" and "T-45-RedEff" The technological composition of the transformation pathways is shown in Figure 2.2.

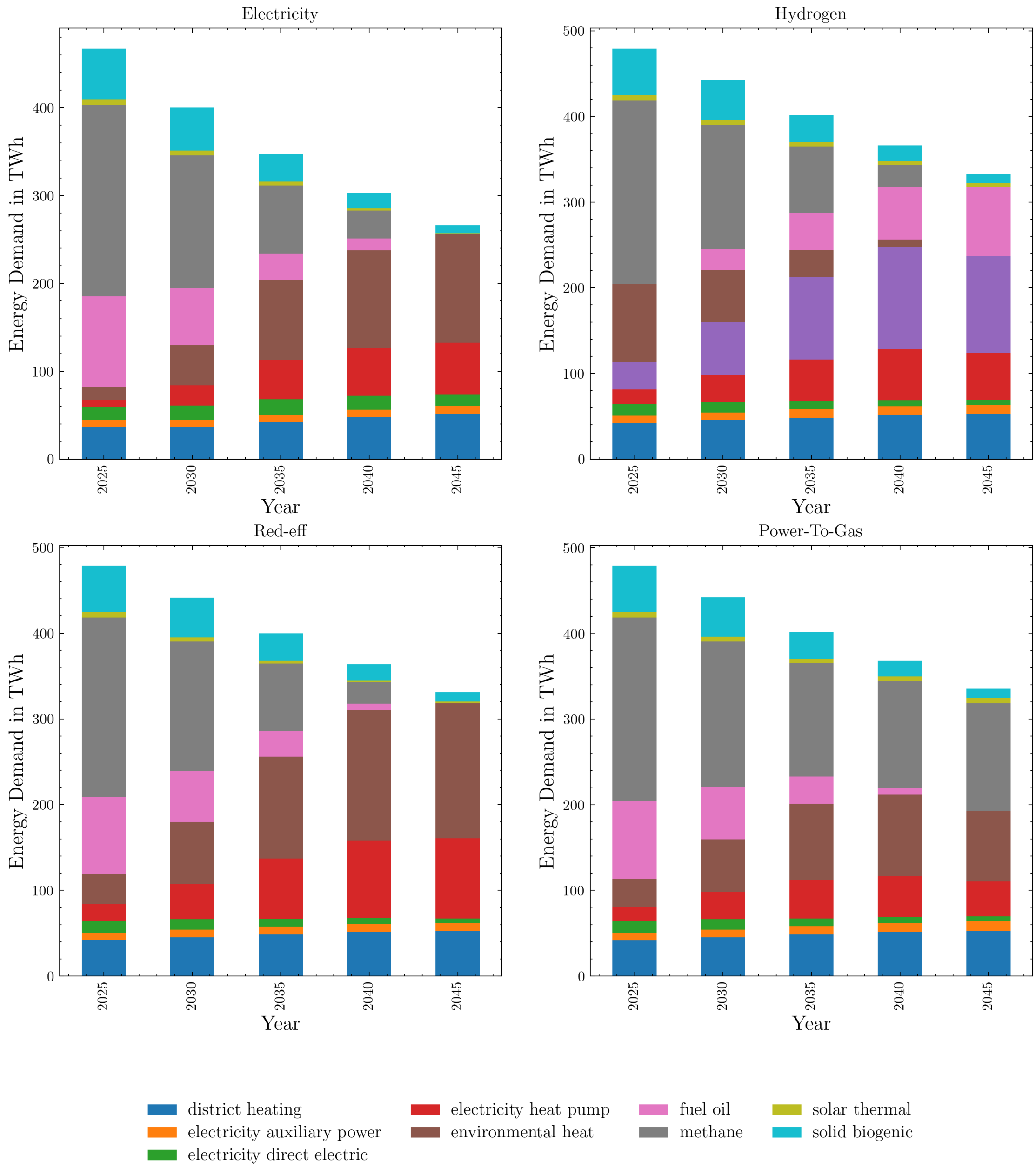


Figure 2.2: Long term scenarios for the energy supply of buildings based on [74].

Upon assessment, it is evident that the sector's target is being met, and that the ambitious objective of achieving a 65 % share of renewable energy by 2024 is on track. Achieving the 2030 targets is projected to be exceptionally challenging in all scenarios except for T45-electricity. Notably, the installation of hybrid heat pumps is expected to be limited to the time frame between 2024 and 2030. Additionally, the availability of biomass is scarce and will undergo significant reductions.

Furthermore, the study's findings highlight several key points. Firstly, regardless of the scenario considered, there is a pressing need to almost triple the number of connections in the district heating system. Moreover, there is a notable and substantial expansion of heat pumps across all scenarios.

Based on these findings, several conclusions can be drawn. It is crucial to maximize efficiency, along with a robust expansion of heat pumps and the development of extensive district heating networks. These factors are deemed essential for achieving the goals and targets outlined in the scenarios. The transition towards a sustainable energy system can be facilitated through a focus on high efficiency and the widespread adoption of heat pumps and district heating

According to the findings of this study, heat pumps can be considered as the sole investment option for heat sources. The future of hydrogen in the building sector is challenging to estimate, leading to uncertain influence. Solar thermal experiences a significant decrease in all scenarios or is not invested in new solar thermal plants, making it unnecessary to consider these two energy technologies. Other studies forecast similar transformation paths, in [31] the structure of heat generators is composed of 70 % heat pumps, 14 % district heating, 10 % biomass and 4 % solar thermal.

3 Theoretical Framework

In this chapter, the methodology of this thesis will be presented. The steps to comprehend the methodology are shown in 3.1 and will be explained in the subsequent sections.

First, the energy system model's structure and the parameters of the technologies used for this case are presented. This is followed by the presentation of the underlying cost functions and required parameters to calculate the investment value for each technology. Uncertainties, such as electricity and gas prices for households, are simulated using a Geometric Brownian Motion. After establishing the fundamentals, the actual algorithm, a Monte-Carlo simulation with exercise thresholds, is presented and explained in detail. By applying the used algorithm, it is possible to calculate the distribution of the means and the standard deviation and make investment decisions based on that.

The algorithm employed in this thesis is derived from the work of Locatelli et al. [53], who developed it for use cases in the energy sector. The authors applied the concept for the evaluation of power plants; in the present case, this methodology is applied to the evaluation of technologies in the residential energy sector.

This algorithm was chosen for its simplicity of implementation and ease of comprehension compared to other available methods. Often, as discussed in the introduction, the application of RO in the energy sector struggles to get implemented because there is a lack of understanding of the approach. By employing this method, one can mitigate such impediments, presenting it as a simpler approach compared to other methods, thus facilitating the user's comprehension.

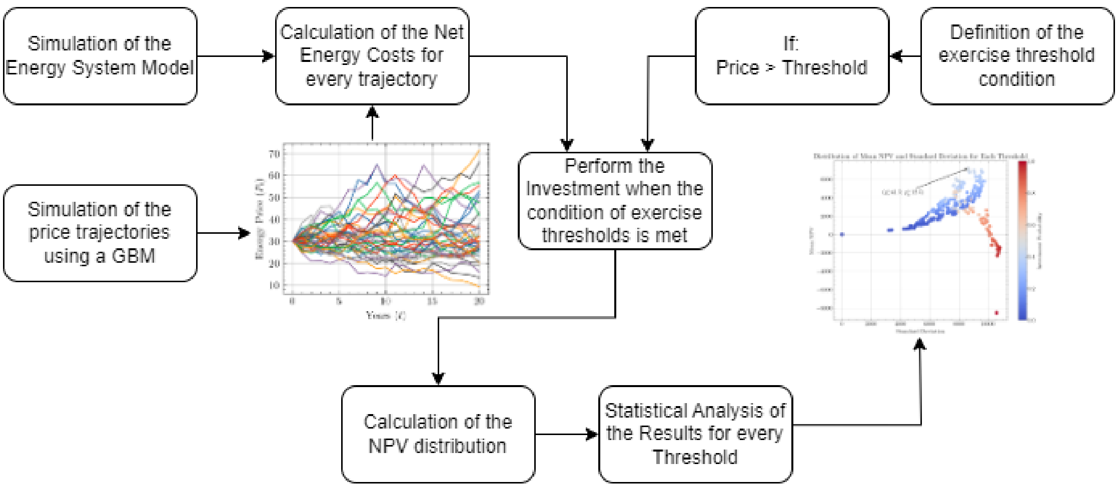


Figure 3.1: Flowchart illustrating the methodological structure of the thesis.

3.1 Configuration of the Examined Building

Figure 3.2 shows the structure of the energy system model. The energy system model is structured into the following parts:

- Surplus Electricity (1): This represents the excess electricity generated by the photovoltaic system, beyond what the building requires for its own consumption. Homeowners receive remuneration for this surplus electricity based on the size of the photovoltaic system, as determined by the Feed-in Tariff set under the Renewable Energy Sources Act.
- Purchased Electricity (2): This is the amount of electricity obtained from the network operator.
- Heat Pump (3) and Hot Water Tank (4): These are used to supply the building with heat. Instantaneous water heaters are employed for providing drinking hot water. The necessary electricity for these systems can be sourced from the grid, the photovoltaic system, or the battery storage system.
- Heat Demand (5): This indicates the amount of heat required by the building.
- Battery Storage System (6): It stores the surplus electricity generated by the photovoltaic system. When there is a demand, this stored electricity is utilized to cover the requirement.
- Natural Gas Boiler (7): This is an alternative method to supply heat to the building.
- Building Electricity Demand: Similar to (8), this represents the required amount of electricity for the building, which can be sourced from the grid, the photovoltaic system, or the battery storage system.
- Photovoltaic System Output (9): This shows the amount of energy generated by the photovoltaic system.
- Amount of Natural gas: The natural gas used to run the boiler comes from (10).

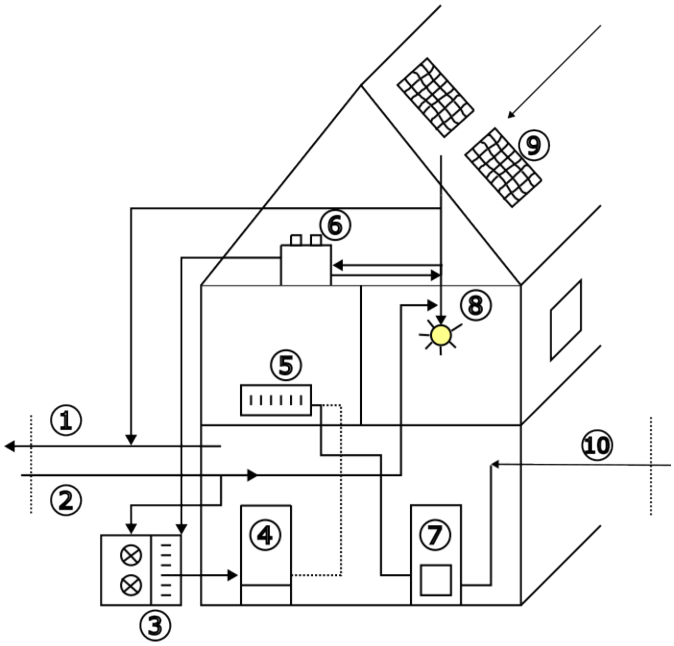


Figure 3.2: Schematic structure of the energy system model.

The energy system model is implemented using the TransiEnt Libary [5]. The TransiEnt Library is a library implemented in the Modelica language for the simulation of coupled energy networks with a high share of renewable energies. The parameters to be determined are building-specific, technology-specific, and parameters used to determine the costs incurred by the investments. The parameters are partly based on own calculations and partly the data is taken from the data sheets of the reference building DE.N.MFH.08.Gen [4], the building represents a typical German apartment building, build in the 90s. The building load profiles used in the work were created using the District Generator. The district generator is a Python tool for creating building-specific heat, electricity and occupancy profiles for residential areas [2].

Building Parameters The used parameters of the reference building are shown in Table 3.1. Heat and electricity demands will differ, depending on which technology option is getting examined. The refurbishment level of one, describes a partially refurbished building.

Table 3.1: Parameters of the building

Building Parameters	
Main Usage of Building	Living
Year of Construction	1989
Location	Potsdam
Used Area	778,1 m^2
Refurbishment Level	Usual Refurbishment [4]
Demands	
Heat demand	39.286 kWh/a
Electricity demand	21.780 kWh/a

Technology Parameters This work examines three different technologies as investment options, the gas boiler serves as a reference investment for the system containing a heat pump. The investment value gained by delaying the timing of investments and incorporating the following technologies: photovoltaic system, heat pump, and battery, will be analyzed. Additionally, the combination of a photovoltaic system and a battery will be assessed since investing in a battery is deemed reasonable when investing in a photovoltaic system. Furthermore, the stand-alone performance of photovoltaic and heat pump systems will be investigated, along with photovoltaic and heat pump as a combination and the integration of all three named technologies. When a heat pump is utilized, the hot water tank becomes part of the system. It is assumed in this work that all technologies are designed for drinking water heating through an instantaneous water heater. The most significant parameters for the energy system model are presented in Table 3.2.

Table 3.2: Parameters of the examined technologies.

System	Parameter	Value
Gas Boiler	Capacity	60 kW
	Boiler Efficiency	1.05
	Calorific Value of Natural Gas	40 MJ/kg
Heat Pump	Nominal heat flow of the heat pump at nominal conditions according to EN14511	13kW
	Anual Performance Factor	2.4
	Nominal electrical power of the backup heater	14.5 kW
	Efficiency of the backup heater	0.95
	Heat pump supply temperature	50°C
Storage	Maximum/Minimum storage temperature	50/45°C
	Volume of the storage	1.788 m^3
Photovoltaic	Capacity	20.8 kWp
	Tilt	0°
	Azimuth	35°
Battery Storage System	Electrical Storage Capacity	20.8 kWh

3.2 Cost Parameters

To evaluate the NPV of the technologies, several parameters need to be considered before performing the calculation, such as the initial capital expenditure (CAPEX) , the maintenance costs (MC) and the lifetime of each technology. Additionally, the discount rate plays a crucial role in the NPV calculation. The discount rate is used to discount future costs and benefits to their present value. It reflects the time value of money and accounts for inflation and opportunity costs. For reasons of simplicity, the discount rate is assumed to be 5 %, for all technology options.

Cost Parameters Photovoltaic Table 3.3 shows possible ways to determine CAPEX and MC values for photovoltaic systems. The lifetime of photovoltaic systems is approximately 25 years [15].

Table 3.3: Cost functions for photovoltaic systems.

Cost Parameters Photovoltaic		
Capex	Developed a function to determine the specific investment costs k: $k = 1.518\,€/\text{kW} \cdot P^{-0.05065}$ where P equals the installed photovoltaic-capacity	2018 - [14]
	Another function for smaller photovoltaic systems in the range of 2-20kW: $k = 2889,5(€/kW) * P^{-0,16}$ where P equals the installed photovoltaic-capacity	2019 - [13]
	In this study, the prices for 19561 photovoltaic systems are examined from 2011 to 2022. From the data obtained, price ranges and mean values of the costs for the corresponding plant size can be derived. The mean specific investment costs 2022 were 1392 €/kW	2023 - [47]
MC	[14] also developed values for the operational costs: 10 €/kW for maintenance costs, $40€/a$ for other variable costs for example meter rent and up to 60 €/a for insurance	2018 - [14]
	[13] developed this function to determine the annually operational costs: $b = 148€(+21€ \text{ if P} > 8kW) + 5€ * P$	2018 - [13]

Cost Parameters Heat pump The lifetime of air heat pumps usually ranges between 15-25 years [26, 33]. In [64], the following equation to determine the CAPEX costs for heat pumps is developed:

$$CAPEX_{heatpump} = 4591,7 * P^{0.6532} \tag{3.1}$$

A maintenance cost of 1 % of the CAPEX costs is applied, which aligns with the range chosen by Greif [76]. The operation of the heat pump includes a hot water tank, and an electric instantaneous water heater is integrated into the system to ensure a continuous supply of heat to the building when the heat pump's capacity is insufficient. Since the instantaneous water heater is often included in the price of the heat pump, the costs of it will not be added separately. For the hot water tank, a regression equation is formed with the price data of a manufacturer [1], to determine the price for the underlying system, an insulation of the tank of the efficiency class B is assumed. This results in the following equation:

$$CAPEX_{hotwatertank} = 1.13 * V - 11.23 \tag{3.2}$$

where V is the volume of the tank in liters.

Cost Parameters Battery Storage The lifetime of battery storage systems is typically observed to be around 15 years, as referenced in the study by Kost et al. (2013) [22]. For this thesis, the focus is on the cost of a lithium-ion battery. Regarding the CAPEX costs for lithium-ion batteries, the costs can vary based on the size of the systems. Existing sources suggest that CAPEX typically falls within the range of 500 - 1200 €/kWh [35, 41, 75]. According to Sensfuß [75], the MC for battery storage systems is negligible for photovoltaic systems below 30 kWp and therefore are not taken into consideration.

