Characterization of the digital holographic wavefront sensor

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ABSTRACT
Correction of atmospheric effects on the propagation of laser light can be achieved with adaptive optics (AO) by relying on adequate wavefront sensors. For free-space laser communications and for tracking of high-speed airborne objects, conventional wavefront sensing methods e.g. those based on the Shack-Hartmann sensor (SHS), are not always effective. Partial obscuration and saturation of the detector due to strong turbulence lead to errors in wavefront reconstruction. Another drawback of Shack-Hartmann wavefront-sensing is the time-consuming readout of the whole detector and subsequent matrix-vector multiplication necessary to reconstruct the wavefront. We characterize a promising modal alternative: digital holographic wavefront sensor (DHWS). We examine the performance of the sensor for single-, and multimode operation and its dependence on the detector size, scintillation, residual tip/tilt and misalignments.

Keywords: Digital holographic wavefront sensor, computer-generated hologram, spatial light modulator, adaptive optics, turbulence

1. INTRODUCTION
The most widely applied wavefront sensor for the measurement of turbulence induced phase errors is the Shack-Hartmann sensor (SHS). This sensor has several advantages, the most significant being its clear principle of operation. On the other hand it has some limitations, namely the sensor relies on a time consuming matrix-vector-multiply algorithm to reconstruct the wavefront, and secondly the error rate of the wavefront reconstruction algorithm becomes significant with the onset of strong scintillation which causes obscuration or saturation of sections of the detector.

Neil et al. proposed a new approach for wavefront measurement based on the modal-sensing principle. Their holographic sensor exhibits a linear response to the amplitude of a given Zernike mode present in the wavefront; it directly measures the strengths of the Zernike modes, hence avoiding extra processing time. Since then several approaches have been proposed with regard to the core of this sensor, i.e. the diffractive optical element (DOE) which contains the holograms of one or several Zernike modes. The two main categories of these approaches are concerned with the type and generation method of the hologram, i.e. analogue, using a holographic plate, or digital, using a multiplexed computer generated hologram (CGH). The former approach has the advantage of low cost and high spatial resolution, while the latter offers more flexibility, as well as the possibility to perform post-processing on the hologram before it is used for aberration measurement. Additionally, the advent of low-cost, low-power-consumption liquid crystal spatial light modulators (SLM) has already lead to their utilization in AO systems, both as wavefront sensors and wavefront correctors.

In this paper we study the dependencies and limitations of the DHWS for measuring several aberration modes. The core of this sensor is a digital grating, realized as a CGH, encoded in a spatial light modulator. The modal decomposition of the wavefront into the Zernike polynomials is carried out by diffraction of the incoming light at the grating. Time consuming wavefront reconstruction is therefore avoided. First we present the DHWS realized only for one aberration: defocus. Secondly we test the DHWS in a more realistic situation in which we are interested in retrieving more than one aberration from the beam. The response curves of the sensor are obtained, showing an approximately linear behaviour with the input amplitude over a pre-designed region, making it suitable for ultrafast closed-loop adaptive optics systems. We present also the effect of scintillation, generated by an intensity mask, and the response of the sensor to tip/tilt and misalignment.
2. REALIZATION OF THE DHWS

The holographic wavefront sensor treated in this paper is a modal detector that makes use of a multiplexed hologram, previously coded with the desired aberration modes to be sensed. The operational principle of HWS relies on the theory of holography\cite{15}: To measure the phase of an object beam it is necessary to produce the interference of such a beam with a reference one. This interference, once stored into a physical medium, will have the properties of a phase grating. The real image of the recorded object will be reconstructed perfectly when the wavefront of the incoming beam at the phase grating is conjugated in phase with the one of the reference beam used previously, otherwise aberrations will be present in the image.

This behaviour can be used to analyse the wavefront of an incoming beam. To measure the amplitudes of modes/aberrations of the beam, a multiplex hologram of at least two holograms is generated in the following way: The first hologram corresponds to the interference between a spherical wave described by \( O_n(x, y) \) convergent at point \( P_1(x_n, y_n, z) \) and a reference wave \( R_n(x, y) \) containing the wavefront aberration \( a_nZ_n \). The second hologram is produced by the interference of a spherical wave \( O_{-n}(x, y) \) convergent at point \( P_{-n}(x_{-n}, y_{-n}, z) \) and a reference wave \( R_{-n}(x, y) \) corresponding to the same mode but with opposite amplitude, \(-a_nZ_n\). These two holograms are recorded for every aberration mode that we wish to encode in the sensor, and the distance \( z \) is constant for all the holograms. The principle is shown in Fig. 1(a).

![Figure 1](image1.png)

**Figure 1:** (a) Recording scheme of one aberration mode, (b) multiplex method in which every pixel of the final hologram takes randomly the value of one of the individual holograms, and (c) resulting multiplex hologram for defocus, based on two saved holograms corresponding to opposite amplitudes of the aberration.

The amplitude transmittance function is obtained by the combination of both holograms,

\[
H_1(x, y) = |O_n(x, y) + R_n(x, y)|^2 + |O_{-n}(x, y) + R_{-n}(x, y)|^2 \\
= O_n(x, y)^2 + R_n(x, y)^2 + O_{-n}(x, y)R_n(x, y) + O_n(x, y)R_{-n}(x, y)^* \\
+ O_{-n}(x, y)^2 + R_{-n}(x, y)^2 + O_{-n}(x, y)R_{-n}(x, y) + O_{-n}(x, y)R_{-n}(x, y)^*.
\]

(1)

For the case of a CGH rather than optical hologram, the zero-order terms in Eq. (1) can be neglected, which increases the diffraction efficiency. We consider only the terms that will generate the real image, then

\[
CGH_n(x, y) = O_n(x, y)R_n^*(x, y) + O_{-n}(x, y)R_{-n}^*(x, y) \\
= \exp (ikr_n) \exp [-ika_nZ_n(x, y)] + \exp (ikr_{-n}) \exp [-ik(-a_n)Z_n(x, y)],
\]

(2)

where \( r_{n,-n} = [(x - x_{n,-n})^2 + (y - y_{n,-n})^2 + z]^{1/2} \).

