An analytical model to predict the compressive damage of concrete plates under contact detonation

TU Huan\textsuperscript{a}, FUNG Tat Ching\textsuperscript{a}, TAN Kang Hai\textsuperscript{a,*}, RIEDEL Werner\textsuperscript{a,b}

\textsuperscript{a}School of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639798, Singapore

\textsuperscript{b}Fraunhofer EMI, Ernst-Zermelo-Str. 4, 79104 Freiburg, Germany

* Corresponding author.
E-mail address: ckhtan@ntu.edu.sg

Abstract

Explosions threaten the safety of buildings and occupants by inducing severe damage to concrete structural elements. The effects of close-in and contact loading situations on RC plates have been investigated in literature and the common approaches employ field test results condensed in engineering charts based on cube-root scaling. In this paper, a new analytical model based on a physical framework is developed to (a) investigate shock wave attenuation along the depth of the concrete plate and (b) predict the size of crater damage. The analytical model for shock wave pressure is discussed in comparison to prediction data from established numerical models and existing experimental results. Moreover, consistency of the newly proposed model with established scaling rules is verified with respect to crater size.

Keywords

Contact detonation; Concrete plates; Shock wave; Crater damage; Scaling.
1. Introduction

While explosion effects on buildings have long since been a task in the military context [1], world-wide terrorism has shifted the attention to civilian infrastructure. Placing high explosive (HE) at a small stand-off distance or even in contact can destroy concrete structural components, initiate collapse and is hazardous by debris and air shock loading. It is therefore important to study and model the mechanisms of concrete failure to design structural elements for specific protective requirements.

1.1 Discriminating between far-field and close-in explosion effects

The explosion effect to the concrete plate is highly dependent on charge weight and its relative distance to the target. The scaled distance $Z$ in Eq. (1) is a common parameter to assessing the blast loading effect on concrete members.

$$Z = \frac{R}{\sqrt{W}}$$

where $R$ is the standoff distance (m) and $W$ is the weight of the charge (kg).

According to scaled distance $Z$, a blast effect can be classified as a far-field or near-field event. For a far-field blast, where $Z$ is greater than an empirically found, material-dependent limit value of $Z_{\text{lim}}$, the blast load can be considered as a uniform load across the member surface initiating global response in flexure and/or shear failure. For a near-field event, where $Z$ is less than $Z_{\text{lim}}$, blast loading is nonuniform with high intensity and members are subjected to a severe localized damage, such as cratering and spalling. An empirical threshold value $Z_{\text{lim}} \approx 0.5$ was reported in reference [2] to differentiate between these two blast effects for reinforced concrete (RC) slabs.

Due to marked differences in blast effect and failure modes on concrete plates between a far-field and a near-field blast, the modeling approaches are not the same. When the scaled distance is larger than $Z_{\text{lim}}$, global response of concrete members (including the boundary conditions) plays a dominant role on the failure mode. In this case, the single-degree-of-freedom (SDOF) method is a suitable approximation to predict the dynamic response of reinforced concrete walls and slabs subjected to blast loading [3]. However, when scaled distance $Z \leq 0.5$, there are very few theoretical methods to predict the damage due to complicated mechanisms involved. Current approaches for $Z \leq 0.5$ are mainly based on experimental and numerical studies [4]. Shi et al. [5] conducted a series of close-in detonation tests to investigate the damage mechanisms of RC slabs and the characteristic of spalling damage was presented. Based on the test results, the degree of spalling damage is influenced by the height to diameter ratio of the cylindrical charge. Wang et al. [6] carried out close-in blast tests to study the damage modes of square RC slabs at different scaled distances. They also conducted
numerical studies to compare with test results. They found that the failure mode of RC slab changes from overall flexure to localized failure with a decrease in scaled distance. In this regard, contact explosion is the extreme example of near-field explosions. As an explosive is directly placed on the surface of a concrete plate, intense shock waves with tremendous local energy are transmitted into the concrete plate from the detonation charge. Several prediction models for concrete plates under contact explosion were proposed, mostly empirically based on experimental data and cube-root scaling (1). UFC [1] provides a method to predict breaching and spalling damage of RC walls under close-in and contact explosion. Beppu et al. [7] investigated damage mode of concrete plates strengthened with FRP laminates under contact explosion and compared Morishita’s empirical formulas with test results [8]. Li et al. [9] studied the spalling phenomenon and fragment distribution of normal strength concrete (NSC) and ultra-high-performance concrete (UHPC) slabs under contact explosion through field tests.