Cost Parameters Gas Boiler In the study conducted by Streblow et al. [77], it is indicated that the MC account for 3 % of the investment cost. The same paper presents the following formula to calculate the CAPEX cost.:

$$CAPEX_{gasboiler} = 61.17(€/kW) * P + 4794(€/kW) \tag{3.3}$$

3.3 Technological Learning Effects

Since the RO approach enables the option to invest in the future is mandatory to also determine the cost developments of the determined technologies. Learning curves provide a method to analyze cost development over time, indicating that industrially produced good costs decrease by a constant percentage with each cumulative doubling of volume [69]. This relationship was first described by Wright et al. [85] in 1936, using the example of the aircraft industry.

Different technologies show distinct learning curves, leading to varied developments in investment costs. For instance, when the number of produced units doubles and costs decrease by 20 %, the learning rate is considered to be 20 %. By incorporating these learning curves into the analysis, a better understanding of how the selected technologies change over time is obtained. This is a crucial factor in making informed decisions within the RO framework. This relationship can be expressed by the following equations [32, 85]:

$$C(x_t) = C(x_0)(\frac{x_t}{x_0})^{-b} \tag{3.4}$$

and

$$LR = 1 - 2^{-b} \tag{3.5}$$

where, the relationship between the quantity x_t produced at time t, the cost $C(x_t)$ compared to the output quantity at reference point x_0 and the corresponding costs $C(x_0)$ and the learning parameter b. When Equation 3.5 is rearranged to express variable b, and subsequently inserted into Equation 3.4, the resulting logarithmically decreasing cost function is derived for each respective technology. The specific investment costs for the used technologies, as shown in Figure 3.3, are determined by adjusting the learning rates to the underlying case. The corresponding initial values or investment values at time $t = 0$ are obtained using the previously developed cost function and building data.

Learning rates for photovoltaic systems vary significantly in published studies. To establish a representative value, an average learning rate of 23 % is determined according to Rubin et al. [70]. Other studies report learning rates within a range of 15-23 % [73, 75, 42]. Considering the age of the data, a learning rate of 10 % is applied for photovoltaic systems.

For residential energy systems, studies report varying learning rates for lithium-ion batteries. Schmidt et al. [73] determine a learning rate of $12 \pm 4\%$, while Lowe and Drummond [56] suggest a rate of 10%. Mauler et al. [60] review several studies published after 2015, which indicate learning rates falling within the range of 15-21 %. In this study, a learning rate of 10 % is utilized. The decision is based on the consideration that the studies referenced are a few years old, and the price development starts from the present, suggesting the application of a slightly lower learning rate.

The literature on learning rates for heat pumps is sparse, leading to limited available data for accurate estimation. Due to this data limitation, it is best to be cautious and conservative in determining the learning rate for heat pumps. By comparing the available data with other technologies, a suitable approach is to set the learning rate for heat pumps at 10 %. This choice aligns with the observed learning rates of the other two technologies.

Figure 3.3 shows the price development of the specific investment cost for each technology, considering the discussed learning rates. For calculating the most recently developed cost functions are taken: 1200 €/kW for photovoltaic systems [47], for heat pumps 1100 €/kW [64], battery storage systems with 800 €/kW, for the natural gas boiler and the hot water tank prices are expected to remain constant, since both technologies are already very established and only very limited technological changes are to be expected. In addition, this also takes into account the effect that subsidies, especially for the gas boiler, will not take place if the federal government sticks to its current plans for the energy transition. For the gas boiler and the hot water tank, the respective specific investment costs of 140 €/kW and 1150 €/m^3 are used for the following analyses.

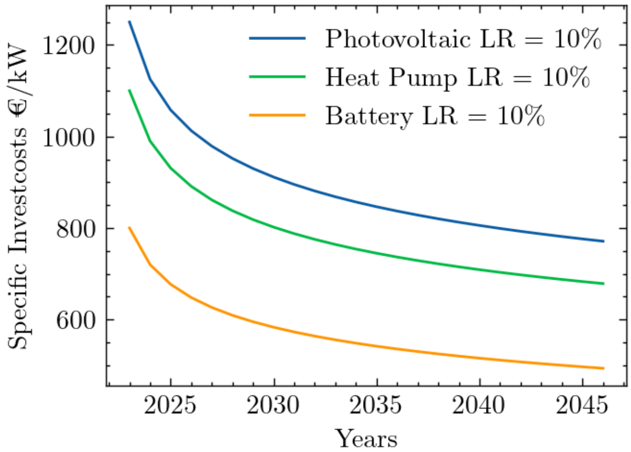


Figure 3.3: Development of the invest cost with the discussed Learning Rates (LR) from 2023 to 2050.

3.4 Modelling and Development of Energy Prices

In the following sections, an overview of the energy market's development will be presented to create a reasonably accurate model for the uncertain variables: electricity and gas prices for household consumers. Energy prices are known to be volatile and subject to fluctuations, as evidenced by recent events such as the Ukraine war and the resulting supply shortages, and the COVID 19 pandemic.

To model the energy prices, the method of choice is the Geometric Brownian Motion (GBM). The GBM is the most common stochastic process used to model price uncertainties in renewable energy applications. It provides flexibility in capturing both mean-reverting and trending behaviors observed in electricity and gas markets. Drawbacks and why this method is suitable for the underlying case will be discussed after the derivation of the method.

3.4.1 Modeling of the Energy Price Trends

The GBM is a stochastic process commonly used to model the price movements of assets, including energy prices, such as stocks, commodities, and currencies. When referring to the "spot price", it denotes the current price for household energy. This term is commonly used in financial markets. The derivation is founded upon the derivation presented in

[38]. The differential equation for this model is:

$$dP_t = \mu P_t dt + \sigma P_t dW_t \tag{3.6}$$

where dP_t is the change in the spot price from t to $t + dt$, μ describes the drift and σ the volatility of the energy prices, dW_t is the increment of a standard Wiener process, so $dW_t \sim N(0, \sqrt{dt})$. The two terms of the equation consist of the drift term $\mu P_t dt$ and the stochastic term $\sigma P_t dW_t$. Both the drift and stochastic terms are proportional to the spot price level at time t. By increasing the stochastic and the drift term, the range of possible price movements will also increase. When the drift term is higher, the price will go up more quickly with each step. To induce a decrease in price, the drift term can be assigned a negative value: $\mu < 0$.

By considering a logarithmic function $x_t = \ln P(t)$ for the price and applying Ito's Lemma to x_t, the following equation is obtained:

$$d_{xt} = (\mu - \frac{\sigma^2}{2} dt) + \sigma dW_t \tag{3.7}$$

By solving this equation for x_t the spot price P_t at time t can be determined, where P_0 is the starting spot price at $t = 0$.

$$P_t = P_0 e^{((\mu - \frac{\sigma^2}{2} t) + \sigma W_t)} \tag{3.8}$$

The variable W_t is a normally distributed random variable with a mean of zero and a variance of t.

One limitation of this approach is that it does not account for the variation in energy prices throughout the day. Additionally, the GBM follows a log-normal distribution, which may not fully reflect energy prices' tendency for mean-reversion, spikes, and jumps [72]. However, it has been argued that these effects can be ignored when considering long-term planning investments and the GBM does not lead to significant errors [51, 34], if electricity spot prices for short-term planning are relevant for the model, another method or a combination of those must be chosen [38, 10].

3.4.2 Development of the Energy Household Prices

To model the evolving energy prices using the previously shown method, assumptions about the volatility and drift of the prices are necessary. The war in the Ukraine and the resulting changes in the framework conditions for Germany's energy supply have led to a massive peak in electricity prices in the energy market, and therefore for increased household prices for electricity and gas. These conditions also impacted the prices for household consumers, with both electricity and gas prices increasing by almost double, as seen in 3.4.

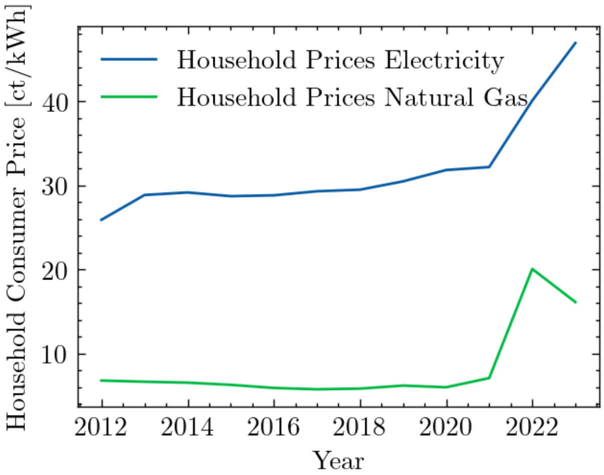


Figure 3.4: Energy Price Development in Households from 2012 until 2023.

Depending on the scenario, the Institute of Energy Economics at the University of Cologne [3] assumes that electricity prices will fall to their pre-war level by 2026. Once electricity prices are back on their pre-war level, they will grow by a small amount each year. To model this using a GBM, two consecutive processes are performed. The first, until 2026, with a strong downward drift with a negative value for μ. Then, a smaller positive value for μ is chosen to represent the gradual increase over time. The same process is used for the modeling of natural gas prices. First the prices will decrease to pre-war level, afterwards they are assumed to increase again. The significant difference between these two is that the gas prices have a stronger upwards drift, due to the increased pricing of greenhouse gases. [7, 78, 3, 64]. Figure 3.5 shows the possible development of the gas and electricity prices. In the work, the price trends developed in the studies are used. As shown in the figure, the price driver, especially for natural gas, is the $CO2$ pricing as specified in the Fuel Emissions Trading Act.

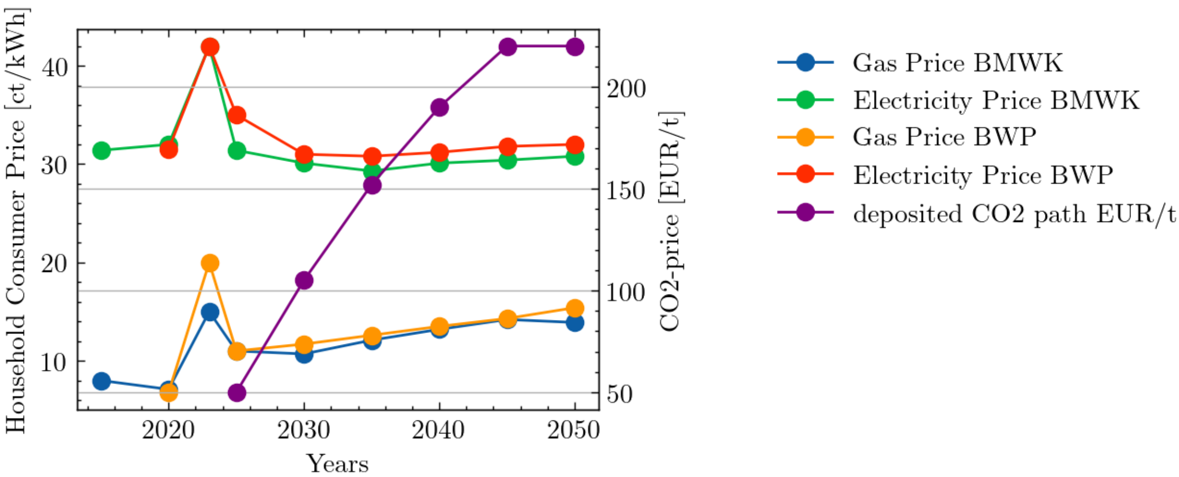


Figure 3.5: Schematic energy price developments based on the studies conducted by the BMWK [78] and the German Heat Pump Association (BWP) [64].

3.5 Cash Flow Valuation

For the evaluation of how much value the option to wait brings to an investment, equation 3.10 is introduced. The option to invest delays the investment until more information about the market and technological conditions is available. In this work two cases need to be considered:

If the option is to invest in a photovoltaic system, or a photovoltaic system including battery storage, only the investment into components of those systems is considered. On the contrary, no investment into any technology will be made, the electricity will still be taken from the grid. The term $CAPEX_{ref}$ will be neglected in these cases. The contrary system is from this point on referred to as the reference system.

To get a realistic case, that allows the evaluation of systems containing a heat pump, both CAPEX terms of NPV_{AIV} need to be considered. In other words, the reference system, of the systems containing a heat pump as a technology option, $CAPEX_{ref}$ needs to be taken into account.

The value we receive when applying these concepts for our use cases is called NPV_{AIV} where AIV stands for Added Investment Value. This equation, which evaluates the added value of delaying the investment, will be explained step by step, considering the underlying assumptions and distinctions between the different cases.

$$NPV_{AIV} = \frac{CAPEX^{j}_{\text{ref}}}{(1+DR)^{j}} - \frac{CAPEX^{j}_{\text{tech}}}{(1+DR)^{j}} + \sum_{t=1}^{\text{lifetime}} \left(\frac{OPEX^{j+t}\text{ref}}{(1+DR)^{j+t}} - \frac{OPEX^{j+t}\text{tech}}{(1+DR)^{j+t}} \right) \tag{3.9}$$

Using an example to gain a deeper understanding of this concept, the discount rate $(1+DR)^{j}$ can be neglected. By assuming that the investment in a heat pump costs 20 monetary units and the investment in a new gas boiler costs 5 monetary units. But by using the heat pump only 2 monetary units are needed to satisfy the energy demand, instead of 4 monetary units when using the boiler. Assuming a lifetime of 8 years. The equation would look like this:

$$NPV_{AIV} = \underbrace{5-20}_{=\text{-}15} + \sum_{t=8}^{\text{lifetime}} \underbrace{32-16}_{=16} \tag{3.10}$$

It becomes evident, that once the CAPEX terms exceed the OPEX terms, investing in the heat pump system becomes beneficial. The OPEX term increases annually due to energy and cost savings achieved through the heat pump, leading to a positive NPV_{AIV} after approximately 8 years.

By deferring the investment, for instance 5 years, the monetary units might be at 5 investing into the boiler. The energy saving per year would increase by one, which would lead to a higher value of NPV_{AIV} after 8 years.

This example shows,how the **option of waiting** can increase the investment value in the underlying case.

OPEX Term The OPEX term in equation 3.10 includes the operational and maintenance costs of a system. In this work, it contains the maintenance costs (as described in the previous section) and yearly energy costs, expressed as Net Energy Costs (NEC). The general expression of the OPEX term in this work is as follows:

Table 3.4: Equations to calculate the Net Energy Costs for all of the respective technologies.

Equation	NEC of the Technologies	NEC of the Reference Technology
NEC_{pv}	$E^t_{\text{imp}}P^t_{\text{e}} - E^t_{\text{exp}}P_{\text{FIT}}$	$E^t_{\text{imp}}P^t_{\text{e}}$
$NEC^t_{\text{pv\&battery}}$	$E^t_{\text{imp}}P^t_{\text{e}} - E^t_{\text{exp}}P_{\text{FIT}}$	$E^t_{\text{imp}}P^t_{\text{e}}$
NEC^t_{hp}	$E^t_{\text{imp}}P^t_{\text{e}}$	$E^t_{\text{imp}}P^t_{\text{e}} + Q^t_{\text{imp}}P^t_{\text{g}}$
$NEC^t_{\text{hp\&pv}}$	$E^t_{\text{imp}}P^t_{\text{e}} - E^t_{\text{exp}}P_{\text{FIT}}$	$E^t_{\text{imp}}P^t_{\text{e}} + Q^t_{\text{imp}}P^t_{\text{g}}$
$NEC^t_{\text{hp\&pv}}$	$E^t_{\text{imp}}P^t_{\text{e}} - E^t_{\text{exp}}P_{\text{FIT}}$	$E^t_{\text{imp}}P^t_{\text{e}} + Q^t_{\text{imp}}P^t_{\text{g}}$

$$OPEX = \frac{NEC^{j+t} + MC}{(1 + DR)^{j+t}} \tag{3.11}$$

where MC represents the maintenance costs, j denotes the point in year in which the investment has been executed and the variable t is the number of periods over which the OPEX costs are discounted, t equals the lifetime of the respective technology.

The calculation of NEC depends on the respective technology options. For instance, investing in a standalone photovoltaic system, the heat demand of the building is unaffected, so there is no need to invest into a natural gas boiler. The required electricity will be drawn from the grid.

In Table 3.4, the provided equations can be found for the calculation of the NECs. In this context, E^t_{imp} (kWh/a) describes the electricity utilized to satisfy the demand of the building, while Q^t_{imp} (kWh/a) denotes the amount of heat used for the supply of the building. The household prices for gas and electricity P^t_g and P^t_e (€/kWh) are taken as the state variables of the considered systems, displaying the given uncertainty of the household prices. The NECs are calculated for every price trajectory. Additionally, the Feed-in Tarif (FiT) must be considered for all technology options containing a photovoltaic system. The FiT represents the payments received for exporting the surplus electricity to the grid and is denoted as P_{FIT} (€/kWh).

When the power generated by the photovoltaic system exceeds the building's electricity demand, there are several options, how this surplus electricity can be used: storing the surplus energy in the battery storage system, feeding it into the grid and obtaining the FiT, or utilizing it in the heat pump. Combining these options of utilizing the surplus electricity is possible. For all systems using a photovoltaic system, the term $-E^t_{\text{exp}}P_{\text{FIT}}$ is incorporated. This term represents the monetary compensation received by the house owner for exporting surplus electricity back to the grid.

