Fraunhofer
ITWM

M. Burger, K. Dreßler, A. Marquardt, M. Speckert

Calculating invariant loads for system simulation in vehicle engineering

Berichte des Fraunhofer ITWM, Nr. 161 (2009)

ISSN 1434-9973

Bericht 161 (2009)

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und
Wirtschaftsmathematik ITWM
Fraunhofer-Platz 1

67663 Kaiserslautern
Germany

Telefon: 06 31/3 16 00-0
Telefax: 06 31/3 16 00-10 99
E-Mail: info@itwm.fraunhofer.de
Internet: www.itwm.fraunhofer.de

Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

MULTIBODY DYNAMICS 2009, ECCOMAS Thematic Conference
K. Arczewski, J. Frączek, M. Wojtyra (eds.)
Warsaw, Poland, 29 June–2 July 2009

CALCULATING INVARIANT LOADS FOR SYSTEM SIMULATION IN VEHICLE ENGINEERING

Michael Burger⋆**, Klaus Dreßler**⋆**, Albert Marquardt**⋆ **and Michael Speckert**⋆

⋆Department Mathematical Methods for Dynamics and Durability
Fraunhofer Institut für Techno- und Wirtschaftsmathematik
Fraunhofer Platz 1, 67663 Kaiserslautern, Germany
e-mail: michael.burger@itwm.fraunhofer.de,
klaus.dressler@itwm.fraunhofer.de,
albert.marquardt@itwm.fraunhofer.de,
michael.speckert@itwm.fraunhofer.de,
web page: http://www.itwm.fraunhofer.de/de/mdf/indexmdf/

Keywords: Iterative learning control, optimal control theory, differential algebraic equations (DAEs)

Abstract. *For the numerical simulation of a mechanical multibody system (MBS), dynamical loads are needed as input data, such as a road profile. With given input quantities, the equations of motion of the system can be integrated. Output quantities for further investigations are calculated from the integration results. In this paper, we consider the corresponding inverse problem: We assume, that a dynamical system and some reference output signals are given. The general task is to derive an input signal, such that the system simulation produces the desired reference output. We present the state-of-the-art method in industrial applications, the iterative learning control method (ILC) and give an application example from automotive industry. Then, we discuss three alternative methods based on optimal control theory for differential algebraic equations (DAEs) and give an overview of their general scheme.*

1 INTRODUCTION

Numerical system simulation plays an important role in vehicle engineering. Virtual prototyping of mechanical systems can accelerate the development process enormously and reduces costs.
In order to simulate the motion of a multibody vehicle model, dynamic loads are needed as input data. Such load data is called *invariant*, if it is independent of the specific system under consideration. A convenient example for invariant loads is a digital road profile used for driving simulation of a vehicle.
Typically, output quantities such as wheel forces, accelerations or relative displacements in the vehicle are measured. However, those quantities are not invariant but highly dependent on the specific vehicle variant, that was used for the measurement. The general task is now to derive and calculate invariant input loads such that they can be used to simulate other vehicle variants, which may only exist as computer models. Mathematically, this leads to a control problem, see section 2 for a general formulation.
In this paper we present some approaches for dealing with this problem. State-of-the art in industrial applications is the so called *iterative learning control (ILC)* method. We give a description of that approach and an application case from automotive industry. The iterative learning control is a pure black-box method, only the input/output behaviour of the considered system is needed. This makes it also applicable to situations, for which it is hard or even impossible to get an equation, which describes the system properly, such as servo-hydraulic test-rigs in the laboratory. See [4] for a detailed description and applications. But the method lacks of precise mathematical justification, i.e. , there are no general statements about important properties like accuracy, stability, and convergence. Of course, this can be seen as a consequence of the minimal system knowledge requirement.
In contrast to the case of a servo-hydraulic test rig in the laboratory, for virtual test rigs or more generally speaking for numerical system simulation on a computer, the system is well-known as multibody system model. Therefore, it seems natural to make use of this information: our aim is to develop mathematical methods as alternatives to the ILC-approach. The general assumption is an - at least structural knowledge - of the model equations.
The methods, we are currently working on and we want to present here, are based on the theory of *optimal-control for DAEs*. The first approach is known as trajectory prescribed path control in literature, see [6]. The second method is an approach, using the calculus of variations ([5]). Both methods augment the system equation and always lead to a differential algebraic equation (DAE), which has to be solved, even if the system equation is originally an ordinary differantial equation (ODE). For the numerical solution of a DAE, the (differentiation)- index is an important property, see section 2 for a definition. We give results about the index of the resulting DAE of the first two methods, see Lemmas 4.2 and 4.7.
The third alternative transfers the continous optimal control problem to a finite-dimensional optimization problem. In literature it is known as *multiple shooting method for optimal control of DAEs* ([11], [12]).
For each approach, we give a short overview of its general scheme, and we apply it to simple test problem, an N-mass-spring-damper-system. We show, that under some assumption, the optimal control problem for this system is solvable with the best possible result, see Theorem 4.6.

2 The optimal control problem for dynamical systems

We formulate the optimal control problem for a general dynamical system: the *state* of the system is represented by a vector $x(t) \in \mathbb{R}^{n_x}$, let further $u(t) \in \mathbb{R}^{n_u}$ denote some *input or control* quantities for the system. The dynamics of the system are described - most generally speaking - by a nonlinear *differential-algebraic equation* (DAE):

$$F(t, x(t), \dot{x}(t), u(t)) = 0, \quad t \in [0; T], \quad x(t = 0) = x_0, \quad \dot{x}(t = 0) = v_0, \tag{1}$$

where $F : \mathbb{R} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$ is sufficiently often differentiable and $\frac{\partial F}{\partial \dot{x}}$ is allowed to be idetically singular. If u is unknown, this equation is underdetermined.

Often, the dynamical system is given in the form of a semi-explicit DAE:

$$\begin{aligned} \dot{x}_d &= f_d(t, x_d, x_a, u) \\ 0 &= f_a(t, x_d, x_a, u) \end{aligned} \tag{2}$$

with differential variables x_d and algebraic variables x_a. The equations of motion of a (constrained) mechanical multibody system (our main interest) is a special case of such a semi-explicit DAE and has the general form:

$$\begin{aligned} \dot{q} &= v \\ M(q)\dot{v} &= f(t, q, v, u) - G^T \lambda \\ 0 &= g(q), \end{aligned} \tag{3}$$

with position coordinates q, velocities v, inputs u, and Lagrange-multipliers λ, i.e., $x = (q, v, \lambda)$, and $x_d = (q, v), x_a = \lambda$ respectively, $G(q) := \dfrac{\partial g}{\partial q}$.

We now assume, that the system outputs are given as a function of the state vector and possibly the input vector:

$$y(t) := g_{out}(x(t), u(t)) \tag{4}$$

where $g_{out} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \longrightarrow \mathbb{R}^{n_y}$. We further assume, that the desired *reference outputs*, typically gained by measurement, are given as functions of time: $y_{ref}(t) \in \mathbb{R}^{n_y}$.

This leads to the following optimal-control problem(OCP):
Minimize the *cost functional*

$$J[x, u] := \|y - y_{ref}\|^2_{L^2} = \int_0^T \left(g_{out}(x(t), u(t)) - y_{ref}(t)\right)^2 dt \tag{5}$$

w.r.t. to the *input/control* u, subject to Eq. (1), (2), (3) respectively. The L^2- norm could also be replaced by another suitable norm.

In general, all methods, which we describe here, require a numerical solution of a DAE, (1), (2), (3). Even, if the system equation is given as an ODE, two approaches lead to a DAE, cf. sec. 4.1 and 4.2. A well-known concept to classify DAEs is the *index*. In literature, various definitions can be found, following [10], we introduce the *differentiation-index* to be the minimal number k, such that

$$\begin{aligned} F(x(t), \dot{x}(t), u(t)) &= 0 \\ \frac{d}{dt} F(x(t), \dot{x}(t), u(t)) &= 0 \\ &\vdots \\ \frac{d^k}{dt^k} F(x(t), \dot{x}(t), u(t)) &= 0 \end{aligned} \tag{6}$$

can be transformed into an ODE only by algebraic transformations. It is a well-known fact, that *higher-index* DAEs, i.e., $k \geq 2$ are (numerically) hard to solve.

The system equation of a constrained multibody system, eq. (3), has differentiation index 3, provided

$$GM^{-1}G^T \tag{7}$$

exists and is invertible.

A last general assumption for the rest of the paper is, that all functions are sufficiently often differentiable.

3 Iterative Learning Control

In this section, we describe the state-of-the-art method for industrial applications, the iterative learning control method. The method is widely used to derive drive-signals for durability test rigs in vehicle industry, cf. [4]. We give an outline of the general procedure followed by an application case from automotive industry. As stated in the introduction, the ILC does not use any information about the system equation (1), only the system's input/output behaviour is needed.

3.1 The general ILC-procedure

The general ILC-procedure is divided into two steps: First, there is an identification process, in which the *frequency response function*, denoted by H, is estimated in the frequency domain. This is accomplished by a system excitation with white or pink noise as input. Secondly, one goes through a Newton-kind iteration process, in which the input is updated until the error is sufficiently small; with each input iterate u_i, the system has to be simulated in order to produce the corresponding system output, y_i, with whom in turn the input-update Δu_i is computed:

$$\begin{aligned} \Delta u_i &= H^{-1}(y_{ref} - y_i), \\ u_{i+1} &= u_i + \Delta u_i, \\ u_{i+1} &\stackrel{sim}{\longrightarrow} y_{i+1}, \qquad\qquad i = 0, 1, 2, ... \end{aligned} \tag{8}$$

figure (1) gives an schematic overview.