1.2 Scaling analysis

In this context, scaled models are useful as they can replace large tests in complex scenarios [10]. By using the Buckingham-π-theorem, key dimensionless parameters during dimensional analysis can be determined. To compare blast loading produced by different explosive weights and/or standoff distances, replica scaling and the cube root scaling law have been widely used, e.g. the scaled distance \( Z \) (Eq. (1)) as a representative parameter. Researchers from different countries adopted scaling analysis to investigate the response of structures under contact detonation. Forsén [11] conducted full and 1/4 scale contact detonation tests on concrete plates. Based on his results, a scale independent relationship between charge weight and crater damage size has been observed, viz., a linear relationship between crater depth normalized by plate thickness and cube root of charge weight over the plate thickness. Schaufelberger et al. [12] reviewed critical scaling rules used for concrete structures under blast loading and reiterated the scaling analysis done by Forsén to confirm the scale independent relationship between crater damage and charge weight. Gebbeken et al. [13] proposed, in line with replica scaling relations, an empirical engineering tool to predict the size of local damage on concrete structures subjected to contact and very close-in blast loading based on extensive tests and numerical simulation results. A similar approach was applied by UFC [1] to propose threshold damage curves for spalling and breaching curves, which could provide qualitative damage predictions on concrete structures under near-field and contact blast effects.
1.3 Damage modes

For contact detonations, intense shock waves created in an explosive will transmit into a concrete plate, and the target will suffer severe local damage. A crater on the impacted face and spalling on the rear face are two common damage patterns that occur on both plain and reinforced concrete plates. Damage modes have been observed in various field tests [7], [9], [14] and are schematically shown in Fig. 1(b). The depth and diameter of the formed crater and spalling are key parameters to assessing the extent of local damage. The crater formation is due to the shock waves near the impacted face that exceed the dynamic compressive strength of concrete by orders of magnitude, thus crushing the material. As the pressure in concrete induced by explosive detonation can reach tens of GPa, the traditional way to consider the dynamic compressive strength of concrete by using a Dynamic Increase Factor (DIF) is inappropriate. Hence, van Amelsfort and Weerheijm [15] conducted several field tests by using concrete with compressive strength from 25 MPa to 30 MPa to investigate the failure of concrete plates under contact detonation. Based on the results, they recommended stress levels for total crushing of concrete of 400 MPa to 1000 MPa. Due to shock dissipation by pore crushing and spherical propagation, the amplitude of the compression wave will decrease quickly with running length. After propagating through the concrete plate, the compression pulse will reflect as a rarefaction wave upon the rear face. Spalling will occur if superposition of release waves following the compression and resulting from reflection lead to net tensile stresses exceeding the dynamic strength of concrete.

Numerical investigations using Finite Element Analyses of loading, response and damage modes in concrete plates under contact detonation have been conducted by some researchers. Grunwald et al. [16] simulated a reinforced concrete plate under contact detonation effect by using the commercial hydrocode ANSYS-Autodyn [11] and compared the numerical results with the test result (Fig. 2). This class of finite element models solves a set of partial differential equations derived from the conservation equations for mass, momentum and energy in an explicit time-stepping scheme. There is good agreement between numerical predictions and test results on crater and spalling damage.
Fig. 1. (a) Diagram for contact detonation, (b) cross section of a concrete plate showing crater and spalling damage, (c) top view of a damaged concrete plate.

Fig. 2. Comparison of damage between simulation and experiment, reprinted with permission from [16].
Although conducting experiments and developing numerical models are two commonly-used approaches to investigate local damage of concrete under contact detonation effect, both approaches have their respective shortcomings. Empirical prediction methods lack theoretical rigor. In addition, conducting a field test is costly and test results are subject to environmental variations in humidity and temperature. Repeatability of explosive charge itself is also subject to a certain amount of stochastic scattering for the detonation process. For numerical-based studies, predictive models for a given concrete quality and explosive properties must be available. Due to resolution of fine mesh in the model with coupled simulations between fluid dynamics and Lagrangian meshes for explosive gases and solid members, running the numerical simulation model is often time-consuming and requires substantial expertise on the user.

Hence, developing a physics-based analytical model to predict damage on concrete plates under contact detonation is promising and warranted. This paper presents a new model based on detonation physics, shock wave propagation, dynamic concrete behavior and observations from experiments. The damage prediction focuses on the dimensions of craters on the impacted face, while spalling at the rear face will also be studied in the future. After developing the analytical model, the predictive capacity of the proposed method for crater damage is verified with published experimental results. To make the analytical treatment more comprehensive, scalability of the proposed model will be addressed.

2. An analytical model of concrete compressive damage under contact detonation

2.1 General principle

When a concrete plate is subjected to a contact detonation effect, explosion energy is transmitted through the contact face in the form of a shock wave as shown in Fig. 1. Different types of waves occur in the concrete after detonation, viz. a hemispherical shock wave, a following shear wave and circular Rayleigh waves on the surface. Fig. 3, adapted from Meyers’ work [17], shows the propagation of stress waves in the solid during contact detonation. Concrete directly under the cylindrical charge is subjected to planar shock compression, with propagation speed $U_s$ and under a state of plane strain (Fig. 3). Additionally, shear and release waves are generated at the edges of the interface between the concrete and the explosive. These waves travel over time with the speed of $C_T$ into the conical area. Thus, an approximate relationship for the conical area in 2 dimensions is derived as follows: half base to height ratio of the area is equal to shear wave velocity $C_T$ normalized by shock wave velocity $U_s$ (Eq. (2a)). Therefore, calculation of the shock transmission at the interface between the charge and the plate
follows the plane strain assumption. Shear wave velocity in concrete depends on the density and shear modulus of concrete and it is calculated by Eq. (2b). To investigate the contact detonation effect on the concrete plate, a step-by-step analysis is conducted as shown in Fig. 4.

\[
\frac{0.5d}{h} = \frac{C_T}{U_s} \tag{2a}
\]

where \(d\) is the diameter of the charge; \(h\) is the height of the cone in Fig.3; \(C_T\) is the shear wave speed in concrete; \(U_s\) is the shock velocity.

\[
C_T = \frac{G}{\sqrt{\rho_0}} \tag{2b}
\]

where \(G\) is the shear modulus of concrete and \(\rho_0\) is the initial density of concrete.