Both the photovoltaic system with and without the battery can employ the same equations to calculate the NEC. Notably, the heat demand of the building is not influenced by the investment, as it exclusively impacts the electricity consumption, the type and amount of heat supply remains the same. Therefore, the heat consumption for the reference case can be neglected.

In all cases where heat pumps are used, the gas supply to the building is stopped, and the heat supply to the building depends entirely on electricity. In the scenario where only a heat pump is used in a standalone system, electricity is drawn from the grid to satisfy both heat and electricity demands. As the heat pump impacts both electricity and heat demands, it must be compared to the NEC costs of electricity and gas from the reference system.

At this stage, one might assume that the results are comparable, since the equations for the different technologies and reference systems have great similarities. However, the inclusion of battery storage systems in the building results in a more favorable self-consumption rate, leading to reduced power consumption from the grid. Similarly, the utilization of both photovoltaic and a heat pump yields cost improvements compared to using only a heat pump. This is because surplus electricity that would otherwise be exported to the grid can now be directly employed. For a comprehensive assessment of the cost distribution associated with each combination, a detailed breakdown of the NEC is provided in Figure 3.6.

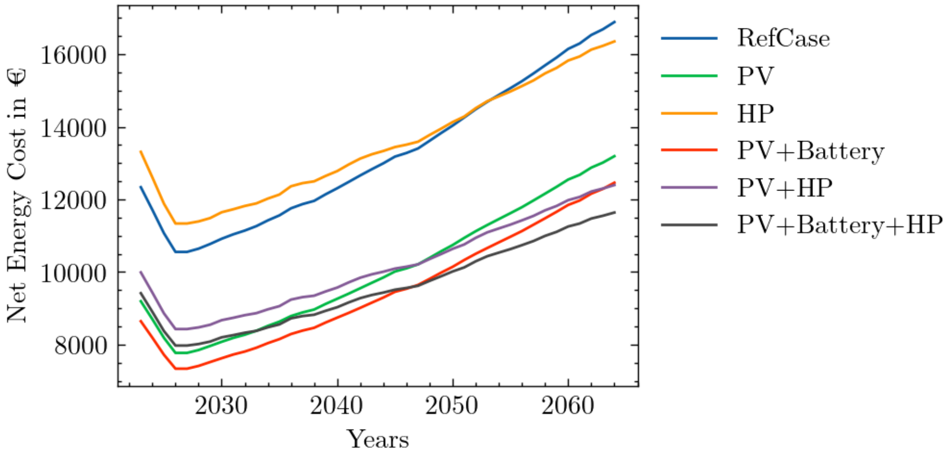


Figure 3.6: Net Energy Costs of the Technologies

CAPEX Term The CAPEX Term consists of two main elements $CAPEX^j_{\text{tech}}$, refers to the CAPEX of the respective technology option, while j is the year in which the investment is performed. If j increases the CAPEX value decreases as described in Section 3.3. Similarly, this principle applies to the reference system's CAPEX, denoted as $CAPEX^j_{\text{ref}}$. The term including the CAPEX costs of both technologies can be expressed as:

$$CAPEX_{\text{Term}} = \frac{CAPEX^j_{\text{ref}}}{(1+DR)^j} - \frac{CAPEX^j_{\text{tech}}}{(1+DR)^j} \tag{3.12}$$

The exception to the change in this term occurs when investing exclusively in a photovoltaic system, which has no impact on the heat inflow and outflow of the building. In the reference case, where electricity is exclusively drawn from the grid, no additional investment is necessary. Thus, in such situations, $CAPEX^j_{\text{ref}}$ equals zero, since no investment into a gas boiler is performed, resulting in the following equation:

$$CAPEX_{\text{Term}} = -\frac{CAPEX^j_{\text{tech}}}{(1+DR)^j} \tag{3.13}$$

Crucial assumptions The crucial assumptions made in the last sections, on the way to the equation of NPV_{AIV}, and the ones that did not get mentioned, will be named now:

- The calculation of the CAPEX and OPEX costs got discussed in Section 3.2. The CAPEX values of the technology options will decrease, due to the technological learning effects, while the assigned MC stay constant over time.
- The time horizon, in which the investment decision is going to be made, is set to 15 years. At that point more recent information about the developments of the energy prices, but also the CAPEX values should be available.
- The assumption is made that the lifetime of the technology option is equivalent to the one of the compared reference system. To have a comparability of the systems, the lifetime is assumed to be 20 years across all systems.
- The discount rate is 5 % in the whole work.
- The natural gas boiler is never examined as a stand-alone system and is only taken as a reference system, for all systems containing a heat pump.
- The used values of the demands of the building, and the used and generated energy of all technologies remain the same at each point of observation.
- Seasonal effects are not considered, because the opportunity to perform investment decisions only occurs once a year and there is also a long observation period, that is why the effects are negligible.
- Surplus energy generated by the photovoltaic system will always be imported into the grid with a fixed price 8,2 ct/kWh which displays the current compensation for photovoltaic system for the next 20 years [84].
- The degradation of performance is neglected for all studied systems.

3.6 Real Option Analysis

3.6.1 Single State Variable

Locatelli et al. [53] proposed a method that utilizes exercise thresholds to determine when to make an investment. According to the authors, this method involves defining an interval of exercise thresholds and using Monte Carlo simulations to calculate the probability distributions of NPV for each threshold. For each distribution, the mean NPV and the standard deviation is calculated. By analyzing these NPV probability distributions, the threshold that offers the best distribution in terms of mean NPV, minimum standard deviation, and other relevant criteria can be selected.

In this work, three investment options are explored: investing in a photovoltaic system, a heat pump, or battery storage. The available options for the investor are to invest (immediate investment), deferring the investment decision (waiting), or abandoning the project. Figure 3.7 illustrates the structure of the implemented algorithm.

In this context, the exercise threshold depends on two state variables: the electricity price p_e and the gas price p_g. The term "state variables" refers to variables that describe the stochastic process in the Monte Carlo simulation. These variables are used to model uncertainties of the model. To gain a better understanding, the method is initially explained using a single state variable, before implementing both thresholds: "single state" and "two or more states."

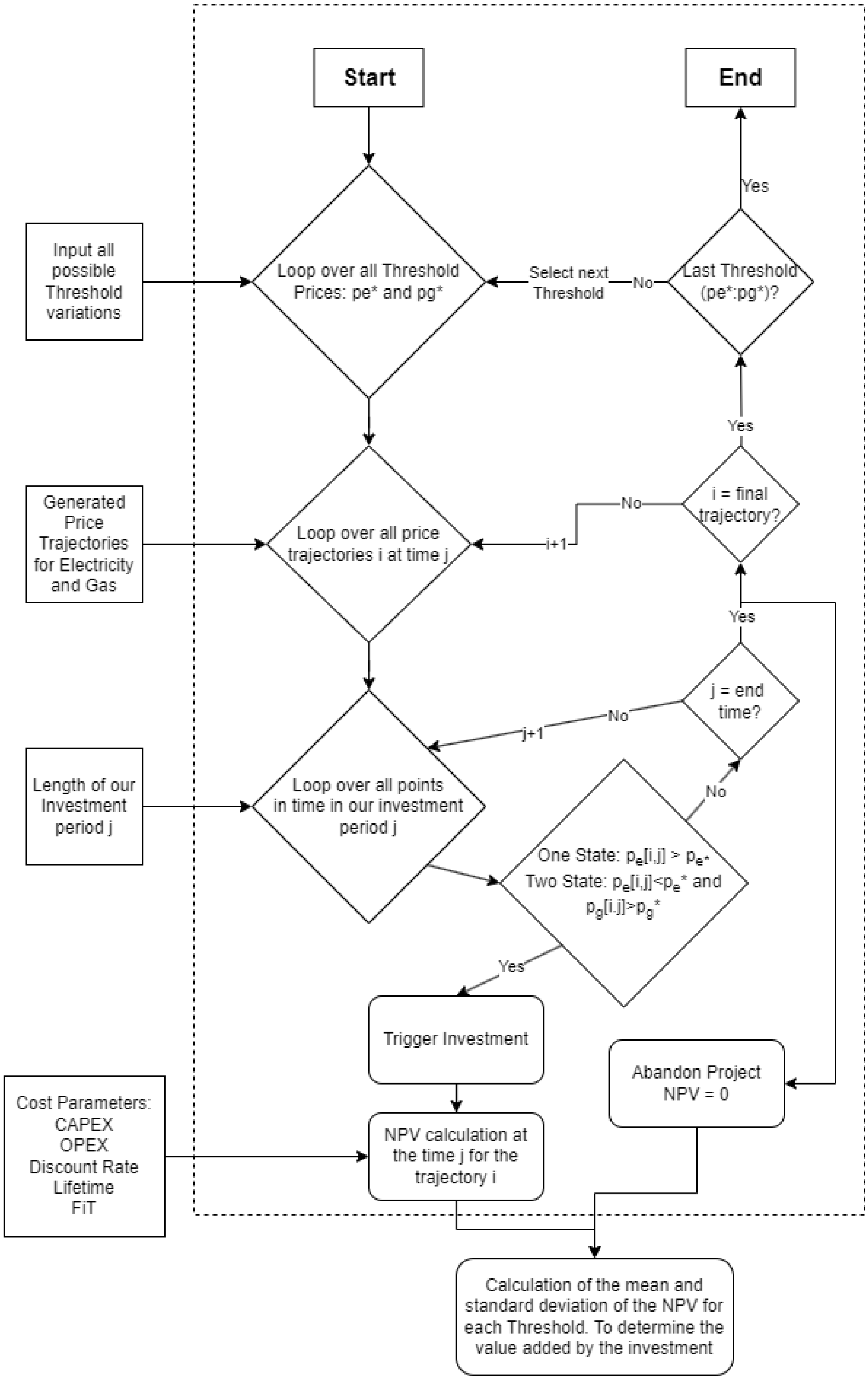


Figure 3.7: Flow chart showing the programming structure of the implemented algorithm. Based on [53, 16].

The method will be further explained using an example, where investing into a photovoltaic system in combination with a battery storage system will be considered. The applicable state variable in this case is the electricity price p_e:

1. Starting with defining an interval of p_e^*. It is important to choose a lower value for the lower bound than the initial electricity price P_0. For the higher bound a value that its unlikely to ever be reached must be chosen. For instance, in this example, a threshold range of $p_e^* = (20, 80)$ ct/kWh is chosen.

2. The condition for an investment to get triggered equals $p_e \geq p_e^*$

3. Calculate the NPV probability distribution for every price trajectory, starting from $p_e^* = 20$ ct/kWh continuing until the threshold of $p_e^* = 80$ ct/kWh is reached. In this example, 100 Monte Carlo simulations are conducted.

4. For each of those simulations, the relevant indicators, the mean and the standard deviation of the NPV are calculated. For each simulation, the three available options are considered. If the investment is triggered by the state variable reaching the threshold value, the decision maker **invests**. If the threshold is not reached in the first year, the process is repeated every year, until the threshold is reached, using the **wait** option. If the threshold is not reached, the investment is **abandoned**, which is reflected in the valuation of the results in a mean NPV of zero.

5. Finally, the mean and standard deviation of the NPV is calculated for each threshold, allowing appropriate investment decisions to be derived from the analysis.

If the electricity consumption decreases compared to the grid, high electricity prices are favorable for a photovoltaic system. In this case, less electricity needs to be bought from the grid due to the self-consumption of the installed photovoltaic system. Therefore, the condition for when to invest in the respective system is set as: $p_e \geq p_e^*$, reflecting the favorable scenario where electricity prices exceed a certain threshold value.

Figure 3.8, shows the distribution of the mean and standard deviation of the NPV, for each threshold value. The plot can be separated in three areas: "Invest Immediately", "Increased Value by waiting" and "Never invest", which will be discussed in detail now:

1. First, the "Invest Immediately" condition, with a predefined exercise threshold of $p_e \geq p_e^*$, is always satisfied when the electricity price is below or equal to the starting price P_0, meaning that every price trajectory meets this condition today. The mean and standard deviation of the NPV of today will be calculated at this point, which is equivalent to a traditional DCF approach of evaluating investments.

2. Secondly, the values that are affected by waiting for the threshold values and increasing the value of the investment can be seen. This is the effect of the option to wait. When waiting for a certain threshold there is a gain in value by waiting for the investment, with the downside that at the first step for values greater than 35, there is only an investment probability of 50 %. The other 50 % of trajectories never reach the defined threshold value, and therefore no investment is performed. For values greater than 48 ct/kWh there is a decrease in the investment value when comparing it to the DCF value. The number of investments made at that specific threshold is so low that the investments not taken, along with the NPV being zero,

contribute to a decrease in the value of the option to wait. The following conclusions can be drawn from the standard deviation plot: The higher the increased value of the mean NPV, the higher the risk (displayed as the standard deviation), but since the revenue is also higher, for this case it should be recommended to invest for the threshold with the highest mean NPV. Because the mean NPV is bigger than the standard deviation, and therefore the investment will always be positive, which is not the case if the investment is performed today.

3. Lastly, for thresholds greater than 60 ct/kWh, very few state variables reach the threshold value. As a result, either no more investments are triggered, or the number of thresholds that satisfy the condition is too small, leading to a large share of mean NPVs being terminated with no investment value (i.e. an NPV of zero).

After the RO analysis with exercise thresholds, the investor can use the mean and standard deviation of the NPV findings to decide: invest now, wait, or abandon the project. Furthermore, depending on the ideas of the investor, a particular threshold value can be chosen: a high mean NPV is connected to a high standard deviation and therefore a high risk, a lower mean NPV leads to a lower standard deviation and therefore a lower risk. Another possible case would be there is no improvement in waiting for a later point in time. In these cases it is recommended to invest immediately if the mean NPV exceeds zero.

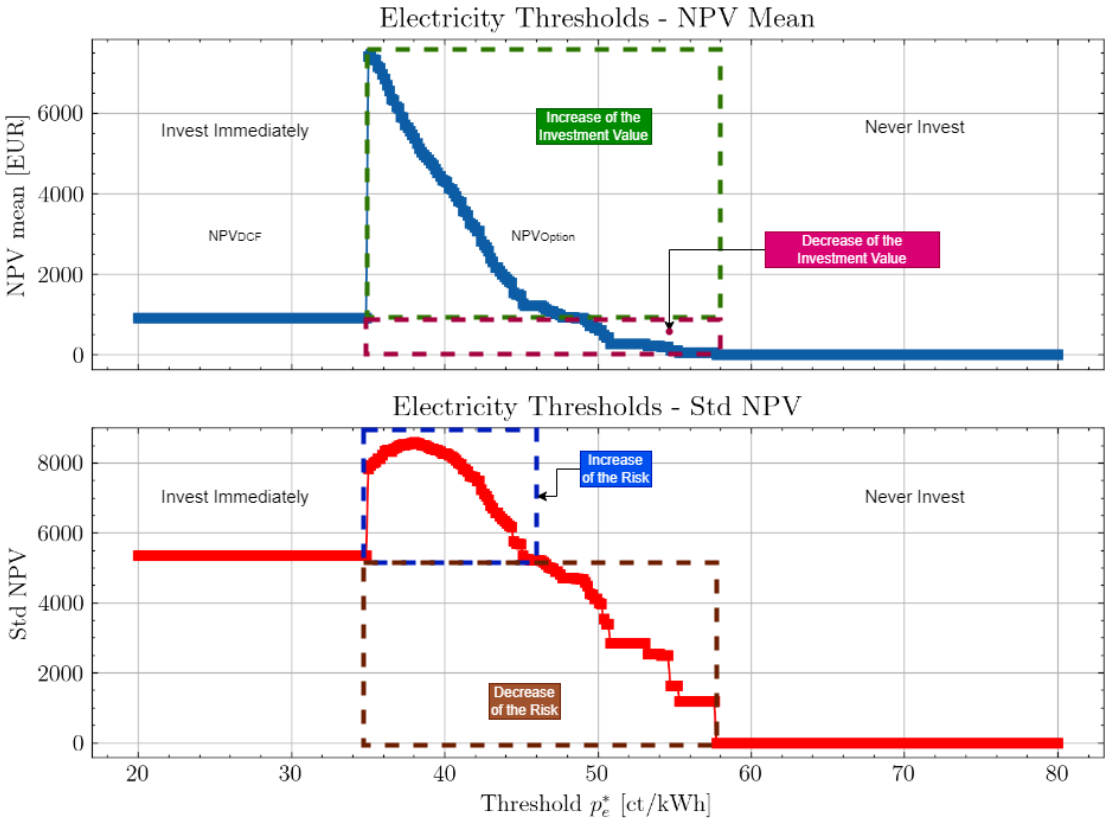


Figure 3.8: Mean and Standard Deviation of the NPV distribution for each Threshold. Using a photovoltaic system in combination with a battery storage system, with a single state variable as an example.

3.6.2 Two or more State Variables

The analysis can also be conducted for two state variables, which is necessary in the underlying case. It is essential for cases in which, the gas and electricity price, influence the profitability of the investment. For two state variables, the exercise threshold consists of a pair of thresholds $(p_e^*; p_g^*)$.