For the estimation process, it is assumed, that there is a *linear, time-discrete* relationship between input and output:

$$y(k\Delta t) = H(q)u(k\Delta t) \qquad k = 1, 2, \ldots \tag{9}$$

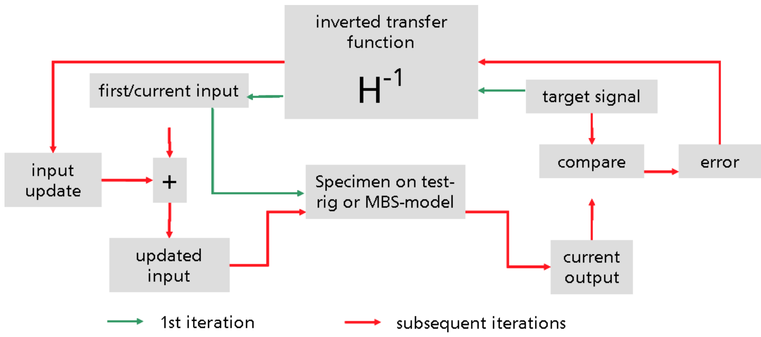

Figure 1: ILC procedure

where q is the so-called shift operator, i.e., $(q^{-1}u)(t) = u(t-1)$, and H the *transfer function*:

$$H(z) := \sum_{l=1}^{\infty} g(l) z^{-l}, \tag{10}$$

with the *impulse response* g.

Then, by well-known standard routines, the frequency response function, i.e. $H(e^{i\omega})$, is estimated, see the book of Ljung, ([2]), for a detailed description. In the following section, we present an application example from industry, where the ILC-method has been applied successfully.

3.2 A Daimler truck cabin with frame on a virtual test rig

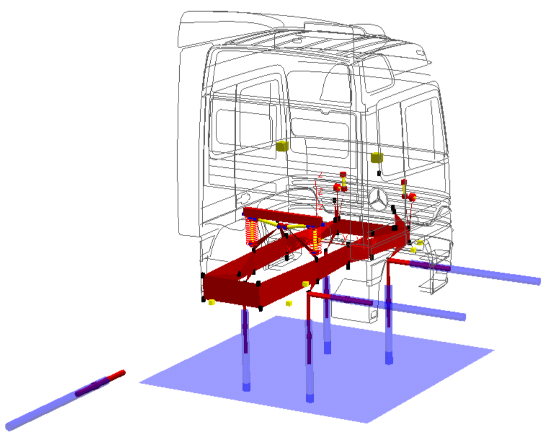

Figure 2: SIMPACK model

In a project with the Daimler AG, cf. [3], we have applied the ILC-method to a truck cabin with frame on a virtual test rig. The truck cabin, the frame and the virtual test rig have been modelled as an MBS-system in the software tool SIMPACK. The frame is mounted on the test rig, i.e., on four vertical cylinders. Additionally, there are two lateral and one longitudinal cylinders connected to the frame. Figure (2) shows the graphics of the model.

The input quantities u of that MBS model are the displacements of the seven test rig cylinders, i.e., $n_u = 7$, as outputs we have defined four spring length, the connections between cabin and frame, i.e., $n_y = 4$. Among others, for those spring lengths, there were reference outputs available gained by measurement. The SIMPACK model has been considered as pure black box model, which produces the corresponding spring-lengths as outputs. The ILC procedure has been performed via MATLAB-routines.

Fig. (3) shows both the measured reference output and the output, that has been generated with the calculated input, after four iteration steps.

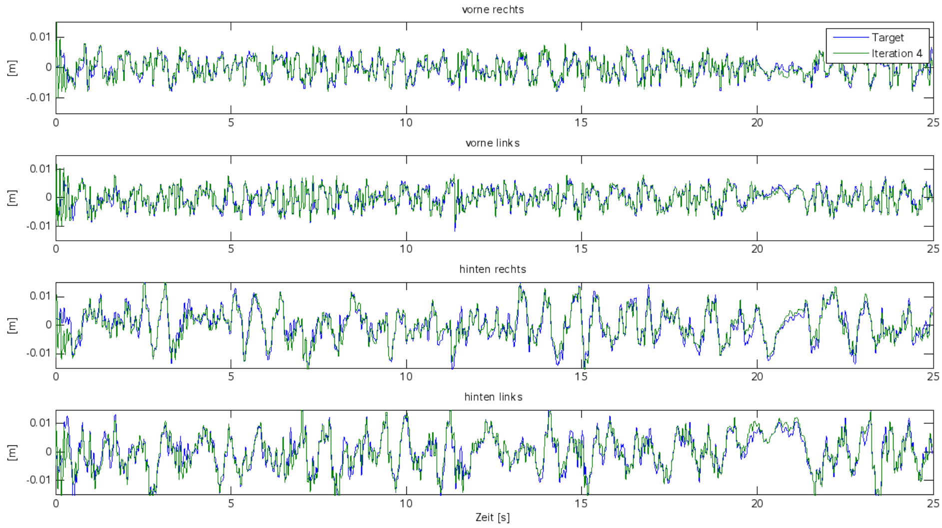

Figure 3: Reference and generated output signals

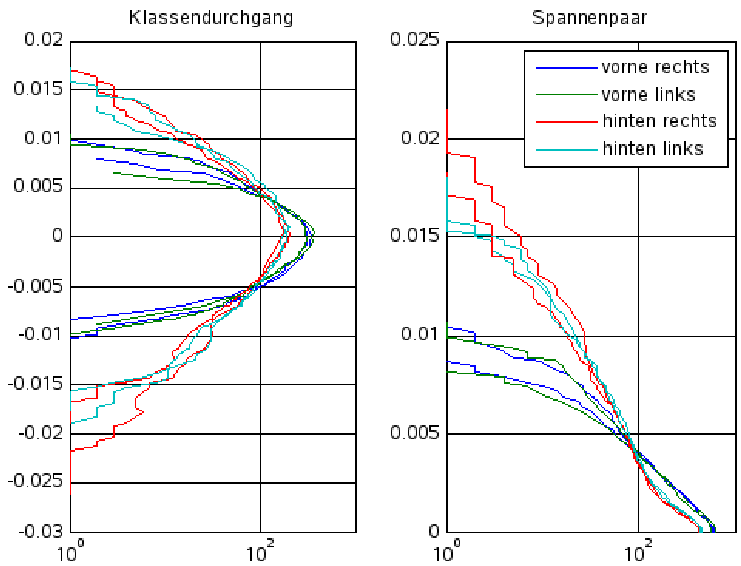

Figure 4: Left: Level crossing diagramm, right: range pair diagramm

Fig. (4) shows a level crossing diagramm to the left and a range pair diagramm to the right of the corresponding quantities. One can see, that with a relatively small number of iteration steps, the calculated ouput signals and the measured ones fit together very well.

4 Alternative Methods

In this section, we present three alternative approaches derived from the optimal control formulation given in section 2.

In order to test the alternative methods, we have choosen a simple mechanical system as benchmark problem: a linear $N-$mass-spring-damper system, $N-$MSD, where the first body is connected to ground only by a spring. Fig. (5) shows two masses.

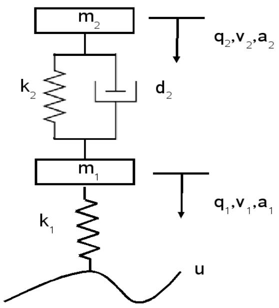

Figure 5: 2-MSD

The input of the system is the end-position of the lowest spring, $n_u = 1$, and the output is the motion of the highest mass, i.e., position, velocity or acceleration.
The equations of motion of that simple system are well-known and have the form:

$$\dot{x} = Ax + b \cdot u, \qquad x(0) = x_0, \tag{11}$$

where $x = (q_N, .., q_1, v_N, .., v_1)^T, b = (0, \dots, 0, 1)^T \in \mathbb{R}^{2N}$, $A \in \mathbb{R}^{2N \times 2N}$.
In this case, the system equation is still an ODE.

4.1 Trajectory prescribed path control methods

The first approach is to require the best achievable result, namely $J[x, u] = \|y - y_{ref}\|^2_{L^2} \equiv 0$, in case of existence, the corresponding u is a global minimizer. This requirement is equivalent to

$$y(t) - y_{ref}(t) = g_{out}(x(t), u(t)) - y_{ref}(t) = 0 \quad \forall t \in [0; T]. \tag{12}$$

Eq. (12) can either be used to solve for one or more components of the state vector x, which can be replaced in the model equation. Then, the latter can be solved for the remaining components *and* u by a DAE-integrator. This, however, requires complete knowledge of Eq. (1) (white-box-approach). Another possibility is to simply add Eq. (12) to the model-equation (1) as a further algebraic constraint-equation, a so called *state- or path constraint*. The resulting equation is a DAE (even, if the system equation was an ODE), which is not underdetermined anymore and again, it can be solved by a DAE-integrator. This approach is known as *trajectory prescribed path control* in literature, cf. [6].

The (differentiation-)index of the resulting DAE, however, can be very high, depending on "where" the input goes into the system and which state variable appears in the output g_{out}. To make this clear, we consider the general constrained mechanical system, eq. (3) with an invertible mass matrix. Moreover, we assume that we have one input and one output, $n_u = n_y = 1$. We denote the jth component of the force function multiplied by the inverse mass matrix by f_j:

$$f_j(t, q, v, u) := \left(M(q)^{-1} f(t, q, v, u)\right)_j \tag{13}$$

If there is an interacting force between body j and i, the corresponding force functions can be split up in

$$\begin{aligned} f_j &= f_{j,i}(q_j, q_i, v_j, v_i) + R_j \quad \text{and} \\ f_i &= f_{i,j}(q_j, q_i, v_j, v_i) + R_i = -f_{j,i}(q_j, q_i, v_j, v_i) + R_i, \end{aligned} \tag{14}$$

with $\frac{\partial R_j}{\partial q_i, v_i} = \frac{\partial R_i}{\partial q_j, v_j} = 0$

We make the following assumption:

Assumption 4.1.