Fig. 3. The formation of different types of stress wave in semi-infinite body after contact detonation; cone of uniaxial strain loading regime directly under the charge.

Fig. 4. Flowchart of analytical steps.
2.2 Explosive detonation

2.2.1 Explosive properties

Generally, TNT is the reference high explosive (HE) in military and civilian applications. Other explosives such as C-4 and PETN can be related to an equivalent mass of TNT according to thermal energy release [18]. Hence, the current study only focuses on TNT as the reference explosive load.

The widely used Chapman-Jouguet (CJ) detonation model [19] assumes that the explosive reaction is instantaneous and there is no reaction zone in the explosive itself. It describes the detonation wave in the explosive propagating with the peak pressure $P_{CJ}$ at the detonation front as shown in Fig. 5. It moves into the unreacted material with detonation velocity $D$. The interface between the unreacted and the reacted explosive moves at a velocity equal to the particle velocity plus the material sound speed ($u_{CJ} + C_E$) [17]. It is assumed that the detonation wave propagation in the explosive is stable. Thus, Eq. (3) must be satisfied. A release wave immediately follows in the products behind the detonation front.

![Fig. 5. Pressure profile of detonation wave in explosive.](image)

$$D = C_E + u_{CJ}$$  \hspace{1cm} (3)

where $C_E$ is the sound velocity of the reacted explosive and $u_{CJ}$ is the particle velocity.

Across the detonation front, the explosive material is converted into gaseous products with extremely high pressure and temperature. The Jones-Wikins-Lee (JWL) equation of state Eq. (4) is commonly used to describe them near the CJ state and at larger expansion [20].

$$P = A(1 - \frac{\omega}{R_1 V})^{-R_1 V} + B \left(1 - \frac{\omega}{R_2 V}\right) e^{-R_2 V} + \frac{\omega E}{V}$$  \hspace{1cm} (4)

where $P$ is the pressure; $A$, $B$ and $C$ are linear coefficients; $R_1$, $R_2$ and $\omega$: nonlinear
coefficients; $V$ is $v/v_0$ (volume of detonation products/volume of undetonated HE). The six constants ($A, B, C, R_1, R_2$ and $\omega$) in the JWL equation are empirical and are generally determined from cylinder-test data [21].

For the explosive products below the CJ state, the JWL isentropic Eq. (5) can be used.

$$P = A e^{-R_1 V} + B e^{-R_2 V} + C V^{-(\omega + 1)} \quad (5)$$

The state of explosive during and after detonation can thus be determined by the combination of the C-J model with the JWL equation of state (EOS). The explosive properties of TNT [21] are summarized in Table 1.

<table>
<thead>
<tr>
<th>Explosive</th>
<th>C-J parameters</th>
<th>JWL EOS parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_0$ (kg/m$^3$)</td>
<td>$P_{CJ}$ (GPa)</td>
</tr>
<tr>
<td>TNT</td>
<td>1630</td>
<td>21</td>
</tr>
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</table>

2.2.2 Detonation wave propagation in the cylindrical charge

In the present work, a cylinder of explosive with a length to diameter ratio ($L/d$) of 1:1 is assumed, which is a common shape for blast applications. However, generalization for other shapes is possible, based on the same principle and approach. Initiation is triggered at the surface opposite to the contact plane between the explosive and the concrete. Both the length and the diameter of the explosive influence the build-up and the peak pressure of the detonation wave. In the present applications the diameter of TNT charges is much larger than the “failure diameter”, which is the minimum diameter for a cylinder charge to propagate a steady detonation [19]. For TNT, this material-specific value is equal to 2.6 mm under unconfined conditions according to [21]. Therefore, for wider cylinders the peak pressure in the explosive is only depending on the length of charge [19]. To investigate its relationship with the charge length, a series of TNT charges with different lengths are simulated using the commercial hydrocode ANSYS-Autodyn™ [25], consistent with the validated modeling approaches in [16]. The TNT charges are simulated using arbitrary Lagrangian-Eulerian (ALE) elements with 1mm size and the JWL equation of state with the same parameters in Table 1. A simulated detonation wave in a cylindrical charge is shown in Fig.6(a). Both the dimensional and normalized peak pressure to charge length relationships are shown in Fig.7, with corresponding relationships in Eqs. (6a) and (6b). The figure shows convergence with increasing length towards the maximum detonation
pressure, which is 21 GPa for TNT.

Fig. 6. (a) Simulated detonation wave in a cylindrical charge by using Autodyne, (b) schematic for detonation wave in a cylindrical charge.