Rising electricity prices negatively affect the heat pump usage due to the overall increasing electricity demand of the building. This is in contrast to the technology options only investing into a photovoltaic system, where the situation is the opposite. High gas prices, on the other hand, are advantageous for investment into the heat pump, because it increases the NEC of the reference system. Therefore, the following condition can be set for two thresholds: $p_e \leq p_e^*$ and $p_g \geq p_g^*$. For every combination of exercise thresholds there is a different NPV probability distribution. The steps to determine the distribution are the following: As an exemplary system, a heat pump in combination with a photovoltaic system is chosen:

1. Definition of the threshold ranges, in which the exercise thresholds can occur. For electricity for values between $20 < p_e^* < 70$ ct/kWh and for gas $5 < p_g^* < 20$ ct/kWh.

2. Divide the intervals into all possible threshold variations. In this case with a step

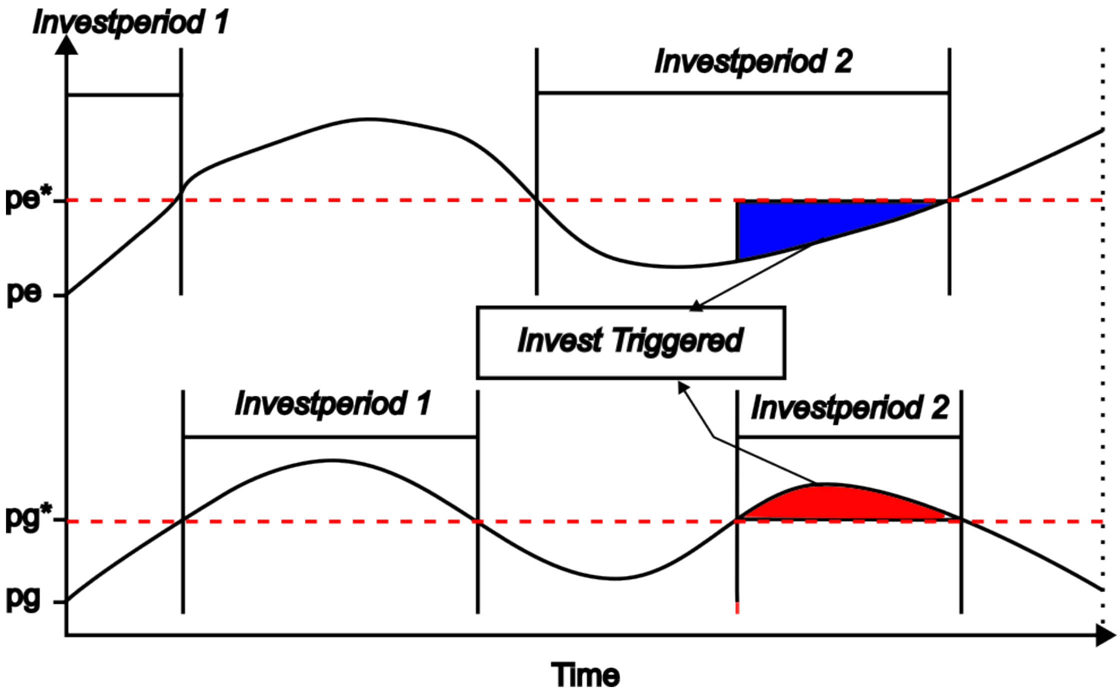


Figure 3.9: Schematic representation of when an investment decision is triggered by two state variables

size of the threshold of one: $(p_e^*; p_g^*) = \{(20; 5), (20; 6) \dots (21; 5), (21; 6) \dots (70; 20)\}$, creating a total of 750 pairs of threshold values.

3. The next steps are analogous to the single state variable analysis. For each threshold pair, the corresponding NPV probability distributions across all simulated price trajectories are generated. In the underlying example, 100 Monte Carlo simulations are employed.

4. From each NPV probability distribution, the mean and standard deviation of the NPV is recorded, allowing appropriate investment decisions to be derived from the analysis.

The described method is illustrated in Figure 3.9. Once both threshold conditions are met, the NPV is calculated, and if the added value by the investment is greater than zero, an investment is triggered. This process is repeated for every energy price trajectory and for every time step for each pair of threshold values. Figure 3.10, shows how to evaluate the results. The main difference in the evaluation of multiple thresholds is the placement of the described zones. The results are now three-dimensional, containing the gas threshold, electricity threshold, and either the mean NPV or the standard deviation. Similar to the evaluation with one threshold, three areas can be obtained: "Immediate Invest," "Improvement Zone," and "Never Invest". The zone of interest for the investor is "Improved Mean NPV and standard deviation" as it provides relevant information on how much value can be gained by delaying the investment and whether the risk increases or decreases.

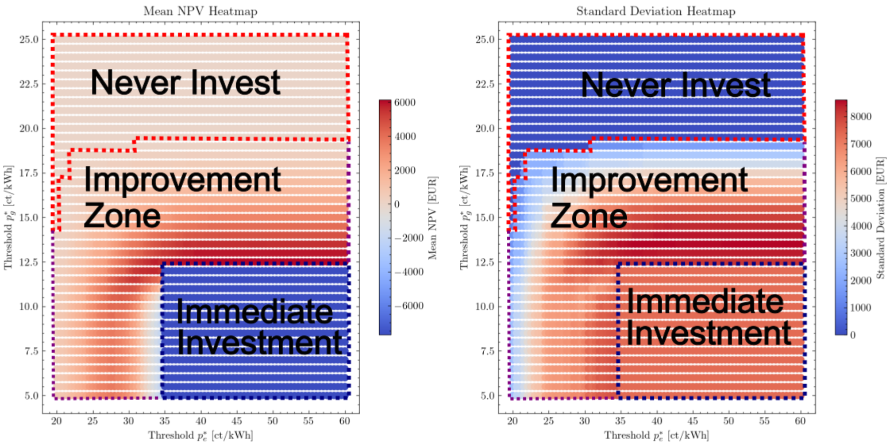


Figure 3.10: Mean and Standard Deviation of the NPV distribution for each Threshold. Using a system that combines a heat pump with a photovoltaic system, with two state variables as an example.

3.7 Statistical Methods

To evaluate the results the mean values, the standard deviation and the investment probability of the NPVs need to be calculated.

Mean Net Present Value The mean NPV is the central metric for determining the expected profitability of the investment. It is calculated by summing the NPV over all simulations and dividing it by the number of scenarios. This process is repeated for each predefined threshold, the mathematical expression for which is:

$$\text{Mean of NPV}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} NPV_{ij} \tag{3.14}$$

where Mean of NPV_i represents the mean NPV for the i-th threshold value, n_i is the total number of simulations and NPV_{ij} represents the NPV of the j-th scenario for the i-th threshold value.

Standard Deviation of the NPV The standard deviation of the NPV measures the dispersion of the NPVs around the calculated mean NPV. It indicates the degree of volatility. A high standard deviation suggests high uncertainties and therefore high risks, while a low standard deviation less volatility and a lower risk. The standard deviation for each threshold looks like this:

$$\text{Standard Deviation of NPV}_i = \sqrt{\frac{1}{n_i} \sum_{j=1}^{n_i} (NPV_{ij} - \text{MeanNPV}_i)^2} \tag{3.15}$$

Investment Probability The investment probability displays the possible investments that are triggered in the range of a certain threshold. The investment probability ranges from 0 to 1, where 0 indicates no investments have been triggered, and 1 indicates there is always an investment taking place. The probability can be expressed as:

$$IP^* = \frac{\text{Number of Price Trajectories where an Investment is Triggered}}{\text{Total Number of Price Trajectories}} \tag{3.16}$$

Flexibility Value The difference of the extended net present value and the traditional net present value defines the flexibility value [50]. The extended net present value is the obtained project value when performing the RO analysis and expresses the value gained from waiting for investment. The mathematical expression is:

$$\text{Flexibility Value} = NPV_{RO} - NPV_{DCF} \tag{3.17}$$

where NPV_{RO} is the NPV of the RO approach, and NPV_{DCF} the NPV of the DCF method.

4 Results

The systematic approach of the algorithm was developed and explained in the preceding chapter 3. This methodology will now be implemented, and the outcomes will be presented in the following sections.

Furthermore, section 4.2 will encompass the uncertainties associated with this study, including the consideration of gas and electricity prices, and the approach adopted to model them. Additionally, this section will investigate the process of selecting simulation parameters and their impact on the obtained results.

Graphical presentations of the calculated results, based on the inputs presented earlier, will be provided in section 4.3.

A sensitivity analysis will be conducted in section 4.4 to demonstrate the impact of different price trajectories on the observed trends.

4.1 Representation and Explanation of the used Parameters

In this section, the key inputs utilized to perform the calculation of NPV_{AIV} are presented.

Parameters of the Energy System Model The values for the calculation of the NECs are derived from the outputs generated by the used energy system model, as shown in Table 4.1. All values in the table are represented in kWh/a. The parameters utilized as inputs for the energy system model to obtain these values were presented in Table 3.2.

Figure 4.1, displays the annual energy demands for each technology option derived from the energy system model's outputs. The figure highlights the electricity and heat consumption patterns for the used technology options and the reference option.

Table 4.1: Data for the net energy cost calculation.

	Electricity demand	Heat demand	Electricity export	PV generated
Reference	21780.3	39286.6	0	0
Heat pump	38032.6	0	0	0
PV	14857.3	39286.6	8981.4	14350
PV+Battery	12611.7	39286.6	6047.3	14350
PV+Heat pump	30079.7	0	6717.0	14350
PV+Battery+Heat pump	27781.1	0	3831.4	14350

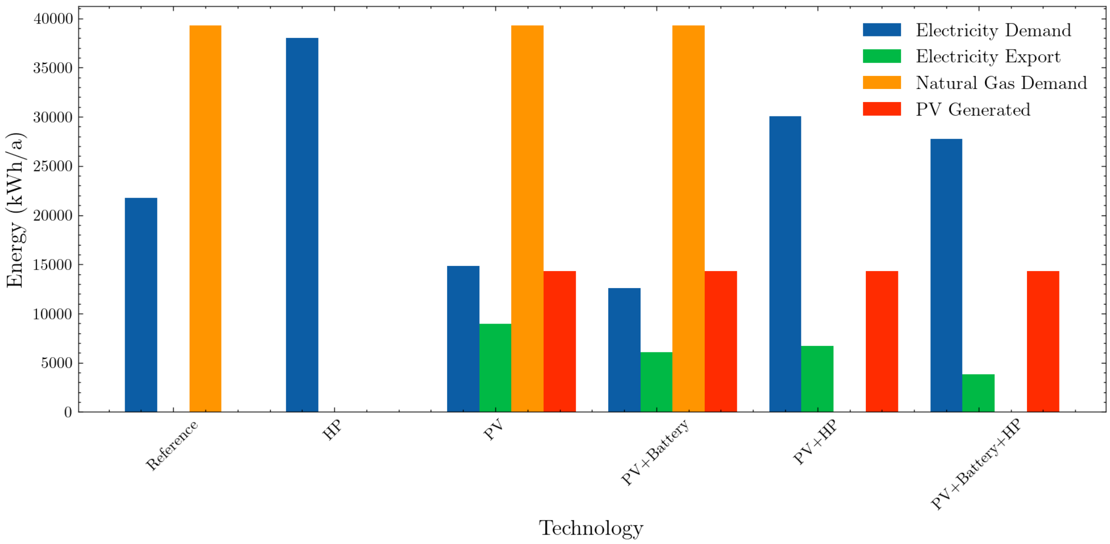


Figure 4.1: Energy demands for all technologies and their respective combinations.

Financial Parameters In this section, the financial parameters used in this work will be discussed, starting with the CAPEX values for the used technology options, as illustrated in Figure 4.2. The technology options considered are photovoltaic, heat pump, battery storage, their respective combinations, and the reference system, i.e., the gas boiler. All technology options have been considered, and it is observed that the gas boiler stands out as the system with the lowest CAPEX costs. However, it is anticipated that the other systems have lower NEC, so they become more profitable at one point.

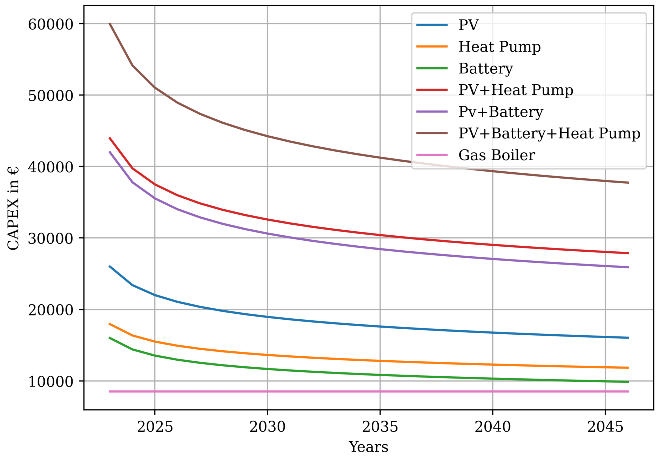


Figure 4.2: Development of the CAPEX costs for all variations. A discussed learning rate of 10 % has been added to the trajectories of all technologies.

Next, the final OPEX values will be presented, comprising the NECs and MCs. The corresponding results are shown in Figure 4.3. For the photovoltaic systems, there are maintenance costs of 248 €. The heat pump incurs maintenance costs of 180 €, and the gas boiler requires maintenance costs that amount to 255 €. As discussed in section 3.2, the battery storage system does not entail any maintenance costs.

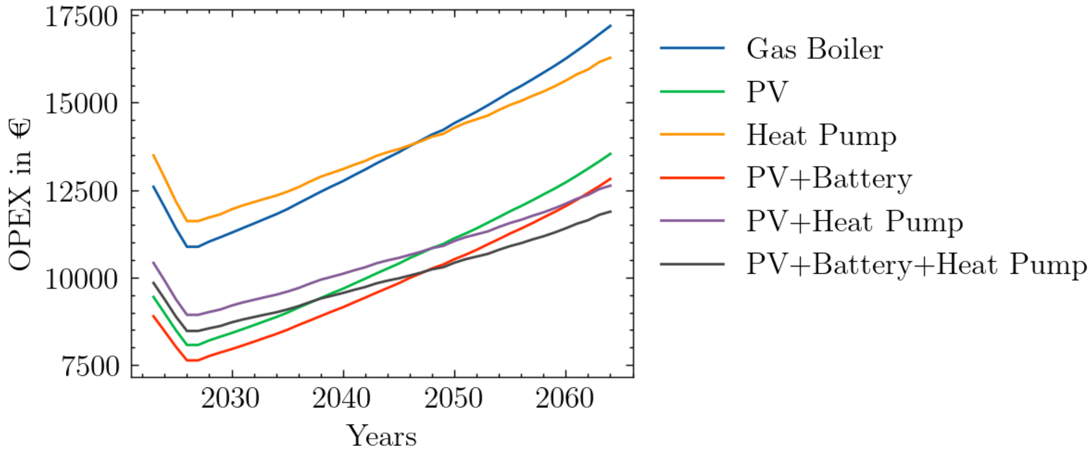


Figure 4.3: Development of the OPEX costs for all variations.

Finally, the remaining financial parameters inputted into the model are the FiT, the discount rate, the lifetime of technologies, and the learning rates. The values of these parameters were previously discussed and are presented again here for completeness. A FiT of 0.08 ct/kWh is assumed, which remains constant over time. The discount rate is set to 5 %. The lifetime of all systems is assumed to be 20 years. The period in which the investment can be made is 15 years.

4.2 Uncertainty Parameter Evaluation

To model the uncertainties in gas and electricity prices for households, a GBM is utilized. The GBM equation, developed to display the respective uncertainties, is expressed as follows:

$$P_t = P_0 e^{((\mu - \frac{\sigma^2}{2} t) + \sigma W_t)} \tag{4.1}$$

Adequate values for the volatility and the drift term need to be chosen to get indicative results. Additionally, the number of simulations conducted significantly influences the outcomes. Another important factor is choosing the interval space and step size for the exercise thresholds. As there are currently only two available studies in the literature, namely Locatelli et al. [53]. and Biggins et al. [16], which employ a similar RO approach, it is crucial to prioritize the determination process of the mentioned parameters before proceeding with the presentation of the use cases. The parameters for a single state variable, the electricity price, will be developed for the photovoltaic and photovoltaic system including a battery storage system. Starting with the volatility of the uncertainties, denoted as σ.

4.2.1 Volatility of the Prices

The volatility in the underlying case can be calculated by taking historic values of the electricity price for households. Either by determining the volatility before the energy prices increased from 2021, and have rather a conservative approach, or by determining the volatility until today. In Table 4.2 the respective volatilities calculated in that way are presented. It is important to note that there are two options for determining the volatility:

Table 4.2: Change in Volatility's in the studies that performed a similar analysis and the underlying case.

	Value for σ
For Electricity Prices from 2012-2021	0.02
For Electricity Prices from 2012- 2023	0.06
For Electricity Market Prices in Locatelli [53]	0.30
For Hydrogen Market Prices in Biggins [16]	0.15

Table 4.3: Parameters to be validated.