- *The input acts only on body i, i.e., $\dfrac{\partial f_i}{\partial u} \neq 0$ and $\dfrac{\partial f_k}{\partial u} = 0 \quad \forall k \neq i$*
- *We want to prescribe the acceleration of body $j > i$, i.e., $g_{out} = g_{out}(\dot{v}_j)$.*
- *There is a connecting chain of bodies, that links bodies j and i. By this formulation, we mean: there is an interacting force between body j and $j-1$, between body $j-1$ and $j-2$,..,between body $i+1$ and i (possibly after renumeration of the bodies). Of course, the prototyping example is the $N-$MSD-system.*
- *For the interacting forces, $i \leq l \leq k \leq j$, at least one of the following conditions holds*

$$\frac{\partial f_{kl}}{\partial q_l} \neq 0 \tag{15}$$

$$\frac{\partial f_{kl}}{\partial v_l} \neq 0 \tag{16}$$

- *If equation (16) is true for all $i \leq l \leq k \leq j$, then the following product of Jacobians is invertible:*

$$\frac{\partial g_{out}}{\partial \dot{v}_j} \frac{\partial f_{j,j-1}}{\partial v_{j-1}} \frac{\partial f_{j-1,j-2}}{\partial v_{j-2}} \cdots \frac{\partial f_{i+1,i}}{\partial v_i} \frac{\partial f_i}{\partial u} \tag{17}$$

- *If (16) is not true for the force function $f_{k,l}$, then the Jacobian $\dfrac{\partial f_{k,l}}{\partial v_l}$ in Eq. (17) has to be replaced by $\dfrac{\partial f_{k,l}}{\partial q_l}$ and the resulting product is assumed to be invertible.*

Lemma 4.2. *Let Assumption 4.1 be fulfilled. Let N denote the number of bodies of the connecting chain, i.e., $N := j - i + 1$ and L the number of force elements between the bodies of the chain, for which eq. (16) is not true. If the system equation of the considered system has differentiation index $D \in \{0, 1, 2, 3\}$ for given input, then the DAE which results by adding the state constraint equation $0 = g_{out} - y_{ref}$ to eq. (3) has differentiation index*

$$\max\{D, N + L\} \tag{18}$$

Proof. Without loss of generality, we assume, that the original system equation is an ODE, i.e., $D = 0$. We consider the case $N = 3$ and set $i = 1, j = i + 2 = 3$. First we assume, that condition (16) is true for all interacting forces. For the output function we have $g_{out} = g_{out}(\dot{v}_3) = g_{out}(f_3 = f_{32} + R_3) = \tilde{g_{out}}(q_2, v_2)$. We differentiate the additional state constraint equation, $0 = g_{out} - y_{ref}$, a first time and get (in the following, we use R as a generic symbol sumarizing terms, which are not of interest):

$$\begin{aligned} 0 &= \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial v_2}\dot{v}_2 + \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial q_2}\dot{q}_2 + R = \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial v_2} f_2 + \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial q_2} v_2 + R \\ &= \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial v_2} f_{21} + \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial q_2} v_2 + R. \end{aligned} \tag{19}$$

A second differentiation yields

$$\begin{aligned} 0 &= \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial v_2}\frac{\partial f_{21}}{\partial v_1}\dot{v}_1 + \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial v_2}\frac{\partial f_{21}}{\partial q_1}\dot{q}_1 + R \\ &= \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial v_2}\frac{\partial f_{21}}{\partial v_1} f_1 + \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial v_2}\frac{\partial f_{21}}{\partial q_1} v_1 + R. \end{aligned} \tag{20}$$

And a third differentiation reveals $\dot{u}$:

$$0 = \frac{\partial g_{out}}{\partial \dot{v}_2}\frac{\partial f_{32}}{\partial v_2}\frac{\partial f_{21}}{\partial v_1}\frac{\partial f_1}{\partial u}\dot{u} + R. \tag{21}$$

By assumption, this equation can be resolved for $\dot{u}$, whence, the whole DAE has a differentiation index $3 = 3 + 0$.
Additionally, one can see, that for each missing damping term, i.e., $\frac{\partial f_{k,l}}{\partial v_l} = 0$, one more differentiation is needed.
A simple induction argument proves the Lemma. □

Remark 4.3. *If we want to prescribe only the velocity of body j, i.e., g_{out} is a function of v_j, the corresponding differentiation index naturally increases by one:* $\max\{D, N + L + 1\}$. *If g_{out} is a function only of the position q_j, the index increases by two:* $\max\{D, N + L + 2\}$

Remark 4.4. *The Lemma above states, that the differentiation index of the resulting DAE of the prescribed path trajectory approach is increasing linearly with the number of "involved" bodies.*

Remark 4.5. *Applied to our benchmark example, an $N-$MSD (which is of course the prototype example of a connecting chain between two bodies), and for $g_{out} = \dot{v}_N$, condition (17) means*

$$d_N \cdot d_{N-1} \cdot \ldots d_2 \neq 0 \tag{22}$$

meaning, that we have dampers between every two masses. The resulting DAE has Index N. If there are L dampers missing, each corresponding d_l in the equation above has to be replaced by the corresponding spring stiffness k_l, and the DAE has index $N + L$. Note, that an additional damper between the first body and ground will not affect the index.

An important property of our benchmark example is the fact, that, with the method of prescribed path trajectory, we can show, that the general task to find an input u, such that the highest mass moves as desired, i.e. follows a prescribed acceleration, velocity or position, has a solution. This solution can be written down explicitly, provided that the reference output, is smooth enough.

The crucial point is, that the system equation (11) is linear. We can transform the corresponding DAE, the system equation and the state constraint equation $g_{out}(q_N, v_N) - y_{ref}(t) = 0$, in the following standard form:

$$E\dot{\tilde{x}} = \tilde{A}\tilde{x} + f(t), \tag{23}$$

with $\tilde{x} = (x, u)^T \in \mathbb{R}^{2N+1}$, a singular matrix E and $f(t) = (0, \ldots, 0, -y_{ref}(t))^T \in \mathbb{R}^{2N+1}$. To guarantee solvability of that DAE, we still have to require the further condition, that the matrix pair $(E, \tilde{A})$ is regular, i.e., there is a $\lambda \in \mathbb{C}$ such that $det(\tilde{A} - \lambda E) \neq 0$.

Theorem 4.6. *If $(E, \tilde{A})$ is regular and the reference output f is smooth enough, then DAE (23) is solvable. The solution can be written down explicitly. The solution-algorithm, given in the proof, reveals directly the index of the DAE.*
As a consequence, the optimal-control problem, we consider for the $N-$MSD has a solution, that fulfills $J[x, u] \equiv 0$.

Proof. The pair $(E, \tilde{A})$ can be transformed to the Weierstraß-canonical form, i.e., there are regular matrices P, Q, such that

$$PEQ = \begin{pmatrix} \tilde{N} & \\ & \mathbb{1} \end{pmatrix} \qquad P\tilde{A}Q = \begin{pmatrix} \mathbb{1} & \\ & J \end{pmatrix} \tag{24}$$

with a nilpotent matrix $\tilde{N}$ and a matrix J in Jordan canonical form.

With the obvious coordinate transformation and a multiplication by P, Eq. (23) splits up into the two independent equations

$$\begin{aligned} \tilde{N}\dot{\overline{x}}_1 &= \overline{x}_1 + (Pf)_1 \\ \dot{\overline{x}}_2 &= J\overline{x}_2 + (Pf)_2 \end{aligned} \tag{25}$$

The solution is

$$\begin{aligned} \overline{x}_1(t) &= -\sum_{\nu=0}^{i} \tilde{N}^i (Pf)_1^{(i)}(t), \\ \overline{x}_2(t) &= \int_0^t e^{J(t-s)}(Pf)_2(s)ds \end{aligned} \tag{26}$$

where i is the index of nilpotency of the matrix $\tilde{N}$, which is obviously also the index of the DAE. See [8] for details. □

For our test-problem, however, we have followed both ways. the following figures show a comparison. For this test, we have choosen a 3-MSD system, we have prescribed the acceleration of the highest mass to be sine-function, i.e., the state-constraint equation added to the system equation is $0 = g_{out}(\dot{v}_3) - y_{ref}(t) = \dot{v}_3 - \sin(t)$. There are no dampers missing, so, according to Lemma (4.2), the index of the resulting DAE is $1 + 2 = 3$, therefore, it can be solved numerically by the DAE-Integrator RADAU5, see [7], without any index reduction.

The error-tolerances of RADAU5 were set to 10^{-4}, both relative and absolute, RADAU5 has taken 118 time steps to solve this simple Index-3-problem.