Fig. 7. Relationship between peak detonation pressure and length of charge.
\[ P = 1.87 \ln(L) + 7.949 \]  \hspace{1cm} (6a)

\[ \frac{P}{P_{CJ}} = 0.089 \ln(L) + 0.3785 \]  \hspace{1cm} (6b)

where \( P \) is the peak detonation pressure (GPa); \( P_{CJ} \) is the peak CJ pressure of explosive; \( L \) is the length of charge (mm).

As observed in the numerical models as well as in detonation tests [19], the detonation propagates in the cylindrical charge with a curved wave front. The curvature and the angle at the edge of the charge are induced by self-confinement in the center and lateral release effects. An empirical approximation for the edge angle of reacting TNT of 83 degrees has been determined by Dorsett et al. [22]. Based on the observation from simulation models and detonation tests, a 2-D detonation wave flow model for a cylindrical charge is proposed and shown in Fig. 6(b) with the following characteristics:

- The detonation velocity \( D \) is equal to the experimental data 6930 m/s as shown in Table 1;
- The peak pressure \( P \) is determined by Eq. (6a). The maximum detonation pressure of TNT \( (P_{CJ}) \) is 21 GPa, which is the upper bound in Eq. (6a);
- The impulse in the explosive is considered to be of triangular shape as shown in Fig. 5. The duration of impulse is assumed to be \( \tau_E = L/D \) (the length of charge/detonation velocity) based on geometrical consideration;
- The edge angle \( \alpha \) is equal to 83 degrees [22] and the radius of the curved detonation front is calculated as \( R = d/2\cos(\alpha) \) (radius of charge/cos(edge angle)).

2.3 Transmission into the concrete

2.3.1 Shock wave principles

After approximating the detonation wave characteristics in the explosive, the next step is to describe the shock wave transmission from the charge into the concrete plate. Along the centerline of the charge extended into the plate, plane strain assumption can be used directly under the explosive (see conical area in Fig. 3). Eqs. (7a–c) apply, which are generally known as the “Rankine-Hugoniot-Equations” [17]. The set of equations is derived from the conservation of mass, momentum and energy and applied to determine the shock states of concrete near the interface.

Mass: \[ \rho_0 U_S = \rho (U_S - u_p) \]  \hspace{1cm} (7a)
Momentum: \[ P - P_0 = \rho_0 U_s u_p \]  
\[ \text{(7b)} \]

Energy: \[ E - E_0 = \frac{1}{2} (P + P_0) (V_0 - V) \]  
\[ \text{(7c)} \]

where \( \rho_0 \) is the initial density; \( \rho \) is the density after shock; \( P \) is the shock pressure; \( P_0 \) is the initial pressure; \( U_s \) is the shock wave velocity; \( u_p \) is the material (or “particle”) velocity; \( E_0 \) is the initial energy; \( E \) is the shocked energy; \( V_0 \) is the initial specific volume = \( 1/\rho_0 \); \( V \) is the specific volume of medium.

Since the number of variables in (7a-c) exceeds the number of analytical relations, an additional equation of state (EOS) must be provided to determine the shock state of concrete. The EOS is frequently described as relation of shock wave velocity \( U_s \) to particle velocity relationship \( u_p \), as shown in Fig.8, and adopted from reference [23]. In concrete, the compressive wave velocity decreases with particle velocity during pore compaction at low particle velocities and later increases when shock pressure overrides the porous dissipation mechanism.

\[ \text{Fig. 8. Experimental data for shock velocity to particle velocity relationship of concrete material in reference [23].} \]

Since pressures and particle velocities of concrete under contact detonation are high, the \( U_s-u_p \) EOS can be simplified to a linear form as shown in Fig.8. The regime considered in this paper is \( u_p < 1600 \text{ m/s or } P < 17.3 \text{GPa} \). Eq. (8) describes the linear fit and parameters \( c_B \) and \( S \) can be derived from Fig.8, which are 1800m/s and 1.8, respectively.
Using the Rankine-Hugoniot-Equations and the linearized $U_s-u_p$ EOS, the shock state of concrete can be fully described whenever one of the three state parameters ($P$, $U_s$ and $u_p$) is determined. This method will be used in the following sections to determine shock wave transmission and pressure attenuation during propagation into the concrete.

2.3.2 Impedance matching at the loading interface

For the case of a contact detonation, the shock wave travels directly from the explosive into the concrete through the contact surface. Nevertheless, the method in the present work can be extended to close-in loading conditions with small air gaps between the explosive and the concrete, by enriching Fig. 9 further by adding characteristics for an additional layer. In the present case of direct contact, impedance matching [17] is a practical method to determine shock wave pressure and particle velocity applied by an explosion on the concrete surface. In the following it is assumed that the initial ambient pressure ($P_0$) in the explosive and concrete is negligible. Then the momentum conservation Eq. (7b) can be simplified as Eq. (9).

$$P = \rho_0 U_s u_p$$  \hspace{1cm} (9)

In the pressure-particle velocity diagram (Fig. 9), there are four curves, viz., explosive Rayleigh line (①), concrete Rayleigh line (②), isentropic release curve of detonation products (③), and its simplified version (④), respectively.