Simulation Parameters	
Number of Simulations	250
Threshold Range	$(0; 150)$
Number of Thresholds	150

either the volatility can be calculated before the energy prices increased from 2021, taking a conservative approach, or it can be calculated up until today. The volatilities for these cases are calculated based on the data in [28]. Additionally, the values used in Biggins and Locatelli for the volatilities are added, to put the values chosen in this work into context. Throughout the analysis, the parameters used in these two papers will be referenced repeatedly to explain why parameters of the same order of magnitude are not suitable for the underlying case and to justify the parametrization of the simulation.

Figure 4.4 illustrates the differences in the implementation of various volatility values for the photovoltaic option. Similarly, Figure 4.5 shows the differences between the photovoltaic and battery storage option. The starting values of the simulations are shown in Table 4.3, they will be continuously developed throughout the parameter evaluation to then form the final simulation parameters.

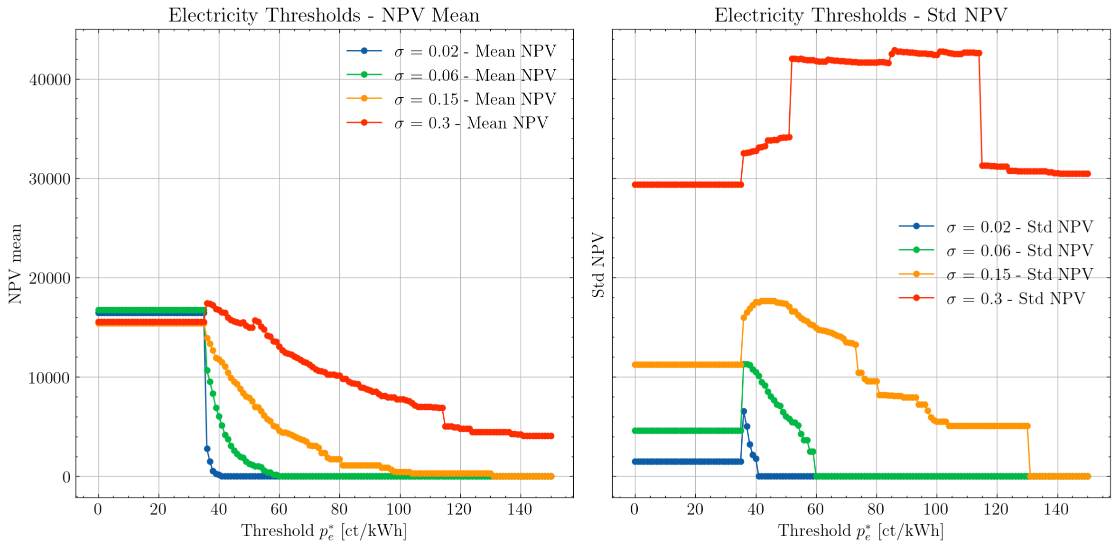


Figure 4.4: Mean and Standard Deviation (Std) distribution over the predefined range of Threshold. For photovoltaic as a stand-alone system.

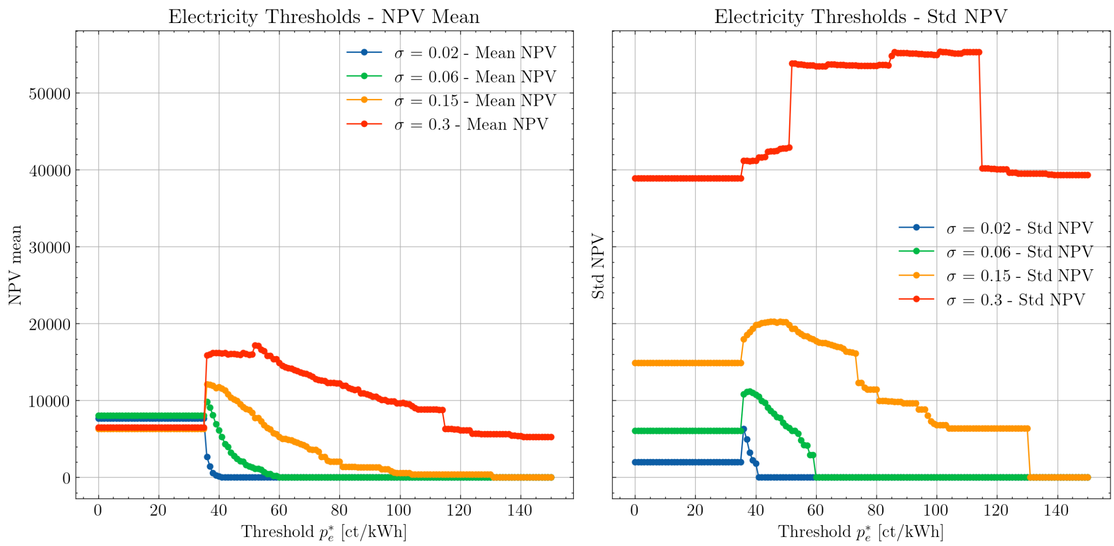


Figure 4.5: Distribution of mean and standard deviation (Std) over the predefined range of the threshold value. For photovoltaic in conjunction with a battery storage system.

The following indications can be gained from the plots, to determine the uncertainty parameters for the underlying case, starting with the plot for the photovoltaic system: The range of possible prices, when choosing a high volatility, like in Biggings or Locatelli, leads to prices that are very unlikely to ever be reached. Opting for one of the suggested volatilities within the range of $0.02 - 0.06$ seems more reasonable. Only in one case, choosing to wait before investing in another photovoltaic option leads to the possibility of a higher mean NPV. In all other cases the recommendation would be to invest immediately, since no value can be gained by waiting. For the photovoltaic and battery storage options, the mean NPV is increasing by waiting for the threshold to trigger the investment. The following conclusion can be drawn from this: The higher the volatility chosen by the investor, the more likely the option to wait will bring value to the investment.

Additionally, it can be observed that the range of the thresholds needs adjustment in further analysis. In the case of $\sigma = 0.02$, there are only four instances where the NPV distribution includes results between the DCF value, the "Immediate investing" scenario, and the "Never Investing" scenario, where the threshold value is consistently greater than the state variable.

Therefore, the next step involves adjusting the number of thresholds and their range. The volatilities utilized in Biggins and Locatelli will be excluded as they have been demonstrated to be unsuitable for this specific case.

4.2.2 Range of Thresholds and Number of Simulations

To select the appropriate threshold parameters, a similar analysis to the previous one is conducted by varying the values. The range of relevant values could be obtained in Figure 4.4 and 4.5, therefore it is possible to define a realistic interval of (20;60) €/kWh. The application is limited to a sigma value of $\sigma = 0.02$, as it has been demonstrated to be the most sensitive case due to its minimal occurrence of values. As the number of values between the scenarios of always investing and never investing decreases, the likelihood of overlooking potential gains that may arise from waiting for the investment to grow in value increases. By exploring four distinct step sizes for the intervals, commencing from 1 and progressively halving in subsequent steps.

Additionally, the variable α_j is introduced, to represent the count of thresholds associated with scenarios of "Immediate investing" or "Never Investing", j represents the number of thresholds used in the defined range. The results of this can be seen in Figure 4.6. The analysis reveals a clear linear relationship between α and the growth in thresholds. When the thresholds double in value, α also doubles accordingly. Moreover, the findings indicate that the results remain relatively stable, and there is no effect on the curves of mean NPVs and standard deviations. By employing a step size of $\Delta P = 0.5$, a satisfactory level of accuracy is achieved by still maintaining a reasonable computation time.

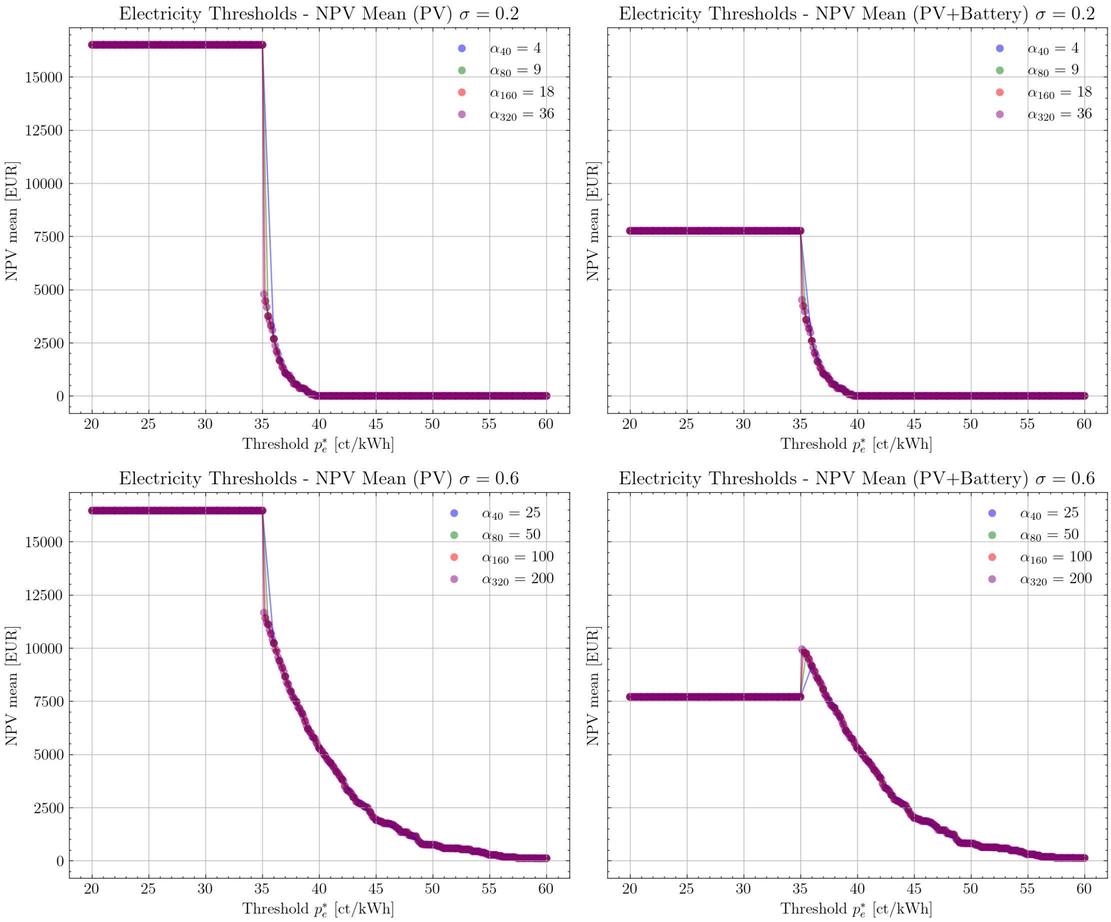


Figure 4.6: Change of the step size ΔP and its influence of the simulation results.

To find a reasonable amount of simulations, with the beforehand determined threshold range, number of thresholds, and volatilities, the impact of the number of simulations on the results is examined. Choosing an increasing number of simulations, as shown in Figure 4.7. The results demonstrate significant volatility when employing less than 100 simulations. To ensure more reliable and stable outcomes, the decision has been made to develop 1000 price trajectories using the GBM.

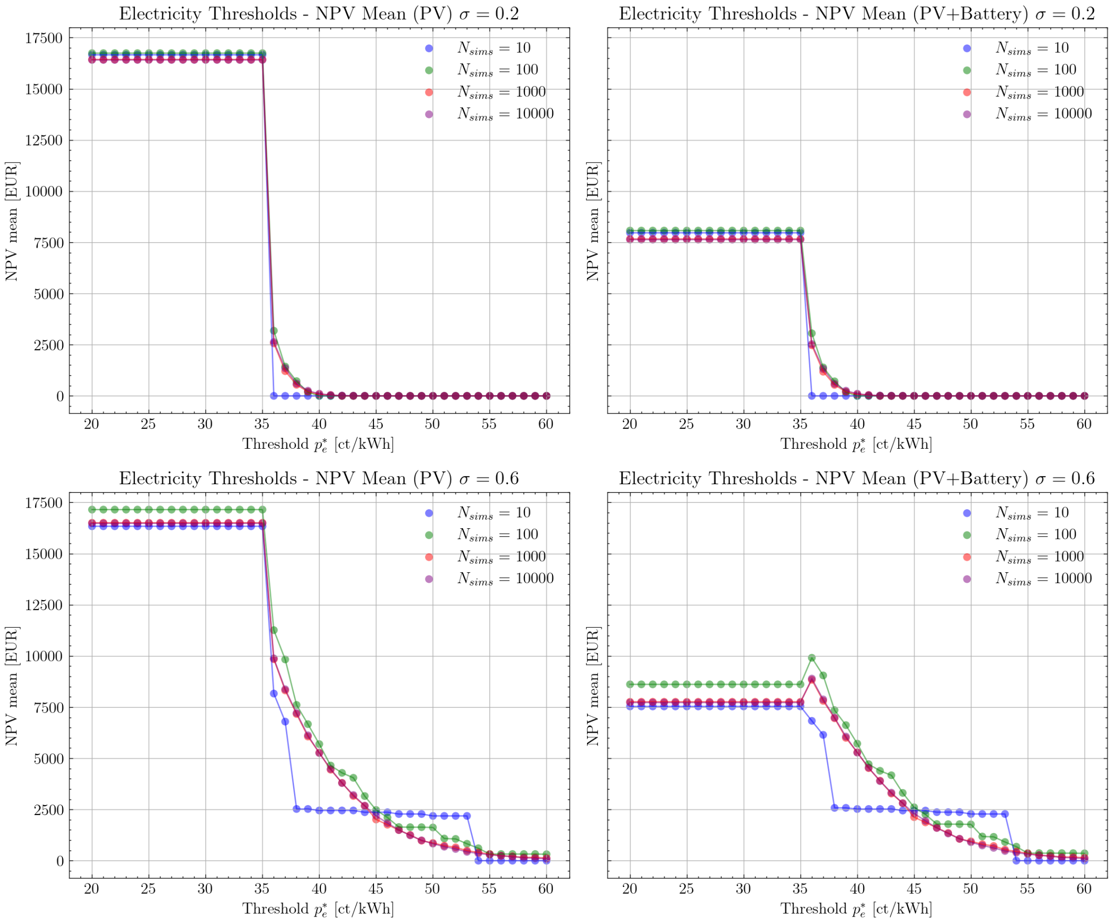


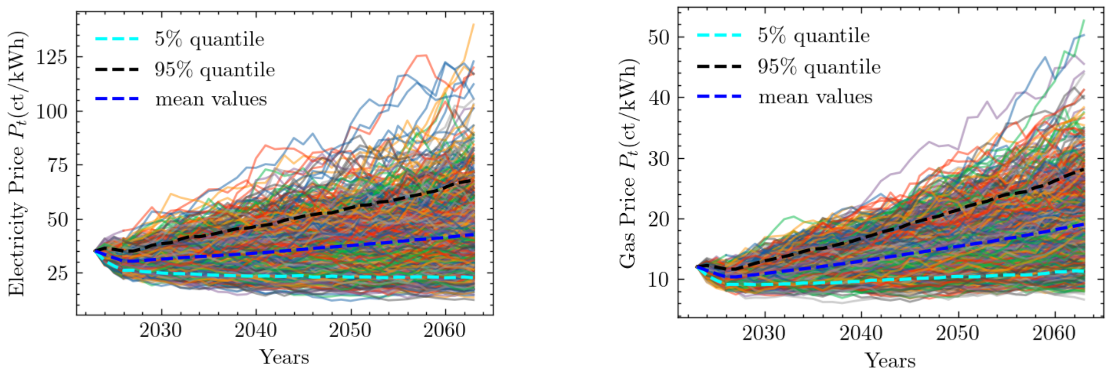
Figure 4.7: Influence of the number of simulations on the results.

The final parameters for simulating the uncertainties of technology options are now chosen. The examination involved observing the influence of different parameters on the results. The remaining aspect is to consider the price developments for the next 5 years, with prognoses indicating a decrease in prices around 2025. This condition is demonstrated through two successive GBM simulations by selecting a negative value for μ_1, representing the drift of the price trajectories.

Until now, only electricity prices were considered in the investigation of systems with a heat pump investment. However, in such cases, it is necessary to factor in not only gas prices but also electricity prices. The knowledge gathered for electricity prices is utilized to derive the simulation parameters for the combination of electricity and gas prices.

To model the increase of energy prices, the pre-war level volatility for electricity is adopted as $\sigma_{elec} = 0.06$. Similarly, the same approach is applied to determine the value of $\sigma_{gas} = 0.05$ [28], leading to the same step size of $\Delta P = 0.5$. The relevant parameters are shown in Table 4.4. The variables not previously introduced include T, representing the number of years the prices trajectories are generated, with one price change generated for each year, P_0 denotes the initial price of the GBM.

The final results of the simulation are shown in Figure 4.8



(a) (b)

Figure 4.8: a) shows the GBM for the electricity prices with the discussed simulation parameters b.) Shows the equivalent, but for the gas prices.

Table 4.4: Final Simulations Parameters, for the GBM, and the Real Options Analysis.