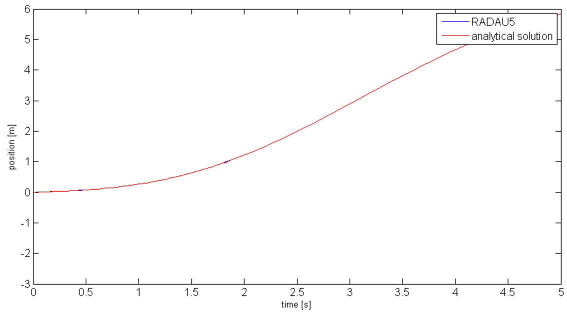

Figure 6: Input

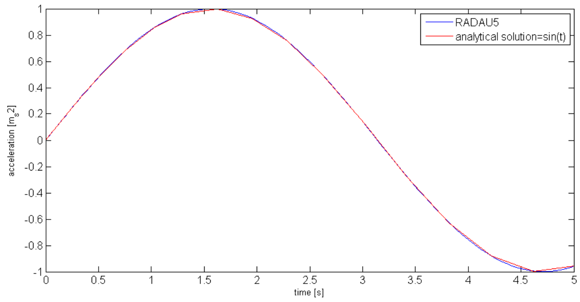

Figure 7: Output, acceleration of the highest mass

4.2 Variational Method

The next approach, we want to discuss here, is a variational approach, in literature also often called indirect optimization. For a detailed overview, see [5], [9].

Recall, that our general task is to minimize the cost functional,

$$J[x,u] := \|y - y_{ref}\|_{L^2}^2 = \int_0^T \big(g_{out}(x(t),u(t)) - y_{ref}(t)\big)^2 dt, \tag{27}$$

subject to the equation, which describes the dynamics for the multibody system. For this section, we assume, that the system equation is an ODE of the form

$$\dot{x} = f(t,x,u), \qquad x(0) = x_0, \tag{28}$$

again with $x = (q,v)^T$. The idea of the variational approach is to derive a necessary condition for u to be a minimizer of (27). We briefly sketch the well-known argumentation: we consider small perturbations of u: $u + \varepsilon\delta u$. If x is a solution of (28) to u, the solution corresponding to the perturbed input is of the form $x + \varepsilon\delta x + \mathcal{O}(\varepsilon^2)$ and therefore

$$\dot{\delta x} = f_x \delta x + f_u \delta u. \tag{29}$$

Linearization of the cost functional yields

$$J[u+\varepsilon\delta u] - J[u] = \varepsilon \int_0^T \big(\varphi_x \delta x + \varphi_u \delta u\big) dt + \mathcal{O}(\varepsilon^2), \tag{30}$$

we have set $\varphi(t,x,u) := g_{out}(x(t),u(t)) - y_{ref}(t)$. Hence, a necessary condition for u to be a minimizer is

$$\int_0^T \big(\varphi_x \delta x + \varphi_u \delta u\big) dt = 0. \tag{31}$$

It is easy to show, that this is equivalent to

$$\int_0^T \big(\mu^T f_u + \varphi_u\big) \delta u dt = 0, \tag{32}$$

provided, that the so called adjoint variable μ fulfills

$$\dot{\mu} = -f_x^T \mu - \varphi_x^T, \qquad \mu(T) = 0. \tag{33}$$

To summarize, a necessary condition for u to be a minimizer is fullfilling the DAE-system:

$$\begin{aligned} \dot{x} &= f(t,x,u), & x(0) &= x_0, \\ \dot{\mu} &= -f_x^T \mu - \varphi_x^T, & \mu(T) &= 0, \\ 0 &= \mu^T f_u + \varphi_u. \end{aligned} \tag{34}$$

For a detailed discussion, see [5].

Eq. (34) is a mixed boundary value DAE-problem (of possibly high index). Hence, it is numerically hard to solve, e.g., one can apply shooting methods, that in turn require derivatives with respect to the end value $\mu(T)$. These derivatives can be obtained by a finite differences approximation, by integrating the corresponding sensitivity DAE or by algorithmic differentiation.

We consider our benchmark problem, the $N-$MSD, see Eq. (11). Again, we want to prescribe the motion of the $N-$th mass

The variational equations (34) for this problem read as follows:

$$\dot{x} = Ax + bu, \qquad x(0) = x_0, \tag{35}$$
$$\dot{\mu} = -A^T \mu - \varphi_x^T, \qquad \mu(T) = 0, \tag{36}$$
$$0 = \mu^T b = \mu_{2N} k_1. \tag{37}$$

For the different output possibilities, we have:

$$\begin{aligned} \text{acceleration: } \varphi(t,x,u) &= (-k_N(x_N - x_{N-1}) - d_N(v_N - v_{N-1}) - y_{ref}(t))^2, \\ \text{velocity: } \varphi(t,x,u) &= (v_N - y_{ref}(t))^2, \\ \text{position: } \varphi(t,x,u) &= (x_N - y_{ref}(t))^2, \end{aligned} \tag{38}$$

whence, for the gradient:

$$\begin{aligned} \text{acceleration: } \varphi_x^T &= (-k_N, k_N, 0, \ldots, 0, -d_N, d_N, 0, \ldots, 0)^T \\ &\qquad \cdot 2(-k_N(x_N - x_{N-1}) - d_N(v_N - v_{N-1}) - y_{ref}(t)), \\ \text{velocity: } \varphi_x^T &= (0, \ldots, 0, 2(v_N - y_{ref}(t)), 0, \ldots, 0)^T, \\ \text{position: } \varphi_x^T &= (2(x_N - y_{ref}(t)), 0, \ldots, 0)^T. \end{aligned} \tag{39}$$

For the index of the DAE system (35)-(37), we have a similar result as for the index for the resultig DAE of the previous method, it grows linearly with the number of bodies:

Lemma 4.7. *Consider the variational equations (35)-(37) for the optimal control problem of the $N-$MSD. Let L denote the number of spring-damper-elements between the first and the last mass, for which the damper is missing, then, the differentiation index of the Eqn. (35)-(37) is*

$$\begin{aligned} \textit{acceleration-level:} \quad & 2(N+L)-1 \\ \textit{velocity-level:} \quad & 2(N+L)+1 \\ \textit{position-level:} \quad & 2(N+L)+3 \end{aligned} \tag{40}$$

Proof. We assume φ on acceleration-level and no damper missing. The result relies basically on the special form of the system matrix A:

$$A = \begin{pmatrix} 0 & \mathbb{1} \\ K & D \end{pmatrix}, \tag{41}$$

where K and D having the following sparse structure:

$$K = \begin{pmatrix} -k_N & k_N & & & & \\ k_N & -(k_N+k_{N-1}) & k_{N-1} & & & \\ & k_{N-1} & -(k_{N-1}+k_{N-2}) & k_{N-2} & & \\ & & & & \ddots & \\ & & & & & \\ & & & & k_2 & -(k_1+k_2) \end{pmatrix} \tag{42}$$

For D, just replace k_j by d_j. The algebraic equation of the variational equation system is $0 = \mu_{2N} k_1$. Recall, that the differential equation for μ is:

$$\dot{\mu} = -A^T \mu - \varphi_x^T. \tag{43}$$

Therefore, we have

$$\dot{\mu}_i = -(A_{i,1..2N})^T \mu - \varphi_{x,i}. \tag{44}$$

Using this and the special structure of A, a simple induction argument shows, that with each time differentiation, one obtains:

$$0 = C \cdot \dot{\mu}_i = C d_i \mu_{i-1} + R, \qquad i = 2N, ..N. \tag{45}$$

Since the first non-vanishing component, seen from the bottom, in φ_x^T is $\varphi_{x,N+2}^T$, again inductively, one has to perform exactly $2N-(N+1)+L = N+L-1$ time differentiations to get the equation

$$0 = -2Cd_N\big(-k_N(x_N - x_{N-}) - d_N(v_N - v_{N-1})\big) + R. \tag{46}$$

Now, consider this equation together with the system equation of the N-MSD, (35), one can apply Lemma 4.2, and the differentiation index turns out to be

$$N + L - 1 + (N-2) + L + 2 = 2(N+L) - 1. \tag{47}$$

The cases with φ on velocity or position level respectively are proven in the same way.

□

We have also applied this method to a 2-MSD-system. The results are as good as of the previous method. However, even for this simple system, a shooting approach for the end-value of the adjoint variable is necessary.

4.3 Direct Optimization - a multiple shooting method

The last approach, we want to present here, is based on a direct optimization of the optimal control problem (5), it is transformed to a finite-dimensional nonlinear programming problem, see [11], [12].
Here, we assume, that the system equation is a DAE in semi-explicit form:

$$\begin{aligned} \dot{x}_d &= f_d(t, x_d, x_a, u), \\ 0 &= f_a(x_d, x_a, u), \end{aligned} \tag{48}$$

with differential variables x_d and algebraic variables x_a.

We give a brief overview of the discretization procedure: First, we introduce a control grid,

$$\pi_u := \{t_1, .., t_M\} \subset [0; T] \qquad t_1 = 0, t_M = T, \tag{49}$$

on which the input u is approximated by splines. E.g., one can think of a piecewise constant or linear approximation on each subinterval $[t_i; t_{i+1}]$. Let $c_1, ..c_{\tilde{M}}$ denote the corresponding spline coefficients.

The next step is to introduce a state grid

$$\pi_x := \{\bar{t}_1, .., \bar{t}_L\} \subset [0; T], \qquad \bar{t}_1 = 0, \bar{t}_L = T. \tag{50}$$

On each subinterval $[\bar{t}_j; \bar{t}_{j+1}]$, the system equation is solved numerically by a suitable integrator. An important condition is, that we have consistent "initial" values $x^j = (x_d^j, x_a^j)$ at each state grid point. Let

$$x_{app}^j = x_{app}^j(t; x^j, c) \tag{51}$$

denote the approximate solution on $[\bar{t}_j; \bar{t}_{j+1}]$, depending on the spline coefficients, which will appear later as a part of the variable to be changed in the optimization process.
The last task is to discretize the cost functional

$$J[x, u] := \int_0^T \varphi(t, x, u) dt. \tag{52}$$

To this end, there are mainly two possible ways. The first one is to approximate the integral by a finite sum, possibly on a third time grid $\pi_J := \{\tilde{t}_1, .., \tilde{t}_K\} \supset \pi_x$:

$$J[x, u] := \int_0^T \varphi(t, x, u) dt \approx \tilde{J}[x, u] := \sum_{i=0}^{K} h_i \varphi(\tilde{t}_i, x(\tilde{t}_i), u(\tilde{t}_i)) \tag{53}$$

with suitable weighting factors h_i.

