

# Marginal blind deconvolution of adaptive-optics corrected images of satellites

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**Abstract:** Conventional blind deconvolution estimates jointly the PSF and the object being imaged, and therefore it is subject to degeneracies. Instead, we use a marginalized approach which has good statistical properties. We show results on AO-corrected observations of satellites. The method will be extended to anisoplanatic imaging.

**OCIS codes:** (100.1455) Blind deconvolution; (100.2000) Digital image processing; (010.1080) Active or adaptive optics; (010.1330) Atmospheric turbulence.

## 1. Introduction

The performance of many optical systems is highly degraded by atmospheric turbulence. This problem can be partially alleviated using adaptive optics (AO). Additionally, the partial correction problem can be tackled using post-processing techniques such as deconvolution.

A possible approach consists of representing the AO point spread function (PSF) through an appropriate basis. Then, the PSF is expressed as a weighted sum of functions, possibly with a sparsity constraint, i.e., with only a small number of non-zero coefficients, thus reducing the number of effective unknowns in the problem. In this work, a marginal blind restoration method (AMIRAL) [1] is used to compute such parameters in order to avoid the degeneracy of the solution encountered in the traditional joint blind deconvolution.

The final goal of this research is to propose solutions for the more general case of a PSF that varies in the field of view, e.g. for wide-field observations with single-conjugate AO. Both the use of a marginal approach and the PSF parameterization will help make the method robust, which is all the more necessary as we extend the method to the anisoplanatic case.

## 2. Method

For tests, two subsets of four AO long-exposure PSFs were taken from a large database of simulated PSFs which were simulated analytically using the PAOLA package [2]. The two simulations from which the subsets were taken differed only in the vertical distribution of turbulence above the telescope. One profile (“atmosphere 1”) had strong ground-level turbulence. The contrary was true for the second profile (“atmosphere 2”). The simulations were carried out for the 3.5-m SOR telescope located in New Mexico. A common value of 1.2” for seeing and 1  $\mu\text{m}$  wavelength was adopted. More information about these and other parameters used in the simulations, as well as the atmospheric conditions assumed, can be found in [3]. Figure 1 shows all 8 PSFs. As test object we used the image of the Hubble Space Telescope (HST). The object was convolved with the PSFs and white Gaussian noise was subsequently added to the blurred image.

The marginal estimator of [1] was applied to identify the correct PSF that creates an image, from a given set of possible PSFs. The principle of marginal estimator is to integrate the object out of the problem by integrating the joint probability of the object and the PSF parameters over all possible values of the object; this reduces the number of unknowns since only the PSF parameters are estimated. Hence, the criterion to minimize reduces to:

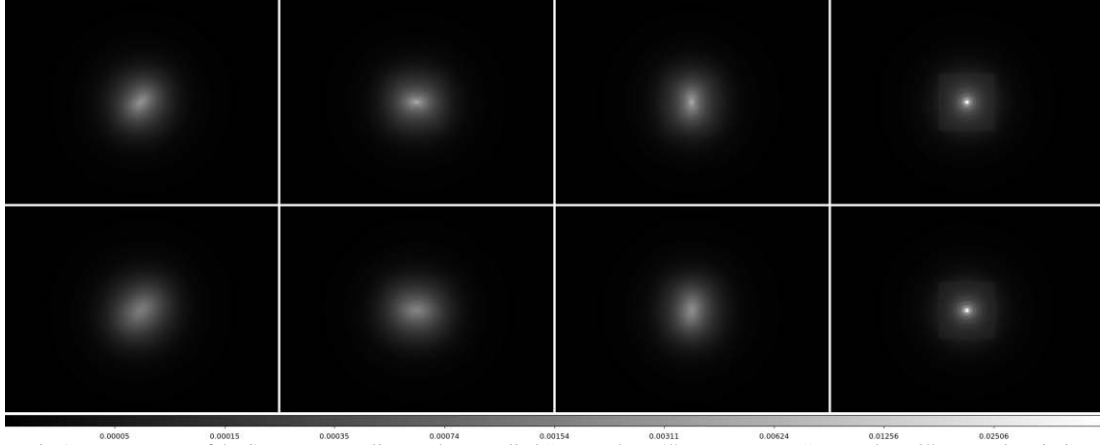


Fig 1. Top row, set of 4 PSFs corresponding to the so-called “atmosphere 1”. Bottom row, “atmosphere 2”. For a description of the simulations we refer the reader to [3]. Images are shown in log scale.

$$J_{ML}(\alpha) = \frac{1}{2} \sum_{\nu} \ln S_o(\nu) + \frac{1}{2} \sum_{\nu} \ln \left( |\tilde{h}(\nu)|^2 + \frac{S_n}{S_o(\nu)} \right) + \frac{1}{2} \sum_{\nu} \frac{1}{S_o(\nu)} \frac{|\tilde{i}(\nu) - \tilde{h}(\nu) \tilde{\sigma}_m(\nu)|^2}{|\tilde{h}(\nu)|^2 + \frac{S_n}{S_o(\nu)}} \quad (1)$$

where  $\tilde{i}(\nu)$  and  $\tilde{h}(\nu)$  are the Fourier transform of the image and the PSF, respectively, the object is assumed to be described by a stationary Gaussian prior probability distribution with mean value  $\tilde{\sigma}_m(\nu)$ ,  $S_o(\nu)$  and  $S_n$  are the power spectral densities (PSD) for the object and the noise, respectively, and the Greek letter  $\nu$  is the frequency index. Note that in equation (1) the object  $\tilde{\sigma}(\nu)$  is not explicitly included; once the PSF parameters have been estimated the final object can be reconstructed using any non-blind deconvolution algorithm, e.g., a Wiener filter.

