High quality digital holographic reconstruction on analog film
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ABSTRACT
High quality real-time digital holographic reconstruction, i.e. at 30 Hz frame rates, has been at the forefront of research and has been hailed as the holy grail of display systems. While these efforts have produced a fascinating array of computer algorithms and technology, many applications of reconstructing high quality digital holograms do not require such high frame rates. In fact, applications such as 3D holographic lithography even require a stationary mask. Typical devices used for digital hologram reconstruction are based on spatial-light-modulator technology and this technology is great for reconstructing arbitrary holograms on the fly; however, it lacks the high spatial resolution achievable by its analog counterpart, holographic film. Analog holographic film is therefore the method of choice for reconstructing high-quality static holograms. The challenge lies in taking a static, high-quality digitally calculated hologram and effectively writing it to holographic film. We have developed a theoretical system based on a tunable phase plate, an intensity adjustable high-coherence laser and a slip-stick based piezo rotation stage to effectively produce a digitally calculated hologram on analog film. The configuration reproduces the individual components, both the amplitude and phase, of the hologram in the Fourier domain. These Fourier components are then individually written on the holographic film after interfering with a reference beam. The system is analogous to writing angularly multiplexed plane waves with individual component phase control.

Keywords: holography, digital holography, hologram, 3D lithography

1. INTRODUCTION
A standard analog holographic film with an average grain size of 20 nm contains up to the equivalent of 2.5 Terapixels of information and costs about 5 U.S. dollars. There are a number of applications in the scientific and industrial communities that would benefit from the production of high-quality digitally calculated holograms. With such a hologram, one could completely remove the imaging optics from a lithography machine and instead of waiting a month for lithography mask to be manufactured and paying ~1 U.S. million for its production, have a system that takes under 1 day and just dollars implement. An example of such a hologram is shown in Fig. 1. But besides the cost and time savings, having access to a holographic lithography mask would push the semiconductor industry in new directions allowing for single-shot 3D patterning of structures and components that would normally be impossible under current exposure techniques. The contents of this paper will focus on generating one-dimensionally calculated holograms on real holographic film. However, these techniques can directly carry over to 2D patterns with little more than additional hardware.

The technique described in this paper was inspired by the methods used in numerically reconstructing digitally acquired holograms and relies on pseudo-holographic reconstruction in Fourier space by building the hologram up from each individual Fourier component. This technique is a subset of angular hologram multiplexing [1]. There are several other methods that utilize quantizing the spatial dimensions of the hologram and writing patterns over each section by means of a spatial light modulator (SLM) [2-4], however these methods still have trouble accessing the entire potential of the hologram because of the limited resolution of the SLM.

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1.1 Theory

The hologram as it is classically known, is constructed by interfering a reference wave of a known structure that shares coherence with an object wave that has been structured by reflecting or scattering off of an object. The object wave propagates until it reaches the plane where the reference wave overlaps with it on either a photographic plate, or a digital camera. The information contained within this interference pattern contains all of the information of the original object wave, including the intensity, phase and polarization component projected onto the reference beam’s state of polarization. The mathematical representation of such a process is elegant in its simplicity:

\[ I = |R + O|^2 = |R|^2 + |O|^2 + R^* O + RO^* \]  

where \( I \) represents the intensity of the interference pattern at the holographic plate or camera, \( O \) is the fully complex form of the object wave and \( R \) is the fully complex form of the reference wave. The graphical representation is shown in Fig. 1(a). What makes this interference pattern exceptional is that when it is illuminated (either in real life on film, or in the digitally acquired world numerically) by the same reference used to record the hologram, the outgoing wavefront contains the complete reconstruction of the original object wave at the plane as such

\[ IR = (|R|^2 + |O|^2)R + |R|^2 O + R^2 O^* \]

It can be seen in Eq. 2 that there are also additional terms in the reconstructed wavefront that are related to, but separable from the recreated object wave. The first term gives rise to an unmodulated background, or DC term which travels in the direction of the incident reconstruction wave. The second term is the hologram term where its intensity is proportional to the intensity of the reference wave, and the last term is known as a virtual hologram which is angularly separated from the hologram. As is well known in digital holographic reconstruction techniques, the Fourier transform of the hologram function, \( I \), gives the wavevector space representation of the hologram and since the hologram is recorded using a single wavelength, the Fourier components are also a one-to-one mapping to the angular direction the light is traveling. With this well-known technique, one can digitally filter the DC term and virtual hologram from the acquired hologram and digitally extract the original object wavefront.