As shown in Fig. 9, the actual shock state of concrete at the contact surface is S1, which is the intersection between concrete Rayleigh line (②) and detonation product
The shock impedance, which is the product of material density and shock wave velocity in the material, is a key parameter to determine the shock state in each material after explosion. The impedance of TNT is equal to $1630 \text{ (kg/m}^3) \times 6930 \text{ (m/s)}$, and the impedance of concrete is equal to $2314 \text{ (kg/m}^3) \times 4500 \text{ (m/s)}$. Thus, the impedance ratio of TNT to concrete is $1.08$, which means that most of the detonation wave is directly transmitted into the plate. Since TNT and concrete are in direct contact, the release curve of the explosive products can be simplified to a linear form with a slope equal to $-\rho_E D$ leading to $S_2$, as shown in Fig. 9. Thus, the equation for curve can be expressed as follows:

$$-\rho_E D u_p + 2P_E = P \quad (10)$$

where $\rho_E$ is the density of explosive, $P$ and $P_E$ are the peak pressure on the concrete surface and in the explosive, respectively.

Combining Eqs. (8), (9) and (10) to Eq. (11) allows the engineer to calculate the concrete particle velocity resulting from transmission of the detonation wave from the explosive into the concrete plate. All other state variables behind the shock front, such as the shock velocity, internal energy etc., can be determined from Eqs. (7) to (9).

$$(\rho_0 c_B + \rho_E D)u_p + \rho_0 S u_p^2 = 2P_E \quad (11)$$

When the detonation wave travels from the explosive to the concrete plate, the duration of impulse will be reduced due to slower shock wave propagation velocity in the latter. The change is assumed to be proportional to the ratio of the shock and detonation wave velocity, respectively, as expressed by Eq. (12):

$$\tau_0 = \tau_E \frac{u_{so}}{D} = \frac{L}{D} \frac{u_{so}}{D} \quad (12)$$

where $D$ and $\tau E$ are the detonation wave velocity and pulse length in the explosive; $L$ is the length of the charge; $U_{so}$ and $\tau_0$ are the initial shock wave velocity and pulse length in the concrete plate.

### 2.4 Propagation across the plate

Concrete is a porous composite material with strongly nonlinear and dissipative shock properties. The $p-\alpha$-EOS proposed by Herrmann [24] is widely used to describe concrete porosity in hydrocode simulations [2], [13], [16], [23], [25]. Under the shock effect, concrete is compressed and internal pores collapse. When the pressure exceeds a certain value, the pores close entirely and do not reopen. This threshold is generally referred to as lockup pressure, above which the concrete behaves as solid material.
2.4.1 Attenuation above the lockup pressure

In the proposed model, the shock wave traveling through the plate is assumed to have a triangular shape, similar to the detonation wave in the explosive (shown in Fig. 5). Fig. 10(a) shows the configuration of a shock wave in the solid media with an instantaneous pressure increase and the release wave, when pressure returns to zero. Both portions propagate in the solid media with different velocities. The velocities at the shock front, the head and tail of release wave are $U_s$, $C(P) + u_p(P)$ and $C_0$, respectively. Parameters $C(P)$ and $C_0$ denote the sound speeds of concrete at a particular pressure and zero pressure, respectively [17]. In a pressure-density ($P-\rho$) projection of the Hugoniot curve (Fig. 11), the Rayleigh line connects the initial and the final states of the concrete material reached by the jump conditions of the shock loading. The shock velocity ($U_s$) is determined by the slope of the Rayleigh line while the sound speed of the release portion at a given pressure $C(P)$ is determined by the slope of the release curve, which can be approximated by the respective value of the Hugoniot curve. It is clearly shown that this local slope is greater than that of the Rayleigh line. Thus, the velocity at the head of release wave $C(P) + u_p(P)$ is greater than the velocity at the shock front $U_s$. The release portion catches the leading front during the propagation and the magnitude of the peak pressure attenuates continually along the propagation path. Based on the principle of shock wave attenuation, the change of the triangular shock wave in concrete is demonstrated in Fig. 10(b).

Fig. 10. (a) Shock wave propagation in a solid, (b) shock wave attenuation in a solid.
In the analytical model, 1-D shock wave propagation is assumed along the center line in the conical portion depicted in Fig. 3. Based on the principle of shock pressure attenuation in the solid material, Eq. (13) is proposed to calculate the change of peak pressure when the pressure is higher than the lockup pressure. This calculation at different distances is an iterative process and the step size $\Delta x$ influences the accuracy. To obtain convergent calculation, pressure-distance curves from numerical models and analytical models using different mesh sizes (10, 5 and 2 mm) are plotted in Appendix Fig. 19. (a) and (b). Both the numerical and analytical results show that a 5 mm step size gives sufficiently converged results and is applied from then on. From the numerical results, the shape of the shock wave is not perfectly triangular and decline of pressure at a high-pressure state is faster than that at a low-pressure state as shown in Fig. 12. To consider the faster attenuation effect for a high-pressure condition, an effective duration of impulse $\tau_{\text{eff}} = 1/3 \times \tau_0$ is used in Eq. (13).

$$\Delta P = \frac{P\Delta x \left( \frac{1}{u_p + \epsilon(P)} - \frac{1}{u_p + \epsilon(P)} + \tau_{\text{eff}} \right)}{\left[ \frac{\Delta x}{\epsilon_0} - \frac{1}{u_p + \epsilon(P)} \right]^2}$$  \hspace{1cm} (13)$$

where $\Delta P$ is the decay of pressure; $\Delta x$ is the mesh size; $\tau_{\text{eff}}$ is the effective
duration= 1/3 × τ0.