Parameters	GBM-Electricity	GBM-Gas
P_0	35	12
σ	0.06	0.05
μ_1, μ_2	-0.05, 0.01	-0.05, 0.02
T_1, T_2	3, 37	3, 37
	Single State Variable	Two State Variables
Threshold Range	$p_e^*(20:60)$	$p_g^*(5;25)$ and $p_e^*(20:60)$
N_{sim}	1000	500
ΔP	0.5	0.5

4.3 Results of the Simulations

In the following section, the results of the analysis of the case studies are presented, providing insights into the variations across the different scenarios by examining the mean and standard deviations of the NPV for various threshold values.

The "single state variable" method is applied for two different configurations: the photovoltaic system as a standalone system and the photovoltaic system with an added battery.

Additionally, the effects on systems with "two state variables" are explored, by considering the photovoltaic systems with an added heat pump and the heat pump as a standalone system. To support the decision-making of investors applying this method, plots have been developed, allowing for the selection of threshold values based on individual requirements. The optimal threshold value should ideally exhibit a favorable mean NPV along with a low standard deviation. Additionally, for the evaluation of the results, the parameters average waiting time and the flexibility value are introduced.

Furthermore, a sensitivity analysis is conducted to assess the impact of market conditions and technical factors on the results. This analysis aims to determine the magnitude of these influences and provide a comprehensive understanding of the dynamics at play.

By analyzing these scenarios, a comprehensive understanding of how threshold values impact the NPV and risk associated with the investment can be gained. These findings will provide valuable guidance for decision-making in optimizing investment strategies and risk management in the context of residential energy technologies. In addition, the findings can be used to determine whether this method is suitable, and enhances the investment valuation of residential energy technologies. The sensitivity analysis should provide valuable insights into the influence of market and technical conditions on investment outcomes, improving the understanding of the broader context of opportunities for investing in technologies used in the residential energy sector.

4.3.1 Photovoltaic

In this section, the results and findings for the photovoltaic system are presented. It will be explored how the systems performance varies under different threshold values and their potential to generate improved financial returns.

Figure 4.9 illustrates the influence of the threshold value, denoted as p_e^*, on the investment value. For the photovoltaic system, it is evident that the "exercise thresholds" method, or the use of ROs, does not enhance the system's value. Interestingly, the investment value decreases when the decision-maker chooses to wait for a specific threshold value. Investing in a photovoltaic system immediately is economically more advantageous than waiting for a specific threshold value. The waiting option does not provide an economic advantage in the case of photovoltaic as a stand-alone system.

Figure 4.10, shows the optimal threshold, which in this work is always the mean NPV that brings the largest of the NPV improvement, $p_{e,pv}^*$, the most profitable threshold value, equals 35 ct/kWh in this case. This value equals our initial electricity price and suggests investing immediately.

4.3.2 Photovoltaic and Battery storage

The second system, incorporating photovoltaic and battery storage, demonstrates a noticeable improvement in mean NPV. The option to wait for a higher price improves the

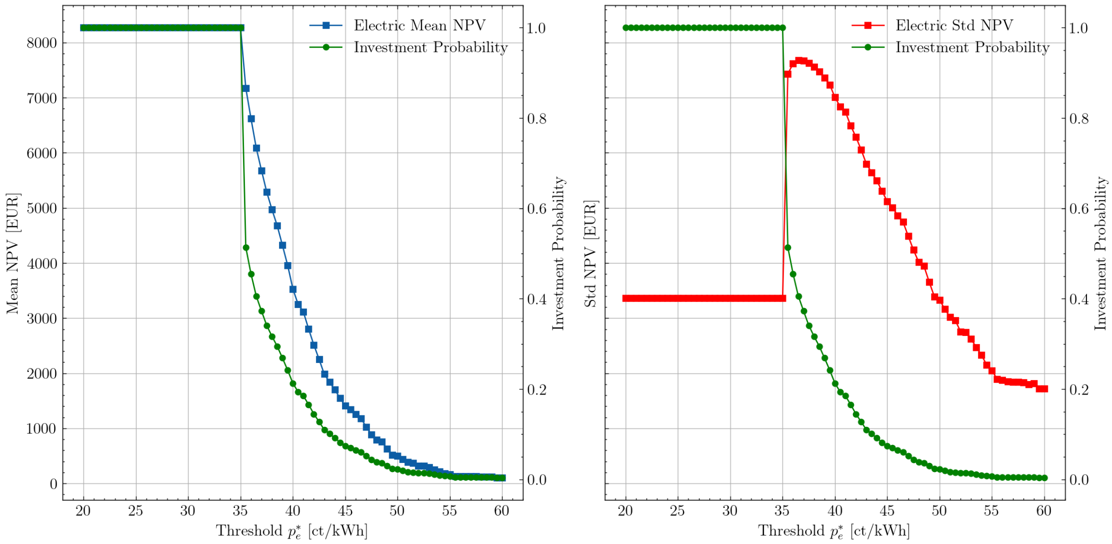


Figure 4.9: Impact of the threshold on the mean and standard deviation of the NPV for the photovoltaic system.

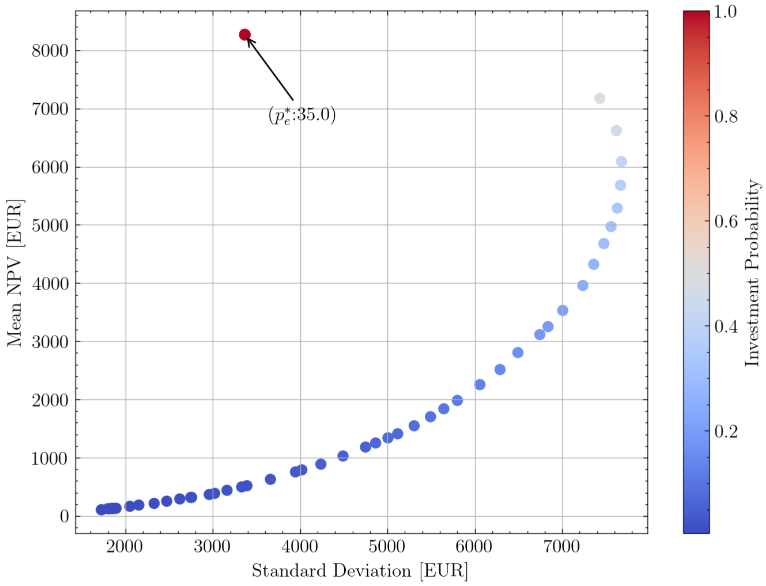


Figure 4.10: Distribution of mean and standard deviation (for photovoltaic as a standalone system) and the "exercise threshold" $p_e^* = 35ct/kWh$ that leads to the highest mean NPV

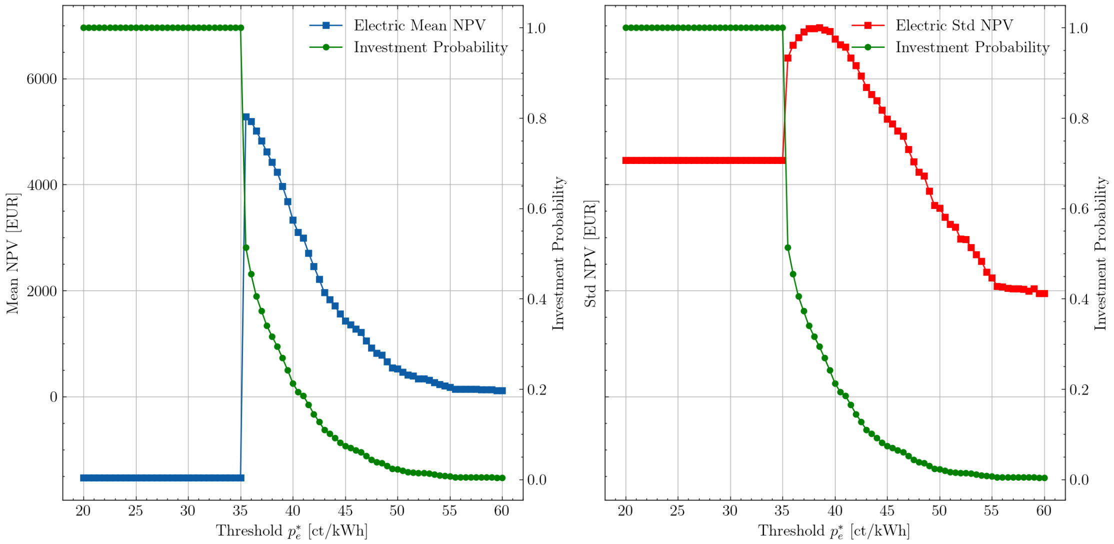


Figure 4.11: Impact of the threshold on the mean and standard deviation of the NPV for the photovoltaic system, in combination with a battery storage system.

value of investing into this option.

Moreover, the DCF value is negative when the waiting option is not considered. This would result in no investment in this system, even if the system has a positive mean NPV when the waiting option is used.

It can be observed that the standard deviation of the parameters evaluating the risk of the investment increases at the same time. This increases the possibility of an unfavorable investment. In summary, a higher NPV can be achieved by the waiting option, under the premise of higher risk. Figure 4.11,

Figure 4.12 displays the optimal threshold value of (p_e^* : 35.5). This value indicates that once the actual electricity price surpasses this value, an investment, results in an increase of the NPV of the project.

4.3.3 Heat Pump

In Figure 4.13, the results for the heat pump scenario are presented. The system is considered as a stand-alone system, with two state variables: the gas and electricity prices, and their respective distribution for each threshold pair. The plot illustrates two main aspects. Firstly, it is evident that outside the DCF-Zone (representing values outside of "investing today"), there is potential for improvement in the achieved mean NPV. This suggests that there are scenarios in which using the option to wait, for values that are outside of this zone, yields higher returns. Secondly, compared to the investigated single threshold systems, an improvement in the standard deviation and therefore lowering the risk of the investment can be obtained. This leads to a lower risk than investing immediately, as a more diverse range of outcomes is observed.

When considering Figure 4.14, the optimal threshold values of $(p_e^{\cdot}29.0; p_g^{\cdot}12.5)$ are showcased, which are leading to the highest mean NPV. However, it's noteworthy that this optimal value is not the most likely to occur, as most of the threshold pairs still result in a negative mean NPV. While the immediate investment would lead to a negative project value, by waiting for the named optimal threshold, the project value is getting

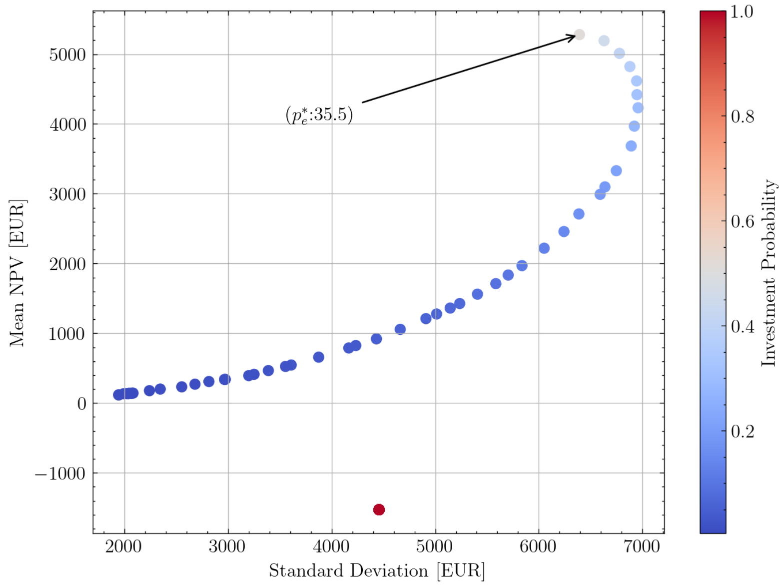


Figure 4.12: Distribution of mean and standard deviation (for photovoltaic and battery storage) and the "exercise threshold" $p_e^* = 35.5ct/kWh$ that leads to the highest mean NPV.

positive.

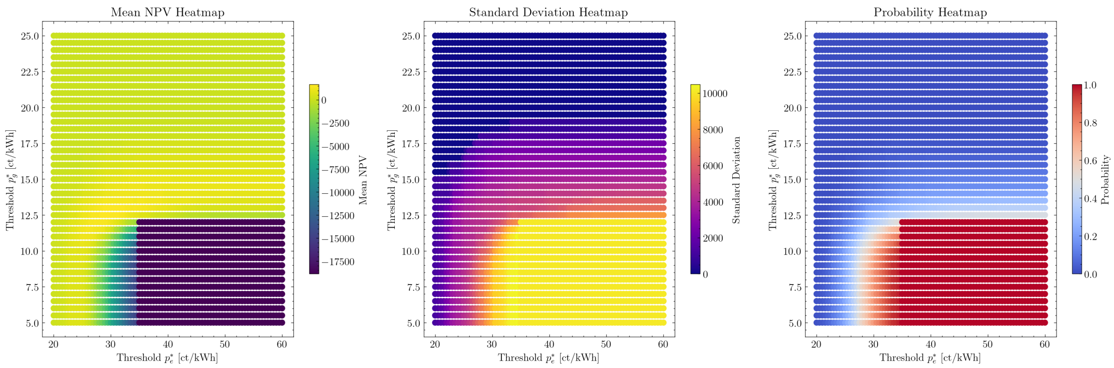


Figure 4.13: Heat map of the results for the heat pump system. Showing the mean and standard deviation of the mean, and the investment probability.

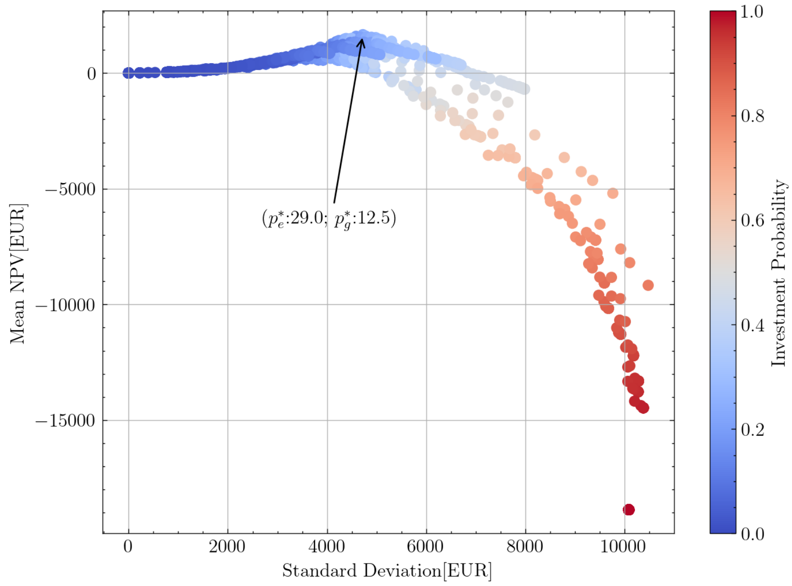


Figure 4.14: Distribution of mean and standard deviation (for heat pump as a stand-alone system) and the "exercise threshold" $p_e^* = 29.0; p_g^* = 12.5$ that leads to the highest mean NPV.

4.3.4 Photovoltaic and Heat Pump

In Figure 4.15, the combined performance of the photovoltaic and heat pump system is illustrated. While the immediate investment would lead to a negative project value, by waiting for the named optimal threshold, the project value is getting positive.

Figure 4.16 showcases the optimal threshold value pair of ($p_e^{;}45.5; p_g^{;}12.5$), which corresponds to the highest mean NPV. It is important to note that although this threshold pair maximizes the mean NPV, it may not be the most likely to occur among all threshold combinations.

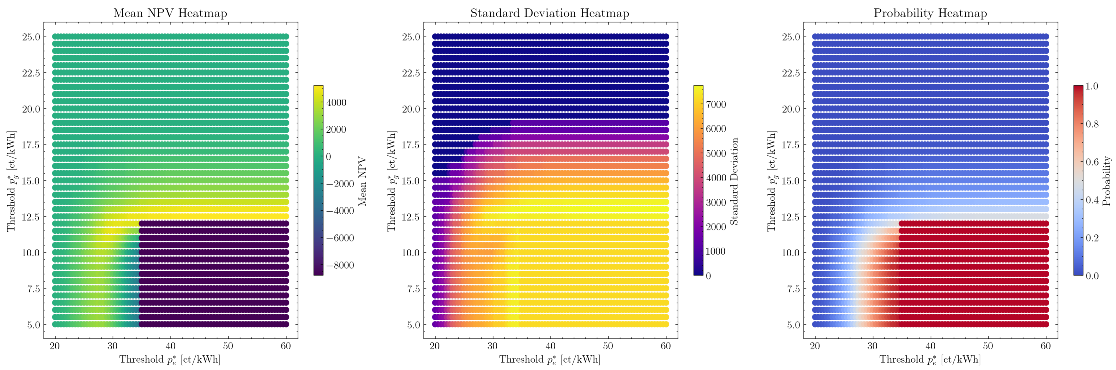


Figure 4.15: Heat map of the results for the heat pump in combination with a photovoltaic system. Showing the mean and standard deviation of the mean, and the investment probability.