To explain the second way, we remark, that the form of our optimal control problem - with the cost functional being an integral - is said to be in *Lagrange form* in literature. It can easily be transformed into a problem in so called *Mayer form*, in which the cost functional is only function of the state variable at the end time: $J[x, u] = \Phi(x(T))$. Of course, such a cost functional has not to be discretized anymore. The transformation is accomplished by introducing an additional state variable x_0 and an additional differential equation

$$\dot{x}_0 = \varphi(t, x, u), \qquad x_0(0) = 0. \tag{54}$$

Adding this differential equation to the system equation and setting

$$\tilde{J}[x,u] := x_0(T), \tag{55}$$

we arrive at an optimal control problem in Mayer form, which is equivalent to our old problem. Now, we are able to state the discretized, finite-dimensional optimal control problem:

$$\begin{aligned} \text{minimize} \quad & \tilde{J}[x,u] \\ \text{w.r.t.} \quad & \zeta := (x_d^1, .., x_d^L, c_1, .., c_{\tilde{M}}) \\ \text{s.t.} \quad & x_{d,app}^1(\tilde{t}_2; x_d^1, c) = x_d^2 \\ & \quad \vdots \\ & x_{d,app}^{L-1}(\tilde{t}_{L-1}; x_d^{L-1}, c) = x_d^L \end{aligned} \tag{56}$$

Note, that the optimization variable ζ only depends on state grid initial values of the differential variable. To be consistent, the initial value of the algebraic variable is locally uniquely determined. The last constraint equations of (56) assure, that the differential part of the approximate solution is continuous.

The whole method is called *direct single shooting method*, if $L = 1$, and *direct multiple shooting method* otherwise. The dimension of the optimization variable ζ is finite, but can be very high, depending on the length of the complete time interval to be considered and on the length of the discretization time grids. Suitable numerical solution methods for such nonlinear programming problems are sequential quadratic programming methods (SQP), see [14]. Those methods, however, use the gradient of the objective function and the Jacobian of the constraints, i.e, as in the variational approach, a sensitivity analysis of the system equation is often necessary.

To reduce the dimension of the optimization variable, so called *moving horizon techniques* are proposed in literature, see [13]. The main idea is to consider short sections of the complete time interval $[0;T]$ and solve local optimal control problems of smaller dimension. The local solutions have to be combined by suitable transient conditions.

We have applied this approach to our benchmark problem, this time a 2-MSD, where the output was the displacement of the highest mass and the reference output was a sine signal again. The following figures show the results.

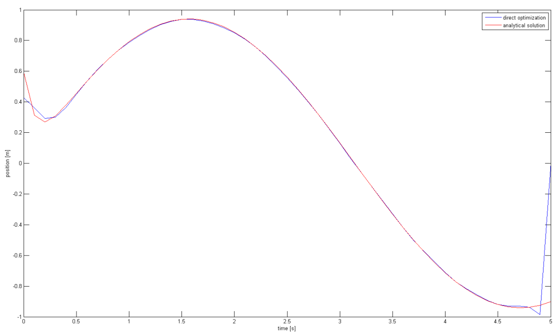

Figure 8: Calculated input and the exact solution

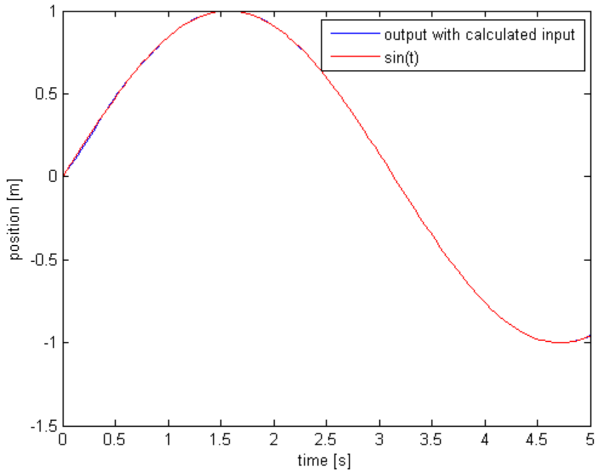

Figure 9: Calculated output and the exact solution, displacement of the highest mass

4.4 Comparison

We have presented three mathematically well-defined approaches for dealing with the optimal control problem described in the introduction. All three methods work very well for our benchmark system, a N-mass-spring-damper system, with small N.

Both the trajectory presribed path control method and the variational approach augment the system equation of the considered dynamical system and always lead to a DAE. The two approaches, however, can suffer from a high differentiation index of the resulting DAE. As we have shown in the Lemmas 4.2, 4.7, without any modifications, for both methods, the differentiation index is increasing with $\mathcal{O}(N)$, where N is the number of involved bodies. It is well-known, that this can lead to severe numerical difficulties or even to a numerical unsolvability of the DAE. The integrator RADAU5 can solve DAEs in semi-explicit form up to index 3. Concerning the variational approach, we have the additional problem, that the underlying DAE is a mixed boundary value problem, so, e.g., a shooting method has to be applied, which in turn requires derivatives, i.e., in this case, Hessians of the right-hand side of the system equation. Note, that one needs Jacobians of the right-hand-side of the system equation only to set up the variational equatons, eq. (34).

The last approach, the direct optimization, has no such index problems as the previous ones. Here, the sytem equation is not affected and the differentiation index remains unchanged. The resulting optimization problem can be solved with a suitable large-scale algorithm, if its dimension is not too high. Otherwise, other techniques, such as moving horizon, are necessary. Additionally, optimization methods usually need gradients of the object function and Jacobians of the constraints, i.e., in this case Jacobians of the right-hand side of the system equation.

As information about the considered dynamical system, the three methods merely require a structural knowledge and right-hand-side evaluations as well as evaluations of the right-hand-side-Jacobians and -Hessians respectively. Hence, an application in connection with a commercial MBS tool could be possible. However, if an index reduction has to be performed for one of the first two methods, surely more information about the system equation is necessary. Concerning the needed information about the system equation, one could tax the first two methods as *grey-box* methods, whereas the third approach is *"dark-grey"*, since only the integration results of the system equation and the corresponding sensitivity equation are needed.

5 Conclusion

In this paper, we have presented the problem of calulating invariant loads for the simulation of dynamical systems in vehicle engineering. Mathematically, the problem is an optimal control problem. We have desribed the state-of-the art solution, the iterative learning control, and have given an application example from the automotive industry, for which the iterative learning control has been applied successfully. However, this method has many drawbacks and does not converge for general nonlinear systems. Therefore, we have presented three alternative approaches, based on the optimal control theory for DAEs. We have successfully applied those methods to simple benchmarks and investigated some of the problems and numerical difficultites occuring there. Currently, we are working on the refinement and implementation of the described methods.

.

REFERENCES

[1] K. Dressler, M. Speckert, G. Bitsch. Virtual Test Rigs. *Multibody Dynamics 2007 ECCOMAS* Thematic Conference, Milano, 2007.

[2] L. Ljung. *System Identification, 2nd edition*. Prentice-Hall, Upper Saddle River, New Jersey, 1999.

[3] N. Weigel, S. Weihe, V. Sing, R. Gerlach, A. Ferreiro de Souza, M. Speckert, A. Marquardt, K. Dreßler. *VIRTUAL ITERATION FOR SET UP OF TRUCK CAB TESTS*. Simpack User Meeting, Bonn, 2007.

[4] J. De Cuyper. *Linear feedback control for durability test rigs in the automotive industry*. PhD-Thesis, KU Leuven, 2006.

[5] A. E. Bryson, Jr., Y.-C. Ho. *Aplied Optimal Control*. Hemisphere Publishing Corporation, Washington, New York, London, 1975.

[6] K.E. Brenan, S.L. Campbell, L.R. Petzold. *Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations*. Classics in Applied Mathematics, SIAM, New York, 1989.

[7] E. Hairer, G. Wanner *Solving Ordinary Differential Equations II*. 2nd ed., Springer, Berlin, Heidelberg, New York, 2002.

[8] P. Kunkel, V. Mehrmann *Differential Algebraic Equations*. European Mathematical Society, 2006.

[9] R. Callies, P. Rentrop. Optimal Control of Rigid-Link Manipulators by Indirect Methods.. *GAMM-Mitt.*, **31**, No. 1, 27–58, 2008.

[10] S.L. Campbell, C.W. Gear. The index of general nonlinear DAEs. *Numerische Mathematik*, **72**,173–196, 1995.

[11] M. Gerdts. Direct Shooting Method for the Numerical Solution of Higher-Index DAE Optimal Control Problems. *Journal of Optimization Theory and Applications*, **117**, No. 2, 267–294, 2003.

[12] M. Gerdts *Numerische Methoden optimaler Steuerprozesse mit differential-algebraischen Gleichungssystemen höheren Indexes und ihre Anwendungen in der Kraftfahrzeugsimulation und Mechanik*. Dissertation, Bayreuther Mathematische Schriften **61**, 2001.

[13] M. Gerdts. A moving horizon technique for the simulation of automobile test-drives. *Z. Angew. Math. Mech*, **83**, No. 3, 147–162, 2003.