By combining Eqs. (7a), (8) and (9), \( P_H - \rho \) (pressure-density) relationship (Eq. (14a)) of the Hugoniot curve can be derived. The local sound speed at pressure \( P \) is determined by Eq. (14b).

\[
P_H = \rho_0 c_B^2 \frac{(\frac{\rho}{\rho_0} - 1)}{[1 - (\frac{\rho}{\rho_0} - 1)(S - 1)]^2}
\]

\[
\mathcal{C}(P)^2 = \left. \frac{\partial P_H}{\partial \rho} \right|_P
\]

where \( P_H \) is the Hugoniot pressure.

2.4.2 Attenuation by pore crushing

Due to attenuation effect, the peak pressure will drop below the lockup pressure after a certain distance and porous dissipation becomes a dominant mechanism. The snow-plow model (Eq. (15)) can be used for a simplified description of shock decay in the porous materials [24]. In Eq. (15) \( P \) is inversely proportional to the square of the running length \( x^2 \).

\[
P = \frac{\beta^2 v_0 I_0^2}{(\beta - 1)x^2}
\]

where \( I_0 \) is the initial impulse; \( \beta = v_0/v_0^{\text{solid}} \) (initial volume of porous material/initial volume of solid material) and \( x \) is the propagation distance.

However, the snow-plow model is based on the assumption of a constant impulse and therefore underestimates the attenuation of shock wave. Hence, a revised \( P-x \) relationship starting from momentum conservation is proposed in this paper:

\[
dm_1 u_{p1} = dm_2 u_{p2}
\]

where \( m_1, u_{p1} \) and \( m_2, u_{p2} \) are concrete mass and particle velocity at two distances \( x_1 \) and \( x_2 \), respectively.

Since the dispersion shape of the compression wave in the concrete below the conical area of Fig. 3 is spherical in shape, Eq. (16a) is changed to:

\[
(\rho_1 x_1^2dr)u_{p1} = (\rho_2 x_2^2dr)u_{p2}
\]

Substituting mass conservation Eq. (7a) into Eq. (16b) to eliminate \( \rho_1 \) and \( \rho_2 \), the relation (Eq. (16c)) is obtained:
\[
\frac{\rho_0 u_{s1} u_{p1}}{u_{s1}-u_{p1}} x_1^2 = \frac{\rho_0 u_{s2} u_{p2}}{u_{s2}-u_{p2}} x_2^2
\] (16c)

Substituting momentum conservation Eq. (9) leads to:

\[
\frac{P_1}{u_{s1}-u_{p1}} x_1^2 = \frac{P_2}{u_{s2}-u_{p2}} x_2^2
\] (16d)

Considering the EOS Eq. (8), which is linearized in the domain of shock compaction, Eq. (16d) is changed to:

\[
\frac{P_1}{P_2} = \frac{x_2^2[\gamma_B+(S-1)u_{p1}]}{x_1^2[\gamma_B+(S-1)u_{p2}]}
\] (16e)

Combining Eqs. (8) and (9), a \(P-u_p\) relationship is derived:

\[
P = \rho_0 (\gamma_B + Su_p) u_p
\] (16f)

Considering Eq. (16f), \(u_p\) in Eq. (16e) is eliminated:

\[
\frac{P_1}{P_2} = \frac{x_2^2[\gamma_B+(S-1)\rho_0]}{x_1^2[\gamma_B+(S-1)\rho_0]}
\] (16g)

Simplifying Eq. (16g) and then calibrating different exponents for the desired power function (the calibration is shown in Appendix Fig. 20), a best approximation for \(P-x\) relationship is derived:

\[
P \propto \frac{1}{x^{2.5}}
\] (16h)

In the snow-plow model, \(P\) is inversely proportional to \(x^2\) based on a constant impulse assumption and the attenuation effect is underestimated. In the present work, a new \(P-x\) relationship (Eq. (16h)) is adopted to replace the snow-plow model.
After deriving the attenuation, peak pressures at different depths can be determined following the calculation steps in Fig. 13 along the centerline in the concrete plate under the charge.

### 2.4.3 Verification of the analytical pressure decay by FEM simulations

To verify the accuracy of the analytical model, a series of FEM simulation models are conducted using the hydrocode simulations (commercial software ANSYS-Autodyn [25]). The models are axisymmetric along the center line of the TNT charge and the concrete plate. The explosive is modeled by an ALE\textsuperscript{1} mesh with the JWL EOS parameters of Table 1. The concrete plate is modeled by a Lagrangian\textsuperscript{2} mesh and the concrete constitutive model (“RHT”) and parameters are adopted from [16] for standard strength concrete ($f_c=35$ MPa). Fig. 14 shows a section of the numerical model with TNT (blue) and concrete plate (green).