Figure 1. Demonstration of how holographic lithography techniques could allow writing of structures on silicon that are in different focal planes.
This angular mapping is at the heart of the routine used to reconstruct digitally calculated holograms on analog film. The pattern developed on the analog holographic film is given by Eq. 1 with the object wave represented in the Fourier domain by

\[ I = \left| R + \sum_k O_k e^{-i\vec{k} \cdot \vec{x}} \right|^2. \]  

(3)

The reference wave first interferes with all of the individual angular components of the object wave before the time averaged intensity is produced. In the recreation method described here however, the individual angular components of the object wave were interfered with the reference, and those individual intensities were summed as

\[ I = \sum_k \left| R + O_k e^{-i\vec{k} \cdot \vec{x}} \right|^2. \]  

(4)

The end result of the intensity recorded by the hologram then has the form

\[ I = \sum_k |R|^2 + \sum_k |O_k|^2 + R^* O + RO^*. \]  

(5)

Note that this formula only comes about by assuming the reference wave is a plane wave. The two terms at the end of Eq. 5 also show up in the classical hologram as calculated in Eq. 1. These terms are the terms responsible for generating the hologram and its phase conjugate so the method of angularly multiplexing the individual Fourier components of the hologram will return the actual object wavefront.

The phase is an important part of the reconstruction scheme and it is worth describing that controlling the actual phase of the beam writing the term \( O_k \) on the hologram is extremely sensitive to vibrations if a steerable mirror system or translating the source of the beam is to be used. Therefore, keeping the object and reference beam fixed while rotating the hologram will provide a more stable result and there is a one-to-one mapping of the patterns written using both methods. The schematic of the principle can be seen in Fig. 3.
Instead of moving directly to writing theses holograms on analog film, an understanding of the noise present in the process due to each written Fourier component will be necessary to measure. As such, a CCD will first replace the holographic film as a recording medium and the hologram will be numerically created after adding point by point. This is a necessary first step. Figure 4 gives the schematic representation of how such a scheme will be implemented. The laser is first expanded and split into two paths by a beam splitter. One path will then be phase modulated by a digitally controlled tunable phase plate. Both paths will interfere at an angle giving rise to an interference pattern on the CCD. The CCD can then be rotated on top of a rotation stage to recreate the different Fourier components on the camera. These images will then be captured and numerically simulate the final hologram to investigate the optimal parameters sets such as the intensity ration between the object and reference path, the integration time and number of Fourier components and the speckle due to any other aberrations introduced in the system.

Figure 4. Trial experimental setup to be implemented to understand the sources of error due to each component of the hologram reconstruction method. The CCD is rotated on a rotation stage to produce the different components of the Fourier transform of the hologram.
2. SIMULATIONS

The general working principle of the use of the above described system is to first have a user input a desired intensity pattern that they would like to reproduce from a plane wave (for an example see Fig. 5(a)). This intensity can then be numerically converted into an appropriate electric field and propagated to the position where the hologram will be calculated as such

\[ \bar{E}(x, y, 0) \rightarrow \bar{E}(x, y, L) = \mathcal{F}\mathcal{F}^{-1} \{ \mathcal{F}\mathcal{F} \{ \bar{E}(x, y, 0) \} e^{-i k z L} \}, \tag{6} \]

where \( \bar{E}(x, y, 0) \) is the initial electric field, \( \bar{E}(x, y, L) \) is the object wave electric field at the hologram plane, \( k_z \) is the \( z \)-component of the wavevector for each Fourier component and \( L \) is the distance in the \( z \)-direction from the initial electric field plane to the hologram plane. A lens phase was added to the object wave,

\[ \bar{E}' = \bar{E} e^{i k x^2 / 4 f}, \tag{7} \]

before it was converted to a hologram, with \( k \) being the magnitude of the wavevector, \( x \) being the coordinate along the hologram and \( f \) being the focal length of the lens chosen in a \( 2f \)-\( 2f \) one-to-one imaging configuration such that \( L = 2f \). A scaled example of a hologram produced from the intensity distribution in Fig. 5(a) is shown in Fig. 5(b). A question remains if the modulation intensity is enough to reproduce the desired intensity of the input. This small modulation intensity comes about from the first two terms in Eq. 5 being much larger than in the classically reproduced hologram. Finally, in Fig. 5(b), the reproduced intensity can be seen.

![Figure 5. (a) Random distribution of Gaussian functions on the microns scale representing the desired intensity to be reproduced from a plane wave. (b) Normalized hologram digitally calculated using the method proposed in this paper. (c) Simulated reconstructed intensity (blue curve) from the hologram calculated in (b). The red curve is the same desired intensity as in (a) scaled and with an offset.](image-url)
The desired intensity is imposed on top of a small background and has a carrier frequency both of which can be attributed to the first two terms in Eq. 5. Despite these discrepancies however, the proposed method for writing a hologram is seen to be effective.

3. CONCLUSIONS

A system to write digitally calculated holograms onto a holographic film was numerically investigated. The proof of concept simulations show that this concept could produce interesting results but must first be carefully implemented. There are a few remaining questions about how well the fidelity of the experiment could reproduce the small signal intensity on top of a large background and how many noise factors influence the final results as the simulations show that the resulting hologram will suffer greatly from noise. Experiments on the system will be carried out in the near future with the prescribed experimental system.

REFERENCES