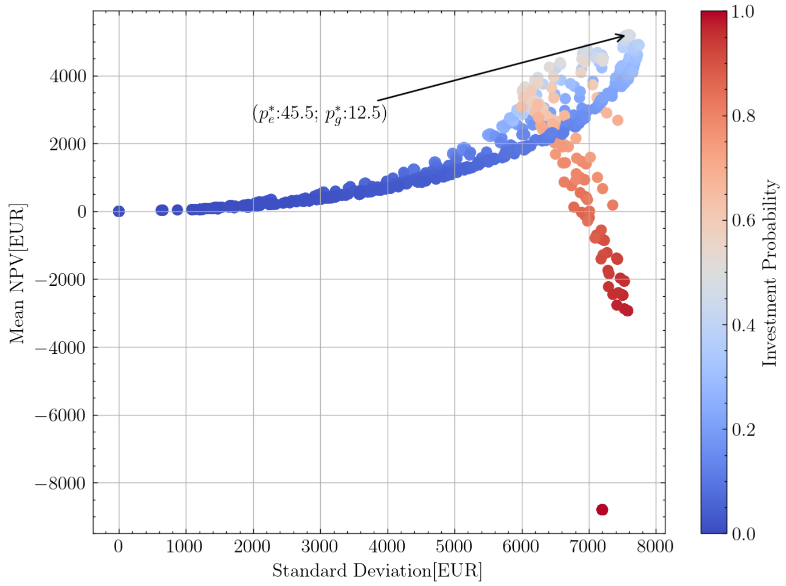


Figure 4.16: Distribution of mean and standard deviation (for heat pump and photovoltaic) and the "exercise threshold" $p_e^* = 45.5; p_g^* = 12.5$ that leads to the highest mean NPV.

4.3.5 Photovoltaic, Heat pump and Battery storage

In Figure 4.17, the results for the combined system of photovoltaic, heat pump, and battery storage are displayed. Similar to the previous scenarios, the RO approach demonstrates its effectiveness in improving this technology option as well. While the immediate investment would lead to a negative project value, by waiting for the named optimal threshold, the project value is getting positive.

Figure 4.18 presents the optimal threshold value pair of ($p_e^{\cdot}$59.5; $p_g^{\cdot}$13). This combination represents the best choice in terms of maximizing the mean NPV. However, it may not be the most probable threshold pair among all possibilities.

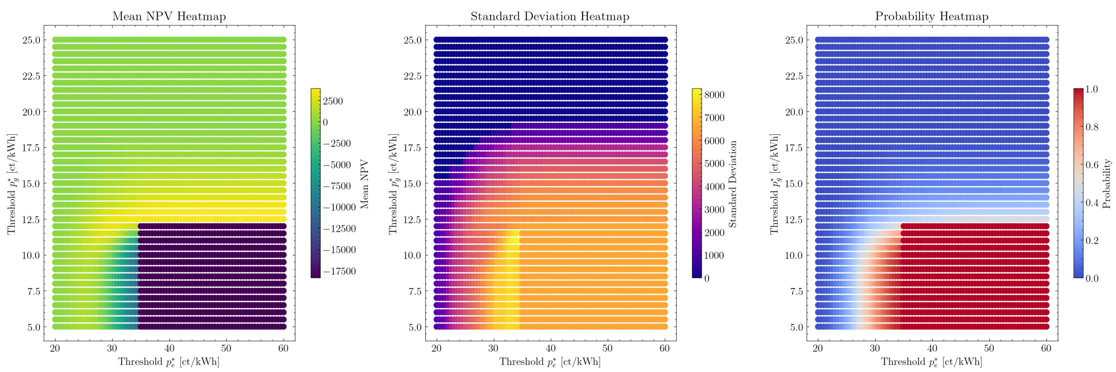


Figure 4.17: Heat map of the results for the heat pump in combination with a photovoltaic and a battery storage system. Showing the mean and standard deviation of the mean, and the investment probability.

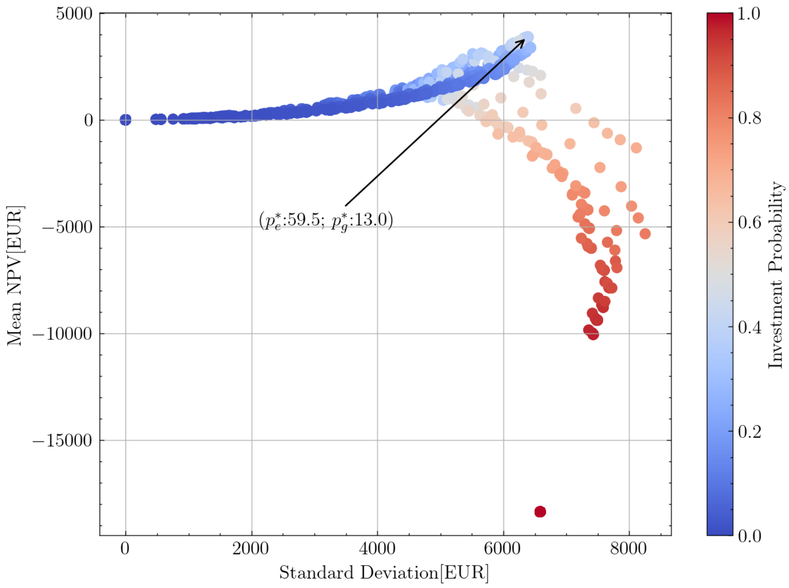


Figure 4.18: Distribution of mean and standard deviation (for heat pump, photovoltaic and battery storage) and the "exercise threshold" $p_e^* = 59.5; p_g^* = 13.0$ that leads to the highest mean NPV.

4.3.6 Summarized Results

In Figure 4.19, the overall improvement for all the considered cases is presented. It reveals that the only system with no improvement is the stand-alone photovoltaic system, for which the best option is the immediate investment, which equals the DCF value of the project. However, for the photovoltaic and battery system, choosing the waiting option brings an improvement of the mean NPV, while also increasing the standard deviation, indicating a higher level of risk. Additionally, when only applying the DCF valuation for this technology option, no investment will be made.

For technology options including a heat pump and therefore using two state variables, the mean NPV of all three options can be increased significantly by the option to wait. The risk can also be reduced for the heat pump as a stand-alone system and the system containing all three technologies. It slightly increases for the photovoltaic and heat pump system.

Table 4.5 provides a more detailed view of the obtained values. This illustrates the development of the RO values achieved through the waiting option, in comparison to the DCF values. Additionally, it presents the corresponding thresholds used to attain these values, along with the investment probability, indicating the probability of the exercise threshold being triggered. Furthermore, the waiting time is described as the average time one needs to wait until the investment is triggered.

It is remarkable that a switch from negative to positive mean NPVs is possible when applying the ROs approach. This approach enables investors to capitalize from flexibility in making investment decisions in uncertain environments. This implies that the method should be considered in the future, given the possible increase in values that can be obtained.

Unlike traditional investment valuation models, such as the DCF approach, which often rely solely on current data and expected cash flows, the ROs approach considers

Table 4.5: The overall results for all technology options.

	Mean DCF	Std DCF	Mean RO	Std RO	Threshold	Investment Probability	Waiting Time	Flexibility Value
PV	8269.7	3366.4	8269.7	3366.4	20.0	1.0	0	0
PV and Battery	-1529.5	4458.3	5280.4	6391.6	35.5	0.5	6	6809.9
Heat Pump	-18874.9	10083.3	1660.5	4722.3	(29.0, 12.5)	0.2	10	20535.4
Heat Pump and PV	-8800.1	7200.4	5207.0	7594.3	(45.5, 12.5)	0.5	9	14007.1
Heat Pump, Pv and Battery	-18364.5	6584.8	3897.3	6384.9	(59.5, 13.5)	0.4	10	22261.8

the potential to capitalize on more favorable conditions in the future. By strategically responding to future changes and uncertainties, investors can delay investment decisions and wait for conditions that may turn initially unfavorable investments into profitable ones. This highlights the value of the ROs approach for residential energy technologies in assessing opportunities that DCF-based models may overlook. This approach opens the opportunity for enhanced long-term evaluation, increased NPVs, reduced risk, and overall, strategic decision-making

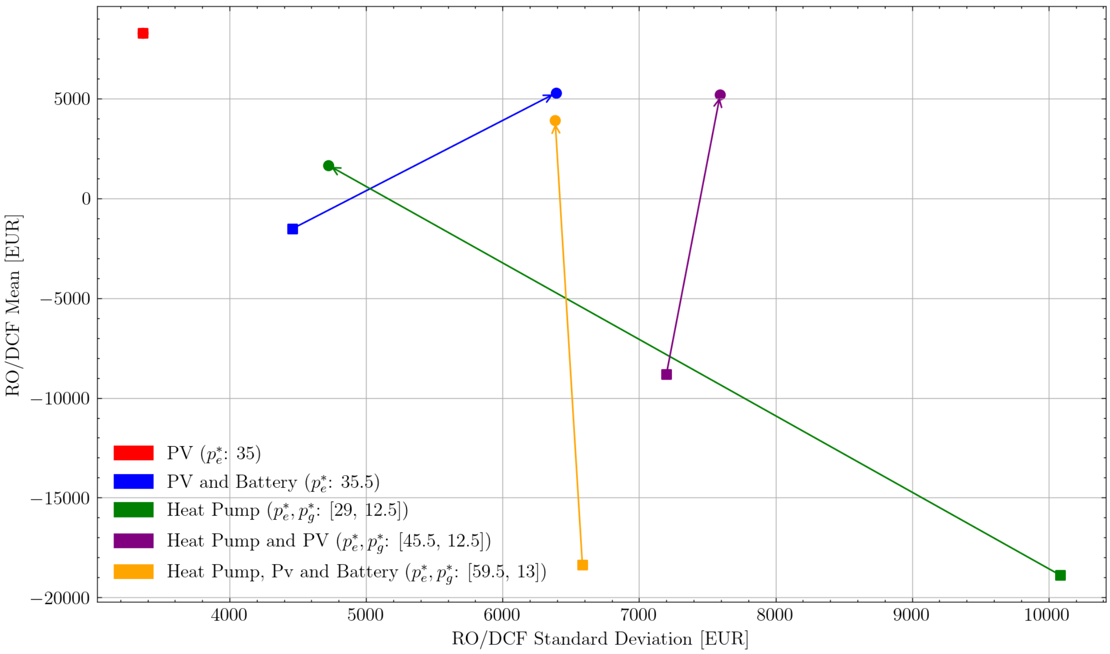


Figure 4.19: Impact on the Net Present Value of the option to wait to invest for all technologies. With the Thresholds generating the maximum mean NPV.

4.4 Sensitivity Analysis

In this section, a sensitivity analysis of the method is conducted to demonstrate the effect of varying inputs, with particular emphasis on the price trajectories and variation in size, of the photovoltaic system.

Two distinct price scenarios are introduced for adjusting the simulations of prices. The aim is to gain insights into how different assumptions may impact the evaluation of residential technology options.

Furthermore, the impact of adjusting the size of the photovoltaic system, both in increasing and decreasing, is explored.

The main objective of this sensitivity analysis is to evaluate the investment project's robustness under various parameter scenarios. It aims to highlight key factors that can influence the risk and the value obtained from waiting for the investment.

4.4.1 Market Sensitivity

In the first scenario, a lower price trend is assumed, based on the findings of a recent study by Thamling et al. [65]. According to the study, household electricity prices are expected to decline in approximately five years and consistently remain slightly below 30 ct/kWh, a level below historical price levels, until 2050. Regarding gas prices, the study forecasts that they will stay about twice as high as before 2022 in the medium and long term, ranging between 11-12 ct/kWh. These adjustments are made to account for the improvement in the security of the supply situation compared to the previous summer. The only modification made to the GBM model is adjusting the drift term to reflect these changes, while keeping all other simulation parameters the same as in the analysis done before.

In the second scenario, a higher price trend is assumed to highlight the contrast that could arise if electricity and gas prices increase. Such a case might occur if the CO2 price does not develop according to the current plans but instead rises again. Recent events have shown that this possibility is not unrealistic. In this case, household electricity prices are assumed to be around 40 ct/kWh until 2050, and gas prices are projected to double to 20 ct/kWh by 2050. It should be noted that these price developments are primarily meant to observe how the mean and standard deviation distribution react to significant price increases and should not be taken for granted.

The results are shown in Figure 4.20. For the photovoltaic system, the immediate investment is still more profitable than waiting with the maximum mean NPV being at $(p_e^{:}35)$ in both cases. The combination of photovoltaic and battery storage shows similar findings, with the maximum mean NPV found at $(p_e^{:}35.5)$ in both cases. Using the RO approach, both cases yield positive mean NPVs compared to a negative DCF value when investing immediately.

Observations of cases with two state variables reveal that even under the lower drift scenario, the RO approach can enhance the investment value. When applying the DCF method, all cases show no profitability, but they become positive when employing the RO approach. Notably, when considering the higher price drift scenario, the RO approach results in greater increases in mean NPVs compared to the lower price drift scenarios. However, it is essential to acknowledge that the standard deviation, and consequently the risk, is higher in these instances.

In Table 4.6, the results of both scenarios are compared. It becomes evident that waiting for the investment is a profitable strategy, even with significantly different price scenarios. Additionally, there is a clear observation that as prices increase, the flexibility values also rise, indicating higher potential benefits. Furthermore, when considering the two state variables, a larger number of values generate positive mean NPVs, further supporting the viability of the investment strategy.

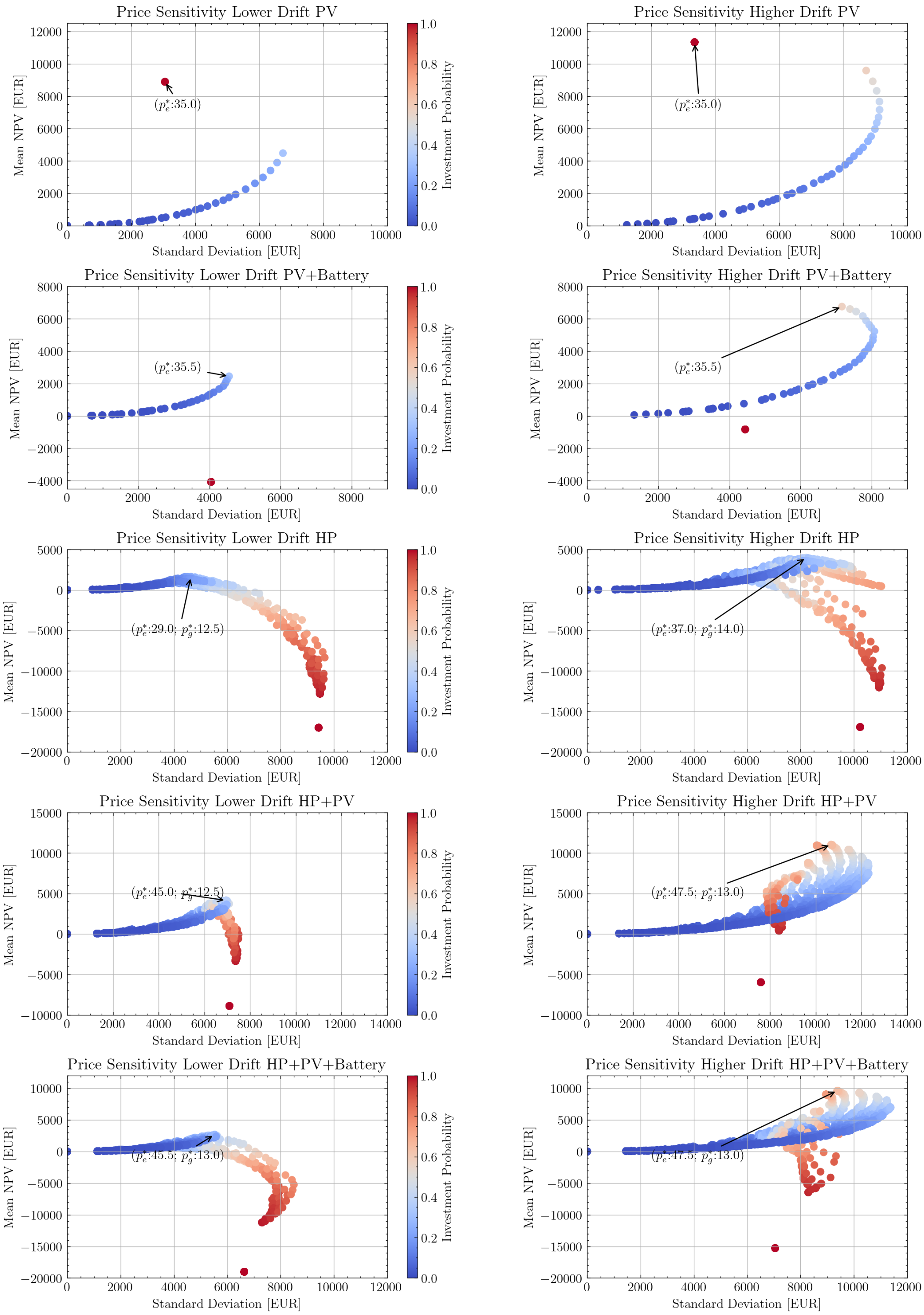


Figure 4.20: Distribution plots for the two market scenarios: lower price drift and higher price drift.

Table 4.6: Results of sensitivity analysis for the low and high price drift scenario.