---

\textsuperscript{1} ALE = arbitrary Lagrangian-Eulerian: the mesh following partly the deformation of the explosive and partly re-aligning the mesh by convection

\textsuperscript{2} body-fixed coordinates
Fig. 14. Axisymmetric FE model in the hydrocode ANSYS-Autody [11] with explosive (blue) and concrete (green).

The pressure-distance plots on the left side of Fig. 15 provide a comparison between the proposed analytical model and the curves from numerical simulation of the peak pressures in the concrete along the distance from the charge at their respective arrival times. The right parts show cross sections of the corresponding simulation models. Charge weights vary from 50 g to 1600 g and the thickness of concrete plates changes from 80 mm to 400 mm. In each model, an array of gauges (blue points) is placed along the center line of the concrete plate with a 5 mm spacing to record the peak pressure at different distances. The pressure readings are compared with analytical prediction for verification.

Fig. 15. (a) Pressure comparison (TNT=50g, thickness of plate=80mm).
The consistent results demonstrate that the peak pressure in the concrete plate from the analytical model is in very good agreement with the numerical model. The next step is to describe the cratering damage resulting from the contact detonation.

2.5 Compressive crater damage at the impacted surface

The pressure in concrete induced by contact detonation reaches 10-20 GPa, exceeding by orders of magnitude the dynamic material strength of concrete and creating crater. Van Amelsfort and Weerheijm recommended from experimental observation a the concrete crushing strength in the range of 400 MPa to 1000 MPa [15]. The analytical model follows their findings and assumes that the concrete crushing strength changes with the static uniaxial compression strength, as Eq. (17).

\[
\sigma_{\text{crushed}} = \frac{f_c}{27.5} (400 \sim 1000)
\]  

where \(\sigma_{\text{crushed}}\) is the concrete crushing strength and \(f_c\) is the static compressive strength.
of concrete.

In the analytical model, the depth of the crater is equal to the propagation distance of shock wave where the peak pressure decays below the concrete crushing strength.

![Diagram of crater formation at front surface.](image)

Fig. 16. Diagram of crater formation at front surface.

\[
C_{\text{radius}} = \sqrt{(R + C_{\text{depth}})^2 - R^2}
\]  

(18a)

\[
C_{\text{diameter}} = 2 \times C_{\text{radius}}
\]  

(18b)

where \(C_{\text{radius}}\) is the radius of crater; \(C_{\text{depth}}\) is the depth of crater; \(R\) is the radius of curvature of shock front in the charge shown in Fig. 6b, \(C_{\text{diameter}}\) is the diameter of crater.

To predict the crater diameter, one further assumption is necessary to define the crater size. The dispersion shape of the shock wave in the concrete plate is spherical. The initial radius of shock wave in the concrete plate is assumed equal to the radius of the curved detonation wave in the explosive (Fig. 6b). With the propagation of shock wave, the spherical surface expands and crater damage is still consistent with the enlarged dispersion shape (Fig. 16). Based on the geometry, the radius of the crater can be determined by using Eq. (18a) after the depth of the crater is calculated.

3. Validation of the analytical model

3.1 Predicted crater dimensions compared to experimental data

The theory and assumptions of the analytical model have been developed in sections 2.1 to 2.5. For the validation, several test results are collected from references and compared with the predictions by the analytical model. However, the test configurations in some cases differ slightly from the assumptions in the analytical model. In references [9], [14], [26], [27] and [28], the specimens for the contact
explosion tests are reinforced rather than plain concrete plates. However, for compressive crater damage, the reinforcing bars contribute little to prevent concrete crushing and crater formation. The explosive used in reference [7] is C-4 and in reference [13] is PETN1.5. These two explosives are converted to equivalent TNT based on the ratio of thermal energy by factors 1.25 and 1.13, respectively, according to references [7] and [13]. The length to diameter ratio of the charges in references [14], [27] and [13] are 3:1, 1:2 and 1:1.4. In order to consider the geometry effect, the duration of the impulse in the charge (τE) is calculated as τE = min(L, d)/D, which corresponds to using the smaller value of the charge length or diameter to calculate the duration. After accounting for these discrepancies, the crater depth and diameter observed from tests and the analytical model are summarized in Table 2. By using Eqs. (17) and (18) the predicted crater damage size is within the range determined by the upper and lower bounds of concrete crushing strength. Thus, for every crater depth and diameter, there are two bound values in Table 2.

Table 2. Crater damage dimension comparison between analytical model (Eqs. (18a) and (18b)) and test results.