[14] A. Barclay, P. Gill, J. B. Rosen. SQP Methods And Their Application To Numerical Optimal Control. *Variational Calculus, Optimal Control and Applications, W.H. Schmidt, K. Heier, L. Bittner, and R. Bulirsch(Eds.)*, **124**, Int. Series Numer. Mathematics, Basel, Birkhäuser, 1998, pp. 207-222.

Published reports of the Fraunhofer ITWM

The PDF-files of the following reports are available under:
www.itwm.fraunhofer.de/de/ zentral__berichte/berichte

1. D. Hietel, K. Steiner, J. Struckmeier
A Finite - Volume Particle Method for Compressible Flows
(19 pages, 1998)

2. M. Feldmann, S. Seibold
Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)

3. Y. Ben-Haim, S. Seibold
Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)

4. F.-Th. Lentes, N. Siedow
Three-dimensional Radiative Heat Transfer in Glass Cooling Processes
(23 pages, 1998)

5. A. Klar, R. Wegener
A hierarchy of models for multilane vehicular traffic
Part I: Modeling
(23 pages, 1998)

Part II: Numerical and stochastic investigations
(17 pages, 1998)

6. A. Klar, N. Siedow
Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes
(24 pages, 1998)

7. I. Choquet
Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium
(24 pages, 1998)

8. J. Ohser, B. Steinbach, C. Lang
Efficient Texture Analysis of Binary Images
(17 pages, 1998)

9. J. Orlik
Homogenization for viscoelasticity of the integral type with aging and shrinkage
(20 pages, 1998)

10. J. Mohring
Helmholtz Resonators with Large Aperture
(21 pages, 1998)

11. H. W. Hamacher, A. Schöbel
On Center Cycles in Grid Graphs
(15 pages, 1998)

12. H. W. Hamacher, K.-H. Küfer
Inverse radiation therapy planning - a multiple objective optimisation approach
(14 pages, 1999)

13. C. Lang, J. Ohser, R. Hilfer
On the Analysis of Spatial Binary Images
(20 pages, 1999)

14. M. Junk
On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes
(24 pages, 1999)

15. M. Junk, S. V. Raghurame Rao
A new discrete velocity method for Navier-Stokes equations
(20 pages, 1999)

16. H. Neunzert
Mathematics as a Key to Key Technologies
(39 pages (4 PDF-Files), 1999)

17. J. Ohser, K. Sandau
Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem
(18 pages, 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel
Solving nonconvex planar location problems by finite dominating sets
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)

19. A. Becker
A Review on Image Distortion Measures
Keywords: Distortion measure, human visual system
(26 pages, 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn
Polyhedral Properties of the Uncapacitated Multiple Allocation Hub Location Problem
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)

21. H. W. Hamacher, A. Schöbel
Design of Zone Tariff Systems in Public Transportation
(30 pages, 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga
The Finite-Volume-Particle Method for Conservation Laws
(16 pages, 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel
Location Software and Interface with GIS and Supply Chain Management
Keywords: facility location, software development, geographical information systems, supply chain management
(48 pages, 2001)

24. H. W. Hamacher, S. A. Tjandra
Mathematical Modelling of Evacuation Problems: A State of Art
(44 pages, 2001)

25. J. Kuhnert, S. Tiwari
Grid free method for solving the Poisson equation
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)

26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier
Simulation of the fiber spinning process
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)

27. A. Zemitis
On interaction of a liquid film with an obstacle
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)

28. I. Ginzburg, K. Steiner
Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)

29. H. Neunzert
»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«
Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalenanalyse, Strömungsmechanik
(18 pages, 2001)

30. J. Kuhnert, S. Tiwari
Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)

31. R. Korn, M. Krekel
Optimal Portfolios with Fixed Consumption or Income Streams
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems
(23 pages, 2002)

32. M. Krekel
Optimal portfolios with a loan dependent credit spread
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)

33. J. Ohser, W. Nagel, K. Schladitz
The Euler number of discretized sets – on the choice of adjacency in homogeneous lattices
Keywords: image analysis, Euler number, neighborhod relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner
Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting

Keywords: Liquid Polymer Moulding, Modelling, Simulation, Infiltration, Front Propagation, non-Newtonian flow in porous media
(43 pages, 2004)

64. T. Hanne, H. Neu
Simulating Human Resources in Software Development Processes
Keywords: Human resource modeling, software process, productivity, human factors, learning curve
(14 pages, 2004)

65. O. Iliev, A. Mikelic, P. Popov
Fluid structure interaction problems in deformable porous media: Toward permeability of deformable porous media
Keywords: fluid-structure interaction, deformable porous media, upscaling, linear elasticity, stokes, finite elements
(28 pages, 2004)

66. F. Gaspar, O. Iliev, F. Lisbona, A. Naumovich, P. Vabishchevich
On numerical solution of 1-D poroelasticity equations in a multilayered domain
Keywords: poroelasticity, multilayered material, finite volume discretization, MAC type grid
(41 pages, 2004)

67. J. Ohser, K. Schladitz, K. Koch, M. Nöthe
Diffraction by image processing and its application in materials science
Keywords: porous microstructure, image analysis, random set, fast Fourier transform, power spectrum, Bartlett spectrum
(13 pages, 2004)

68. H. Neunzert
Mathematics as a Technology: Challenges for the next 10 Years
Keywords: applied mathematics, technology, modelling, simulation, visualization, optimization, glass processing, spinning processes, fiber-fluid interaction, trubulence effects, topological optimization, multicriteria optimization, Uncertainty and Risk, financial mathematics, Malliavin calculus, Monte-Carlo methods, virtual material design, filtration, bio-informatics, system biology
(29 pages, 2004)

69. R. Ewing, O. Iliev, R. Lazarov, A. Naumovich
On convergence of certain finite difference discretizations for 1D poroelasticity interface problems
Keywords: poroelasticity, multilayered material, finite volume discretizations, MAC type grid, error estimates
(26 pages,2004)

70. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On Efficient Simulation of Non-Newtonian Flow in Saturated Porous Media with a Multigrid Adaptive Refinement Solver
Keywords: Nonlinear multigrid, adaptive renement, non-Newtonian in porous media
(25 pages, 2004)

71. J. Kalcsics, S. Nickel, M. Schröder
Towards a Unified Territory Design Approach – Applications, Algorithms and GIS Integration
Keywords: territory desgin, political districting, sales territory alignment, optimization algorithms, Geographical Information Systems
(40 pages, 2005)

72. K. Schladitz, S. Peters, D. Reinel-Bitzer, A. Wiegmann, J. Ohser
Design of acoustic trim based on geometric modeling and flow simulation for non-woven
Keywords: random system of fibers, Poisson line process, flow resistivity, acoustic absorption, Lattice-Boltzmann method, non-woven
(21 pages, 2005)

73. V. Rutka, A. Wiegmann
Explicit Jump Immersed Interface Method for virtual material design of the effective elastic moduli of composite materials
Keywords: virtual material design, explicit jump immersed interface method, effective elastic moduli, composite materials
(22 pages, 2005)

74. T. Hanne
Eine Übersicht zum Scheduling von Baustellen
Keywords: Projektplanung, Scheduling, Bauplanung, Bauindustrie
(32 pages, 2005)

75. J. Linn
The Folgar-Tucker Model as a Differetial Algebraic System for Fiber Orientation Calculation
Keywords: fiber orientation, Folgar–Tucker model, invariants, algebraic constraints, phase space, trace stability
(15 pages, 2005)

76. M. Speckert, K. Dreßler, H. Mauch, A. Lion, G. J. Wierda
Simulation eines neuartigen Prüfsystems für Achserprobungen durch MKS-Modellierung einschließlich Regelung
Keywords: virtual test rig, suspension testing, multibody simulation, modeling hexapod test rig, optimization of test rig configuration
(20 pages, 2005)

77. K.-H. Küfer, M. Monz, A. Scherrer, P. Süss, F. Alonso, A. S. A. Sultan, Th. Bortfeld, D. Craft, Chr. Thieke
Multicriteria optimization in intensity modulated radiotherapy planning
Keywords: multicriteria optimization, extreme solutions, real-time decision making, adaptive approximation schemes, clustering methods, IMRT planning, reverse engineering
(51 pages, 2005)

78. S. Amstutz, H. Andrä
A new algorithm for topology optimization using a level-set method
Keywords: shape optimization, topology optimization, topological sensitivity, level-set
(22 pages, 2005)

79. N. Ettrich
Generation of surface elevation models for urban drainage simulation
Keywords: Flooding, simulation, urban elevation models, laser scanning
(22 pages, 2005)

80. H. Andrä, J. Linn, I. Matei, I. Shklyar, K. Steiner, E. Teichmann
OPTCAST – Entwicklung adäquater Strukturoptimierungsverfahren für Gießereien Technischer Bericht (KURZFASSUNG)
Keywords: Topologieoptimierung, Level-Set-Methode, Gießprozesssimulation, Gießtechnische Restriktionen, CAE-Kette zur Strukturoptimierung
(77 pages, 2005)

81. N. Marheineke, R. Wegener
Fiber Dynamics in Turbulent Flows
Part I: General Modeling Framework
Keywords: fiber-fluid interaction; Cosserat rod; turbulence modeling; Kolmogorov's energy spectrum; double-velocity correlations; differentiable Gaussian fields
(20 pages, 2005)

Part II: Specific Taylor Drag
Keywords: flexible fibers; k-ε turbulence model; fiber-turbulence interaction scales; air drag; random Gaussian aerodynamic force; white noise; stochastic differential equations; ARMA process
(18 pages, 2005)

82. C. H. Lampert, O. Wirjadi
An Optimal Non-Orthogonal Separation of the Anisotropic Gaussian Convolution Filter
Keywords: Anisotropic Gaussian filter, linear filtering, orientation space, nD image processing, separable filters
(25 pages, 2005)