	Low Price Drift Scenario				High Price Drift Scenario			
Waiting Time	Threshold	Investment Probability	Waiting Time	Flexibility Value	Threshold	Investment Probability	Waiting Time	Flexibility Value
PV	35	1.0	0	0.0	20.0	1.0	0	0.0
PV and Battery	35.5	0.3	6	6523.2	35.5	0.6	7	7586.3
Heat Pump	(29.0, 12.5)	0.2	9	18653.6	(37.0, 14.0)	0.4	11	20916.4
Heat Pump and PV	(45.0, 12.5)	0.4	8	13027.0	(47.5, 13)	0.6	9	16997.3
Heat Pump, Pv and Battery	(45.5, 13.0)	0.3	9	21675.8	(47.5, 13)	0.6	9	24892.9

Table 4.7: Change of the CAPEX values when applying the Sensitivity Analysis.

	PV 15kWp	PV 25kWp
Specific Investment Cost	1300	1100
Total Investment Cost	19500	27500

4.4.2 Technology Sensitivity

To demonstrate the influence of the increase and decrease of the power of the photovoltaic system, the methodology will be applied once again, studying two sizes: 15 kWp and 25 kWp. The only parameter that changes is the initial CAPEX cost for both cases, the changes of the costs are summarized in Table 4.7.

Figure 4.21 displays the results of the sensitivity analysis. In all systems, except for the photovoltaic stand-alone system, there is an improvement when using the RO approach. It is worth noting that applying the DCF approach to the photovoltaic stand-alone system would result in no investment being made.

Table 4.8 provides the relevant parameters for both sensitivity analyses. Notably, no significant differences in investment probability and average waiting time can be observed. However, there is a slight increase in flexibility value for the 25kWp system.

From this it can be observed that the plant size has no influence on the evaluation results of the RO approach. The waiting option still adds additional value to the investment regardless of the size of the system.

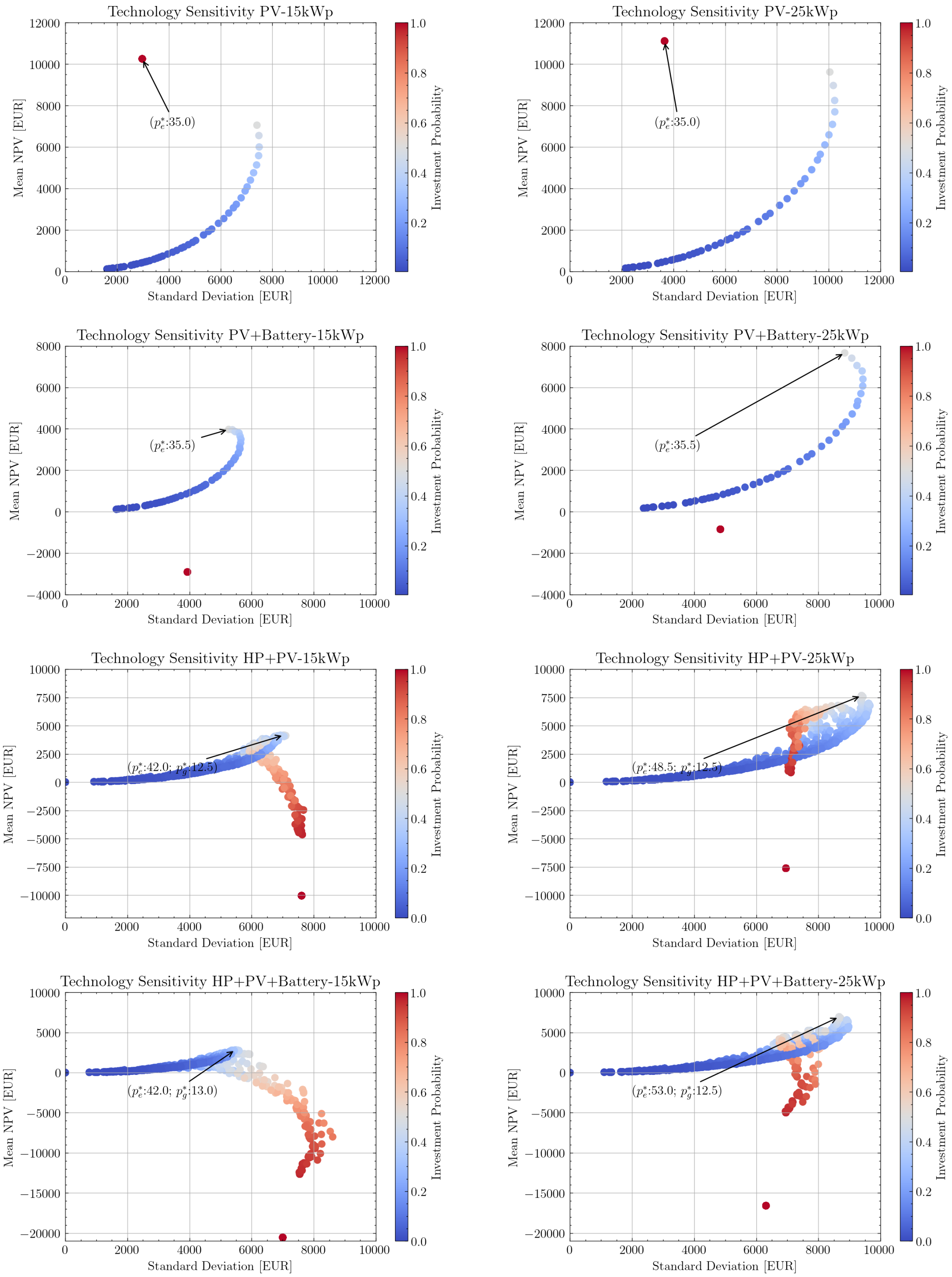


Figure 4.21: Distribution plots for the technology sensitivity analysis for different sizes of photovoltaic systems 15 kWp and 25 kWp.

Table 4.8: Results of sensitivity analysis for the 15 kWp and 25 kWp photovoltaic system.

	PV-15kWp				PV-25kWp			
Waiting Time	Threshold	Investment Probability	Waiting Time	Flexibility Value	Threshold	Investment Probability	Waiting Time	Flexibility Value
PV	35.0	1.0	0	0.0	35.0	1.0	0	0.0
PV and Battery	35.5	0.5	6	6879.7	35.5	0.5	6	8506.6
Heat Pump and PV	(42.0, 12.5)	0.5	9	14202.6	(48.5, 12.5)	0.5	9	15266.2
Heat Pump, Pv and Battery	(42.0, 13.0)	0.4	10	23391.4	(53.5, 12.5)	0.5	9	23526.8

5 Discussion

In this section, the results of the analysis are evaluated and engage in a critical discussion of the Monte Carlo RO approach method employed.

First, the chosen method for evaluating energy technology in residential buildings will be compared to its commonly used counterpart, the DCF analysis. The strengths and weaknesses of the framework introduced will be discussed. The crucial assumptions that influence the results will be pointed out and potential areas for improvement suggested, aiming to enhance the reliability and effectiveness of the proposed framework for residential energy technologies.

Furthermore, the results of the examined scenarios will be discussed. The question will be answered if this method is able to improve the value of the technology options. The influence of the sensitivity analysis and the findings that can be gained from it will be discussed. Additionally, findings from other studies, also applying a RO approach for evaluating residential energy systems are used to compare the findings from the underlying study with the results from other studies.

Finally, the models further possible improvements will be discussed. Having established a powerful prototype for valuing investments for residential energy systems, there are still a lot of areas for improvement.

5.1 Applicability for Residential Energy Technologies

In the beginning of the work, the method was chosen because it is supposed to be simpler than other RO approaches, and therefore can be introduced to a wider audience. After applying the concept for energy technologies in residential buildings, this statement can only be described as true to a certain extent. The reasons for that are: The underlying equations for the NEC must be developed for every case, and require technical understanding, which not every financial department that makes such decisions has. Also, the GBM might be easy to implement, but as shown, the right parameters must be chosen to get valid results. A traditional DCF is way faster to implement, even though the computation is not that high for the ROs case. Even the amount of the parameters that need to get input into the model are almost the same, and only the variables of the GBM are added. There needs to be in-depth knowledge as shown in parameter evaluation on how to handle them and understand what impact these values have.

Locatelli et al. [53] argues at this point that the implementation of the algorithm only takes a few days of work, when being an experienced excel user. According to them, the main work is the collecting of the required input data, that can take days if not months. They argue that for practitioners and investors using the algorithm, the data collection is the most time-consuming part, that might involve the co-operation with subcontractors and payments to consultants. They recommend a trial and error method for implementing simulation parameters. However, this approach can lead to significantly different results, as demonstrated in this study. For less experienced users of the algorithm, it is quite difficult to see whether the results correspond to reality or not.

What can be stated true is that this method is applicable for valuation of residential energy technologies. The information that can be obtained can be very valuable in evaluating investments in this application area. Information about the increase of the NPV and the information about the risk of waiting for the corresponding exercise threshold can be gained. Based on this, all uncertainties associated with this investment can be evaluated or included in the investment evaluation. So, despite the disadvantages that arise from the required knowledge for the application of the methodology, a profit can still be made for the investor through the additional flexibility and the consideration of the uncertainties, when applying this method to value investments.

The method is applicable, but it relies on several assumptions that need to be considered before evaluating the investment. These assumptions include the development of price uncertainties for gas and electricity in this work. Additionally, other relevant factors came into consideration during this work, such as:

- The FiT development: This factor plays a critical role in the evaluation of the investment. It is essential to accurately model the uncertainties associated with FiT changes and their impact on the investment outcomes.
- CAPEX costs for the technologies: The assumption of decreasing CAPEX costs in this work raises an important question about the validity of this assumption. A thorough examination of real-world trends and data is required to confirm whether the CAPEX costs are indeed decreasing as assumed.
- Sensitivity analysis and price path assumptions: The results demonstrate a strong connection with the assumed price path. To enhance the reliability of the analysis, alternative approaches, such as utilizing different price scenarios from reputable literature, and applying the GBM method step-wise with distinct μ values for each step, could be explored.
- Constant discount rate and neglecting inflation: The assumption of a constant discount rate, slightly higher than the current rate, raises questions about the actual influence of the discount rate on the investment valuation. Additionally, the omission of inflation throughout the work could potentially impact the outcome of the results.
- The risk of the uncertainties of two state variables is not connected in this work. What that means is, there is a mathematical connection between gas and electricity prices, and they move up and down together [67, 53]. This connection is not considered in this work.

5.2 Discussion of the Results

Starting with the Photovoltaic system: In the case of this work, it is always recommended to invest immediately in a photovoltaic system under the proposed conditions. There is no additional value to be gained by applying the RO approach.

For the photovoltaic and battery system, an improvement of the mean NPV can be made by performing the RO analysis, by considering the investment time in the future. The investment would not be performed using the DCF approach to evaluate this system.

For all systems using two state variables, the RO analysis increases the NPV. As of today, all of the systems would not be recommended to invest in when using the DCF approach. For the heat pump and photovoltaic, heat pump and battery storage systems, even the risk of the investment can be reduced by applying the RO approach.

In the first sensitivity analysis applying different price scenarios, the following key findings can be obtained: The RO approach still improves the mean NPV for all examined technologies, for both scenarios. Additionally, the lower the assumed price trends, the smaller the improvement of the mean NPV and therefore the value of the option to wait. This goes both ways, the higher the assumed price trends, the greater the improvement of the mean NPV. This is not just of interest to investors, but also to policy makers. These observations can be used to evaluate policy decisions.

The second sensitivity analysis shows the impact of increasing and decreasing the capacity of the photovoltaic system. This sensitivity got employed because out of all the technologies, the photovoltaic system seems to be the most economic. In all cases where photovoltaic was considered a standalone system, the immediate investment was the best option. A size of 15 kWp and 25 kWp was chosen to examine how big the influence of the capacity is. The following key findings can be obtained: The stand-alone photovoltaic system still does not get improved by the RO approach. The greater the capacity of the photovoltaic system, the greater the obtained flexibility value. For systems that include a heat pump, the threshold values that generate a positive mean NPV is significantly smaller.

Overall, the results and the sensitivity analysis prove that the RO approach discussed in this work enhances the evaluation of investments of residential energy technologies. The "option to wait" and "option to abandon" increase the mean NPV in all cases, except for the photovoltaic as a stand-alone system. One big point is the model can not just be used for investors, but also to value political subsidies. With the developed tool, the opportunity to see the difference in changing values when increasing or decreasing the FiT, but also subsidize the different technologies which can be expressed by different CAPEX values. By implementing variations of the parameters, the algorithm can be used to validate the impact of political decisions.

Martinez et al. [58] and Androelli et al. [11] come up with similar results for residential photovoltaic systems. In Martinez et al. [58] an average waiting time of 5 years is estimated to be the optimal waiting time for photovoltaic systems, have the FiT as an uncertainty variable. The findings of Androelli et al. are like the one in this work, the investment for household to invest in a photovoltaic system coupled with small battery storage should be performed immediately. The greater the system, the greater is the value in waiting. They also observe that the higher the volatility of the energy prices, the greater is the option to defer an investment. This relationship was also found in the evaluation of the parameters, if volatilities are chosen too small, the flexibility value will always get smaller, or be equal to zero.

An additional observation is that in certain cases, a regular DCF analysis may result in a negative NPV for the system under consideration, while utilizing the RO approach leads to a profitable outcome. This finding aligns with the conclusions in Penizzotto et al. [68], where postponing the decision on a rooftop photovoltaic project using the RO approach proves beneficial, whereas a traditional NPV analysis would never justify the investment.

6 Outlook and Further Work

The fundamentals of a prototype have been established, and it has been proven to be a powerful tool for evaluating investments of residential energy technologies. A lot of information can be gained that is not available with a traditional DCF approach.

The following improvements can be made to improve the underlying work:

- Better models for the input parameters with higher accuracy. In detail, create improved/different models for energy prices linking the two variables gas and electricity. An increase in the price of electricity also leads to an increase in the price of gas in the model. In addition, other methods for evaluating investment costs can certainly be found. Prices are rather stagnant at the moment, which is not quite reflected in the model.
- The threshold values got chosen manually in this work, taking the threshold that delivers the highest mean NPV. This approach can be optimized taking the mean NPV, the standard deviation and the investment probability into account
- Advance the sensitivity analysis with a higher range to find out which parameters have a particular influence on the residential energy options and which possibilities of subsidies evaluation result from it.
- So far, only one-time investment in all technologies have been considered. Another method, which would be particularly interesting for the evaluation of transformation paths, is to consider several investment points for the respective technologies. This would mean, for example, for the photovoltaic heat pump system: investment of the photovoltaic system first in 5 years, followed 10 years later by the investment in a heat pump, resulting in the greatest increase in flexibility value through the waiting option. This approach also got introduced as a further research perspective [11], where they introduce the idea of sequential investment.
- Evaluation of other uncertainty variables. It is possible to assume multiple uncertainty variables, which can lead to improved accuracy of the model. On the downside, this would increase the complexity of the Monte-Carlo analysis
- It might be interesting to do different kinds of tests for the underlying case. When is there value to be gained when investing in a photovoltaic stand-alone system? What OPEX costs are the maximum where the result is not to invest immediately, but waiting increases the value of the investment? Similar approaches can be done for all Technologies, depending on the interest of the user of these methods.
- Only the respective investment costs of the technologies are considered, so only an economic evaluation is performed. Another option of comparison could be the uses of Energy/primary energy sources of the systems or the energy efficiency of the systems to be installed.

- Instead of looking at one uncertainty variable, in the case of this work the market uncertainties, multiple uncertainties could be considered. For instance, the CAPEX development of the technologies could be handled as uncertainties.

7 Summary

Investments in the energy sector face significant uncertainties due to changing political conditions and volatile energy prices. The traditionally used method for the evaluation of investment in the energy sector, the Discounted Cash Flow method, can not take these uncertainties into account appropriately. Real Options theory provides the tools to take these uncertainties into account. Often, decision-makers do not use this approach because they are not familiar with it and are deterred by the complexity.

The goal of this work is to use a simple to implement Monte Carlo-based Real Options approach to evaluate investment in residential energy technologies. The used algorithm, applies "exercise thresholds" to decide whether to invest now, wait for the investment or to abandon the project. Using this approach, the decision-maker can influence the profitability and the corresponding risk of the investment. This algorithm will be examined to determine if it is applicable for the assessment of residential energy technologies, showing the limitations and crucial parameters, and pointing out the advantages and disadvantages of the commonly used Discounted Cash Flow method. Additionally, the influence of waiting options on the evaluation of residential energy technologies will be examined, to find out whether this approach can improve the Net Present Value of the technologies. For the evaluation of this a prototype will be developed. In the underlying use case, the technologies photovoltaic, heat pump and battery storage and their respective combinations will be considered. The uncertainties in this work are the electricity and the gas price. The uncertainties were modeled using a Geometric Brownian Motion.

The results show that for every residential energy technology, except for the photovoltaic as a stand-alone system, an improvement of the mean Net Present Value can be obtained. Remarkably, all other options are, when evaluated with the Discounted Cash Flow method, not worth investing in. Applying this method and delaying this investment time returns positive mean Net Present Values for all technologies. This shows that this method can enhance the evaluation process of residential energy technologies and can be used as a powerful tool for evaluation for investors and also policy makers.

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