Parametric models for both the PSF and the object PSD are assumed. The PSF is modeled by a weighted sum of modes, i.e.,  $h(x) \approx \sum_j \alpha_j h_j(x)$ , where  $x$  is the spatial index,  $\alpha_j$  are the parameters to be estimated, and  $h_j(x)$  the modes. By modes we mean any set of functions that form a basis where to project the PSF solution. In this work a simple set of simulated PSFs depicted in figure 1 has been used. However, in more complicated scenarios such as anisoplanatic observations, a different basis might be more convenient such as the principal component (PC) decomposition. For the PSD the following 3-parameter model is used [4]:  $S_o(\nu) = \frac{k}{1+(\frac{\nu}{\nu_0})^p}$ . Since the noise is assumed to be Gaussian and

homogeneous,  $S_n = cte$ , hence the total set of parameters to be estimated is  $\{\alpha_j, \nu_0, p, S_n/k\}$ . There is no analytical expression for the solution of equation (1) so the minimization must be done numerically. In this work, a Variable Metric with Limited Memory and Bounded (VMLM-B) has been used [5]. Initial PSF parameters were assumed to be uniformly distributed at the beginning of the minimization.

### 3. Results

Table 1 and figure 2 show the results obtained with AMIRAL for PSFs depicted in figure 1. The marginal method is able to estimate the parameters of interest, here  $\alpha_j$ , corresponding to the correct PSF in 3 out of 4 cases. It currently has problems with the non-elongated one (PSF 4; rightmost panels in figure 1). We believe this is related with the current absence of prior information about PSFs in the method and the fact that we are currently not imposing positivity on the object. Figure 3 shows PSDs for the true object, the solution and the image.

Table 1. Results of AMIRAL for simulated AO PSFs. The highest values of the synthesis coefficients  $\alpha_j$  are highlighted in red.

		ATMOSPHERE 1				ATMOSPHERE 2						ATMOSPHERE 1				ATMOSPHERE 2			
PSF 1	True	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	PSF 3	True	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0
	Estimated	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0		Estimated	0.2	0.0	0.8	0.0	0.4	0.0	0.6	0.0
PSF 2	True	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	PSF 4	True	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0
	Estimated	0.1	0.9	0.0	0.0	0.1	0.9	0.0	0.0		Estimated	0.4	0.5	0.0	0.1	0.4	0.4	0.0	0.2

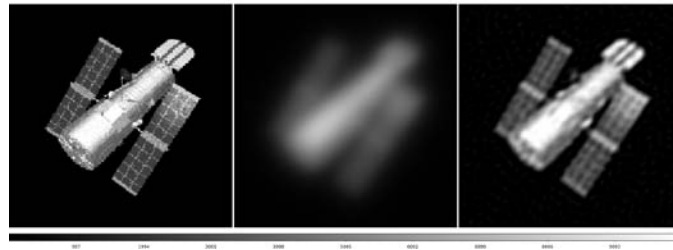


Fig 2. Leftmost panel, ground truth image. Middle panel, blurred image with PSF 1 from atmosphere 2. Rightmost panel, deconvolved image using the marginal method. All panels are shown in linear scale from 0 to  $10^4$ .

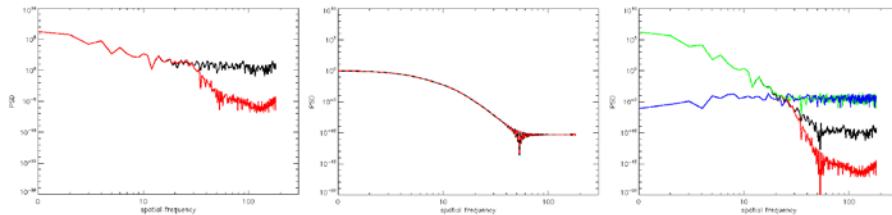


Fig 3. Leftmost panel, PSDs for the true object (black) and deconvolved image (red). Middle panel, squared modulus of the true Optical Transfer Function (OTF, black) and of the estimated one (red). Rightmost panel, PSD of the true object multiplied by the squared modulus of the true OTF (black), PSD of the deconvolved image multiplied by the squared modulus of the estimated OTF (red), PSD of the image (green), and PSD of noise (blue).

#### 4. Conclusions and future directions

It has been shown that AMIRAL is a very promising tool for myopic deconvolution of AO images in the isoplanatic case. However, in order to make it more robust, the criterion to be minimized must be modified to include additional prior information about the PSFs and/or about the object, such as positivity. Our final goal is to extend this method to anisoplanatic conditions, for which several ideas will be incorporated into the current approach. Firstly, in order to reduce the number of parameters, the basis of modes will be based on decompositions in PCs of simulated PSFs [3]. Such a decomposition guarantees that, for a certain number of modes, the reconstruction error will be minimal in comparison with any other possible basis. Secondly, the sparsity concept can be incorporated on the PSF decomposition on a dictionary by use of the  $l_1$  norm, or by group lasso. This should allow us to find the correct atmospheric conditions among those described by two or more dictionaries of PSFs. Finally, optimization procedures which combine the augmented Lagrangian of the problem with proximal mapping operators [6,7] will be used to minimize the criterion with the aforementioned priors, while speeding up the convergence. All these ingredients can be applied to different regions in wide-field AO observations affected by anisoplanatism to extract the PSF at different locations in the field of view. The obtained set of PSFs can then be used to perform a final image restoration using a fast approximation of the shift-variant blur [8].

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