83. H. Andrä, D. Stoyanov
Error indicators in the parallel finite element solver for linear elasticity DDFEM
Keywords: linear elasticity, finite element method, hierarchical shape functions, domain decom-position, parallel implementation, a posteriori error estimates
(21 pages, 2006)

84. M. Schröder, I. Solchenbach
Optimization of Transfer Quality in Regional Public Transit
Keywords: public transit, transfer quality, quadratic assignment problem
(16 pages, 2006)

85. A. Naumovich, F. J. Gaspar
On a multigrid solver for the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: poroelasticity, interface problem, multigrid, operator-dependent prolongation
(11 pages, 2006)

86. S. Panda, R. Wegener, N. Marheineke
Slender Body Theory for the Dynamics of Curved Viscous Fibers
Keywords: curved viscous fibers; fluid dynamics; Navier-Stokes equations; free boundary value problem; asymptotic expansions; slender body theory
(14 pages, 2006)

87. E. Ivanov, H. Andrä, A. Kudryavtsev
Domain Decomposition Approach for Automatic Parallel Generation of Tetrahedral Grids
Key words: Grid Generation, Unstructured Grid, Delaunay Triangulation, Parallel Programming, Domain Decomposition, Load Balancing
(18 pages, 2006)

88. S. Tiwari, S. Antonov, D. Hietel, J. Kuhnert, R. Wegener
A Meshfree Method for Simulations of Interactions between Fluids and Flexible Structures
Key words: Meshfree Method, FPM, Fluid Structure Interaction, Sheet of Paper, Dynamical Coupling
(16 pages, 2006)

89. R. Ciegis , O. Iliev, V. Starikovicius, K. Steiner
Numerical Algorithms for Solving Problems of Multiphase Flows in Porous Media
Keywords: nonlinear algorithms, finite-volume method, software tools, porous media, flows
(16 pages, 2006)

90. D. Niedziela, O. Iliev, A. Latz
On 3D Numerical Simulations of Viscoelastic Fluids

Keywords: non-Newtonian fluids, anisotropic viscosity, integral constitutive equation
(18 pages, 2006)

91. A. Winterfeld
Application of general semi-infinite Programming to Lapidary Cutting Problems
Keywords: large scale optimization, nonlinear programming, general semi-infinite optimization, design centering, clustering
(26 pages, 2006)

92. J. Orlik, A. Ostrovska
Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems
Keywords: hereditary viscoelasticity; kern approximation by interpolation; space-time finite element approximation, stability and a priori estimate
(24 pages, 2006)

93. V. Rutka, A. Wiegmann, H. Andrä
EJIIM for Calculation of effective Elastic Moduli in 3D Linear Elasticity
Keywords: Elliptic PDE, linear elasticity, irregular domain, finite differences, fast solvers, effective elastic moduli
(24 pages, 2006)

94. A. Wiegmann, A. Zemitis
EJ-HEAT: A Fast Explicit Jump Harmonic Averaging Solver for the Effective Heat Conductivity of Composite Materials
Keywords: Stationary heat equation, effective thermal conductivity, explicit jump, discontinuous coefficients, virtual material design, microstructure simulation, EJ-HEAT
(21 pages, 2006)

95. A. Naumovich
On a finite volume discretization of the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: Biot poroelasticity system, interface problems, finite volume discretization, finite difference method
(21 pages, 2006)

96. M. Krekel, J. Wenzel
A unified approach to Credit Default Swaption and Constant Maturity Credit Default Swap valuation
Keywords: LIBOR market model, credit risk, Credit Default Swaption, Constant Maturity Credit Default Swap-method
(43 pages, 2006)

97. A. Dreyer
Interval Methods for Analog Circiuts
Keywords: interval arithmetic, analog circuits, tolerance analysis, parametric linear systems, frequency response, symbolic analysis, CAD, computer algebra
(36 pages, 2006)

98. N. Weigel, S. Weihe, G. Bitsch, K. Dreßler
Usage of Simulation for Design and Optimization of Testing
Keywords: Vehicle test rigs, MBS, control, hydraulics, testing philosophy
(14 pages, 2006)

99. H. Lang, G. Bitsch, K. Dreßler, M. Speckert
Comparison of the solutions of the elastic and elastoplastic boundary value problems
Keywords: Elastic BVP, elastoplastic BVP, variational inequalities, rate-independency, hysteresis, linear kinematic hardening, stop- and play-operator
(21 pages, 2006)

100. M. Speckert, K. Dreßler, H. Mauch
MBS Simulation of a hexapod based suspension test rig
Keywords: Test rig, MBS simulation, suspension, hydraulics, controlling, design optimization
(12 pages, 2006)

101. S. Azizi Sultan, K.-H. Küfer
A dynamic algorithm for beam orientations in multicriteria IMRT planning
Keywords: radiotherapy planning, beam orientation optimization, dynamic approach, evolutionary algorithm, global optimization
(14 pages, 2006)

102. T. Götz, A. Klar, N. Marheineke, R. Wegener
A Stochastic Model for the Fiber Lay-down Process in the Nonwoven Production
Keywords: fiber dynamics, stochastic Hamiltonian system, stochastic averaging
(17 pages, 2006)

103. Ph. Süss, K.-H. Küfer
Balancing control and simplicity: a variable aggregation method in intensity modulated radiation therapy planning
Keywords: IMRT planning, variable aggregation, clustering methods
(22 pages, 2006)

104. A. Beaudry, G. Laporte, T. Melo, S. Nickel
Dynamic transportation of patients in hospitals
Keywords: in-house hospital transportation, dial-a-ride, dynamic mode, tabu search
(37 pages, 2006)

105. Th. Hanne
Applying multiobjective evolutionary algorithms in industrial projects
Keywords: multiobjective evolutionary algorithms, discrete optimization, continuous optimization, electronic circuit design, semi-infinite programming, scheduling
(18 pages, 2006)

106. J. Franke, S. Halim
Wild bootstrap tests for comparing signals and images
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(13 pages, 2007)

107. Z. Drezner, S. Nickel
Solving the ordered one-median problem in the plane
Keywords: planar location, global optimization, ordered median, big triangle small triangle method, bounds, numerical experiments
(21 pages, 2007)

108. Th. Götz, A. Klar, A. Unterreiter, R. Wegener
Numerical evidance for the non-existing of solutions of the equations desribing rotational fiber spinning
Keywords: rotational fiber spinning, viscous fibers, boundary value problem, existence of solutions
(11 pages, 2007)

109. Ph. Süss, K.-H. Küfer
Smooth intensity maps and the Bortfeld-Boyer sequencer
Keywords: probabilistic analysis, intensity modulated radiotherapy treatment (IMRT), IMRT plan application, step-and-shoot sequencing
(8 pages, 2007)

110. E. Ivanov, O. Gluchshenko, H. Andrä, A. Kudryavtsev
Parallel software tool for decomposing and meshing of 3d structures
Keywords: a-priori domain decomposition, unstructured grid, Delaunay mesh generation
(14 pages, 2007)

111. O. Iliev, R. Lazarov, J. Willems
Numerical study of two-grid preconditioners for 1d elliptic problems with highly oscillating discontinuous coefficients
Keywords: two-grid algorithm, oscillating coefficients, preconditioner
(20 pages, 2007)

112. L. Bonilla, T. Götz, A. Klar, N. Marheineke, R. Wegener
Hydrodynamic limit of the Fokker-Planck-equation describing fiber lay-down processes
Keywords: stochastic dierential equations, Fokker-Planck equation, asymptotic expansion, Ornstein-Uhlenbeck process
(17 pages, 2007)

113. S. Rief
Modeling and simulation of the pressing section of a paper machine
Keywords: paper machine, computational fluid dynamics, porous media
(41 pages, 2007)

114. R. Ciegis, O. Iliev, Z. Lakdawala
On parallel numerical algorithms for simulating industrial filtration problems
Keywords: Navier-Stokes-Brinkmann equations, finite volume discretization method, SIMPLE, parallel computing, data decomposition method
(24 pages, 2007)

115. N. Marheineke, R. Wegener
Dynamics of curved viscous fibers with surface tension
Keywords: Slender body theory, curved viscous bers with surface tension, free boundary value problem
(25 pages, 2007)

116. S. Feth, J. Franke, M. Speckert
Resampling-Methoden zur mse-Korrektur und Anwendungen in der Betriebsfestigkeit
Keywords: Weibull, Bootstrap, Maximum-Likelihood, Betriebsfestigkeit
(16 pages, 2007)

117. H. Knaf
Kernel Fisher discriminant functions – a concise and rigorous introduction
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(30 pages, 2007)

118. O. Iliev, I. Rybak
On numerical upscaling for flows in heterogeneous porous media
Keywords: numerical upscaling, heterogeneous porous media, single phase flow, Darcy's law, multiscale problem, effective permeability, multipoint flux approximation, anisotropy
(17 pages, 2007)

119. O. Iliev, I. Rybak
On approximation property of multipoint flux approximation method

Keywords: Multipoint flux approximation, finite volume method, elliptic equation, discontinuous tensor coefficients, anisotropy
(15 pages, 2007)

120. O. Iliev, I. Rybak, J. Willems
On upscaling heat conductivity for a class of industrial problems
Keywords: Multiscale problems, effective heat conductivity, numerical upscaling, domain decomposition
(21 pages, 2007)

121. R. Ewing, O. Iliev, R. Lazarov, I. Rybak
On two-level preconditioners for flow in porous media
Keywords: Multiscale problem, Darcy's law, single phase flow, anisotropic heterogeneous porous media, numerical upscaling, multigrid, domain decomposition, efficient preconditioner
(18 pages, 2007)