<table>
<thead>
<tr>
<th>Blast test</th>
<th>Equivalent TNT mass (g)</th>
<th>Crater depth (mm)</th>
<th>Crater diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test result</td>
<td>Analytical result</td>
<td>Test result</td>
</tr>
<tr>
<td>XJ, Li et al. [26]</td>
<td>200</td>
<td>66</td>
<td>45 ~ 55</td>
</tr>
<tr>
<td>XJ, Li et al. [26]</td>
<td>1600</td>
<td>90</td>
<td>70 ~ 100</td>
</tr>
<tr>
<td>McVay [14]</td>
<td>700</td>
<td>62</td>
<td>50 ~ 75</td>
</tr>
<tr>
<td>J, Li et al. [9]</td>
<td>1000</td>
<td>No data</td>
<td>60 ~ 85</td>
</tr>
<tr>
<td>Beppu et al. [7]</td>
<td>57</td>
<td>27.1</td>
<td>25 ~ 35</td>
</tr>
<tr>
<td>W, Fang et al. [28]</td>
<td>1000</td>
<td>No data</td>
<td>65 ~ 95</td>
</tr>
<tr>
<td>W, Fang et al. [28]</td>
<td>2000</td>
<td>No data</td>
<td>85 ~ 120</td>
</tr>
<tr>
<td>J, Li et al. [27]</td>
<td>100</td>
<td>No data</td>
<td>35 ~ 50</td>
</tr>
<tr>
<td>J, Li et al. [27]</td>
<td>1000</td>
<td>No data</td>
<td>40 ~ 60</td>
</tr>
<tr>
<td>Gebbeken et al. [13]</td>
<td>960</td>
<td>80</td>
<td>70 ~ 95</td>
</tr>
</tbody>
</table>
Fig. 17(a) and 17(b) compare graphically the experimental and analytical results for crater depth and diameter. Empty parts in Fig. 17(a) indicate missing records in the references. For crater depth, all the test results except the first are within the range of analytical predictions. A deviation is observed in a 200 g TNT blast test [26]. However, the depth of crater in the test (66 mm) is close to the analytically predicted upper limit value of 55 mm. Concerning crater diameter, Fig. 17(b) shows six test results in the range of analytical predictions. The percentage of error for all tests is less than 16%.
Hence, the predictions for both the crater depth and the crater diameter are mostly in good agreement with the test results.

### 3.2 Scalability of the proposed model

As mentioned in Section 1, dimensional analysis, replica scaling and the scaled distance $Z$ are commonly accepted and have been verified by extensive tests [1]. For contact detonation on the concrete plate, Forsen [11] and Schaufgelberer et al. [12] described a linear relationship between crater depth $C_{\text{depth}}$ normalized by the plate thickness $T$ and cube root of charge weight $W$ over the plate thickness $T$.

![Analytical data point](image1)

**Fig. 18.** (a) Scale analysis for crater depth predicted by analytical model.

![Analytical data point](image2)

**Fig. 18.** (b) Scale analysis for crater diameter predicted by analytical model.
Hence, the analytical model has been validated against the scaling laws as follows. The corresponding dimensionless relationship between crater depth $C_{\text{depth}}$ and crater diameter $C_{\text{diameter}}$ and cube root of the charge weight $W^{1/3}$, both normalized by the plate thickness $T$, are shown in Fig.18(a) and 18(b). In the analytical results, a linear relationship similar to Forsen’s findings [11] is found, with only small deviations, which would expect to be in the range of experimental scatter.

In summary, the analytical model thus gives a sound, physics-based description of the shock waves both in the TNT charge and the concrete plate, with impact crater predictions in the latter. Hence, the analytical model can be applied in different conditions with various charge weights or plate thicknesses.

4. Summary and conclusions

The formation of crater at the impacted face is a common damage mode for concrete plates under contact detonation. So far, empirically based design charts [1] and engineering tools [13] as well as finite element methods are used for engineering purposes. In this paper, a new analytical model has been developed step-by-step, which has the potential to give fast predictions. The proposed approach is based on the physics of the interaction of the detonation wave and the concrete show response. The most important assumptions of the model are:

- Description of the detonation referring to $P_{\text{CJ}}$ and $D$ as commonly available explosive material parameter;
- Simplified modeling for cylindrical charges with $L/d = 1$ and for detonation front in the explosive;
- Impedance matching and insight from predictive hydrocode models to quantify the shock transmission from the explosive into the concrete;
- Propagation and attenuation due to release wave run-up and porous dissipation, using a linearized equation of state for the governing pressure region;
- Correlation of peak pressures and compressive damage based on experimental observation by [15].

The proposed analytical model is verified and validated in the following against some published available experimental data:

- Very good predictions on the peak pressure along the center line of concrete plates when compared to predictive hydrocode simulations;
- Analytical crater depth and diameter in agreement with test results;
Good agreement with established replica scaling rules for predicted crater damage sizes.

In this paper, the investigation focuses on crater damage on the impacted face of the concrete plate. Another characteristic damage mode under contact detonation is spalling damage on the protected side of the plate. It occurs when the reflected pressure wave and superposition with other release wave portions exceed the dynamic tensile strength of concrete. In the future, the analytical model will be extended to include both impacted face crater and rear face spalling so that it can be used for analytical prediction for perforation.

Acknowledgments

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Appendix

Fig. 19. (a) Convergence analysis for numerical models with different mesh size.
Fig. 19. (b) Convergence analysis for analytical models with different mesh size.

Fig. 20. The calibration of $P$-$x$ relationship.

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