122. M. Brickenstein, A. Dreyer
POLYBORI: A Gröbner basis framework for Boolean polynomials
Keywords: Gröbner basis, formal verification, Boolean polynomials, algebraic cryptoanalysis, satisfiability
(23 pages, 2007)

123. O. Wirjadi
Survey of 3d image segmentation methods
Keywords: image processing, 3d, image segmentation, binarization
(20 pages, 2007)

124. S. Zeytun, A. Gupta
A Comparative Study of the Vasicek and the CIR Model of the Short Rate
Keywords: interest rates, Vasicek model, CIR-model, calibration, parameter estimation
(17 pages, 2007)

125. G. Hanselmann, A. Sarishvili
Heterogeneous redundancy in software quality prediction using a hybrid Bayesian approach
Keywords: reliability prediction, fault prediction, non-homogeneous poisson process, Bayesian model averaging
(17 pages, 2007)

126. V. Maag, M. Berger, A. Winterfeld, K.-H. Küfer
A novel non-linear approach to minimal area rectangular packing
Keywords: rectangular packing, non-overlapping constraints, non-linear optimization, regularization, relaxation
(18 pages, 2007)

127. M. Monz, K.-H. Küfer, T. Bortfeld, C. Thieke
Pareto navigation – systematic multi-criteria-based IMRT treatment plan determination
Keywords: convex, interactive multi-objective optimization, intensity modulated radiotherapy planning
(15 pages, 2007)

128. M. Krause, A. Scherrer
On the role of modeling parameters in IMRT plan optimization
Keywords: intensity-modulated radiotherapy (IMRT), inverse IMRT planning, convex optimization, sensitivity analysis, elasticity, modeling parameters, equivalent uniform dose (EUD)
(18 pages, 2007)

129. A. Wiegmann
Computation of the permeability of porous materials from their microstructure by FFF-Stokes
Keywords: permeability, numerical homogenization, fast Stokes solver
(24 pages, 2007)

130. T. Melo, S. Nickel, F. Saldanha da Gama
Facility Location and Supply Chain Management – A comprehensive review
Keywords: facility location, supply chain management, network design
(54 pages, 2007)

131. T. Hanne, T. Melo, S. Nickel
Bringing robustness to patient flow management through optimized patient transports in hospitals
Keywords: Dial-a-Ride problem, online problem, case study, tabu search, hospital logistics
(23 pages, 2007)

132. R. Ewing, O. Iliev, R. Lazarov, I. Rybak, J. Willems
An efficient approach for upscaling properties of composite materials with high contrast of coefficients
Keywords: effective heat conductivity, permeability of fractured porous media, numerical upscaling, fibrous insulation materials, metal foams
(16 pages, 2008)

133. S. Gelareh, S. Nickel
New approaches to hub location problems in public transport planning
Keywords: integer programming, hub location, transportation, decomposition, heuristic
(25 pages, 2008)

134. G. Thömmes, J. Becker, M. Junk, A. K. Vaikuntam, D. Kehrwald, A. Klar, K. Steiner, A. Wiegmann
A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method
Keywords: Lattice Boltzmann method, Level Set method, free surface, multiphase flow
(28 pages, 2008)

135. J. Orlik
Homogenization in elasto-plasticity
Keywords: multiscale structures, asymptotic homogenization, nonlinear energy
(40 pages, 2008)

136. J. Almquist, H. Schmidt, P. Lang, J. Deitmer, M. Jirstrand, D. Prätzel-Wolters, H. Becker
Determination of interaction between MCT1 and CAII via a mathematical and physiological approach
Keywords: mathematical modeling; model reduction; electrophysiology; pH-sensitive microelectrodes; proton antenna
(20 pages, 2008)

137. E. Savenkov, H. Andrä, O. Iliev
An analysis of one regularization approach for solution of pure Neumann problem
Keywords: pure Neumann problem, elasticity, regularization, finite element method, condition number
(27 pages, 2008)

138. O. Berman, J. Kalcsics, D. Krass, S. Nickel
The ordered gradual covering location problem on a network
Keywords: gradual covering, ordered median function, network location
(32 pages, 2008)

139. S. Gelareh, S. Nickel
Multi-period public transport design: A novel model and solution approaches
Keywords: Integer programming, hub location, public transport, multi-period planning, heuristics
(31 pages, 2008)

140. T. Melo, S. Nickel, F. Saldanha-da-Gama
Network design decisions in supply chain planning
Keywords: supply chain design, integer programming models, location models, heuristics
(20 pages, 2008)

141. C. Lautensack, A. Särkkä, J. Freitag, K. Schladitz
Anisotropy analysis of pressed point processes
Keywords: estimation of compression, isotropy test, nearest neighbour distance, orientation analysis, polar ice, Ripley's K function
(35 pages, 2008)

142. O. Iliev, R. Lazarov, J. Willems
A Graph-Laplacian approach for calculating the effective thermal conductivity of complicated fiber geometries
Keywords: graph laplacian, effective heat conductivity, numerical upscaling, fibrous materials
(14 pages, 2008)

143. J. Linn, T. Stephan, J. Carlsson, R. Bohlin
Fast simulation of quasistatic rod deformations for VR applications
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients
(7 pages, 2008)

144. J. Linn, T. Stephan
Simulation of quasistatic deformations using discrete rod models
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients
(9 pages, 2008)

145. J. Marburger, N. Marheineke, R. Pinnau
Adjoint based optimal control using meshless discretizations
Keywords: Mesh-less methods, particle methods, Eulerian-Lagrangian formulation, optimization strategies, adjoint method, hyperbolic equations
(14 pages, 2008

146. S. Desmettre, J. Gould, A. Szimayer
Own-company stockholding and work effort preferences of an unconstrained executive
Keywords: optimal portfolio choice, executive compensation
(33 pages, 2008)

147. M. Berger, M. schröder, K.-H. Küfer
A constraint programming approach for the two-dimensional rectangular packing problem with orthogonal orientations
Keywords: rectangular packing, orthogonal orientations non-overlapping constraints, constraint propagation
(13 pages, 2008)

148. K. Schladitz, C. Redenbach, T. Sych, M. Godehardt

Microstructural characterisation of open foams using 3d images

Keywords: virtual material design, image analysis, open foams
(30 pages, 2008)

149. E. Fernández, J. Kalcsics, S. Nickel, R. Ríos-Mercado

A novel territory design model arising in the implementation of the WEEE-Directive

Keywords: heuristics, optimization, logistics, recycling
(28 pages, 2008)

150. H. Lang, J. Linn

Lagrangian field theory in space-time for geometrically exact Cosserat rods

Keywords: Cosserat rods, geometrically exact rods, small strain, large deformation, deformable bodies, Lagrangian field theory, variational calculus
(19 pages, 2009)

151. K. Dreßler, M. Speckert, R. Müller, Ch. Weber

Customer loads correlation in truck engineering

Keywords: Customer distribution, safety critical components, quantile estimation, Monte-Carlo methods
(11 pages, 2009)

152. H. Lang, K. Dreßler

An improved multiaxial stress-strain correction model for elastic FE postprocessing

Keywords: Jiang's model of elastoplasticity, stress-strain correction, parameter identification, automatic differentiation, least-squares optimization, Coleman-Li algorithm
(6 pages, 2009)

153. J. Kalcsics, S. Nickel, M. Schröder

A generic geometric approach to territory design and districting

Keywords: Territory design, districting, combinatorial optimization, heuristics, computational geometry
(32 pages, 2009)

154. Th. Fütterer, A. Klar, R. Wegener

An energy conserving numerical scheme for the dynamics of hyperelastic rods

Keywords: Cosserat rod, hyperealstic, energy conservation, finite differences
(16 pages, 2009)

155. A. Wiegmann, L. Cheng, E. Glatt, O. Iliev, S. Rief

Design of pleated filters by computer simulations

Keywords: Solid-gas separation, solid-liquid separation, pleated filter, design, simulation
(21 pages, 2009)

156. A. Klar, N. Marheineke, R. Wegener

Hierarchy of mathematical models for production processes of technical textiles

Keywords: Fiber-fluid interaction, slender-body theory, turbulence modeling, model reduction, stochastic differential equations, Fokker-Planck equation, asymptotic expansions, parameter identification
(21 pages, 2009)

157. E. Glatt, S. Rief, A. Wiegmann, M. Knefel, E. Wegenke

Structure and pressure drop of real and virtual metal wire meshes

Keywords: metal wire mesh, structure simulation, model calibration, CFD simulation, pressure loss
(7 pages, 2009)

158. S. Kruse, M. Müller

Pricing American call options under the assumption of stochastic dividends – An application of the Korn-Rogers model

Keywords: option pricing, American options, dividends, dividend discount model, Black-Scholes model
(22 pages, 2009)

159. H. Lang, J. Linn, M. Arnold

Multibody dynamics simulation of geometrically exact Cosserat rods

Keywords: flexible multibody dynamics, large deformations, finite rotations, constrained mechanical systems, structural dynamics
(20 pages, 2009)

160. P. Jung, S. Leyendecker, J. Linn, M. Ortiz

Discrete Lagrangian mechanics and geometrically exact Cosserat rods

Keywords: special Cosserat rods, Lagrangian mechanics, Noether's theorem, discrete mechanics, frame-indifference, holonomic constraints
(14 pages, 2009)

161. M. Burger, K. Dreßler, A. Marquardt, M. Speckert

Calculating invariant loads for system simulation in vehicle engineering

Keywords: iterative learning control, optimal control theory, differential algebraic equations(DAEs)
(18 pages, 2009)

Status quo: May 2009