The final CGH is obtained by multiplexing the phase of each of the summands of Eq. (2) as shown in Fig. 1(b). For the sake of clarity, Table 1 lists the Zernike polynomials relevant for this paper. By means of the same basis we can represent the phase, \( W(x, y) \) of the incoming electric field, \( R'(x, y) \), as the linear combination

\[
W(x, y) = \sum_{m} b_mZ_m(x, y).
\]

(3)
Table 1: Fringe Zernike polynomials considered in this paper, $\rho$ is the normalized radial distance at the pupil, and $\theta = \tan^{-1} \frac{y}{x}$ is the angular coordinate.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_3^2$</td>
<td>$2\rho^2 - 1$</td>
<td>Defocus</td>
</tr>
<tr>
<td>$Z_4$</td>
<td>$\rho^2 \cos 2\theta$</td>
<td>Astigmatism 0°</td>
</tr>
<tr>
<td>$Z_5$</td>
<td>$\rho^2 \sin 2\theta$</td>
<td>Astigmatism 45°</td>
</tr>
<tr>
<td>$Z_6$</td>
<td>$(2\rho^3 - 2\rho)\cos \theta$</td>
<td>Coma horizontal</td>
</tr>
<tr>
<td>$Z_7$</td>
<td>$(2\rho^3 - 2\rho)\sin \theta$</td>
<td>Coma vertical</td>
</tr>
<tr>
<td>$Z_8$</td>
<td>$6\rho^4 - 6\rho^2 + 1$</td>
<td>Spherical aberration</td>
</tr>
<tr>
<td>$Z_9$</td>
<td>$\rho^3 \cos 3\theta$</td>
<td>Trifoil 0°</td>
</tr>
<tr>
<td>$Z_{10}$</td>
<td>$\rho^3 \sin 3\theta$</td>
<td>Trifoil 30°</td>
</tr>
<tr>
<td>$Z_{11}$</td>
<td>$(4\rho^4 - 3\rho^2)\cos 2\theta$</td>
<td>2nd Astigmatism 0°</td>
</tr>
<tr>
<td>$Z_{12}$</td>
<td>$(4\rho^4 - 3\rho^2)\sin 2\theta$</td>
<td>2nd Astigmatism 45°</td>
</tr>
</tbody>
</table>

If the only component of the wavefront is the mode $Z_n$ with amplitude $b_n$, then the reconstructed intensity distribution in the detector plane, $(x', y')$, is the squared modulus of the Fourier transform of the reconstructed amplitude,

$$I_n = |F\{ \text{CGH}(x, y) R'(x, y) \}|^2$$
$$= |F\{ \exp (ikr_n) \exp [ik(b_n - a_n)Z_n(x, y)] + \exp (ikr_{-n}) \exp [ik(a_n + b_n)Z_n(x, y)] \}|^2. \quad (4)$$

The two images generated are well separated due to the off-axis positions of the spherical waves in the recording process, thus we can neglect the interference terms of the previous equation. Finally, the intensities at the two detectors placed at the focal point are integrated over regions $A_n$ and $A_{-n}$, where $A_n$ is the area of integration of the detectors. When $b_n$ matches one of the aberration modes recorded in the DHWS, i.e. $b_n = a_n$, we will obtain a peak energy at the point $P(x_n, y_n, z)$, that is $I_n > I_{-n}$. In the same way, when $b_n = -a_n$ there will be a peak energy at point $P(x_{-n}, y_{-n}, z)$. For the case in which $-a_n < b_n < a_n$, the intensity will be distributed between the two points.\(^5\) The sensor output, $S$, is the normalized intensity difference,

$$S_n = \frac{I_n - I_{-n}}{I_n + I_{-n}}, \quad (5)$$

which gives a direct detection of the amplitude of the corresponding Zernike mode, $Z_n$. The sensor output for the general case in which $Z_m$ differs from $Z_n$ gives rise to the so-called intermodal cross talk.\(^5,16\) The ability to detect a given Zernike mode is decreased because of the intermodal crosstalk.

When we want to detect several Zernike modes simultaneously, we need to multiplex the holograms represented for each Zernike mode, which will generate extra images in the image plane $(x', y')$ at distance $z$ from the displayed hologram. The CGH thus has the form:

$$\text{CGH}(x, y) = \sum_{n=3}^{N} \text{CGH}_n. \quad (6)$$

We do not implement the Zernike modes $Z_1$, $Z_2$ and $Z_3$ corresponding to piston, tip and tilt. The first can be ignored and the other two must be already corrected by an additional correction system.

For the experimental realization of the DHWS, the multiplexed hologram is displayed on the liquid crystal modulator (SLM Pluto from Holoeye with 1920x1080 elements), Fig. 2(a). To measure the intensity of the spots in each channel we make use of the Kodak Megaplus 6.3i 10 bit camera because of its big detector size, 3070x2048 elements (pixel size = 9 $\mu$m). Large detector area allows us to record several channels simultaneously.
Figure 2: (a) Liquid crystal SLM Pluto from Holoeye with 1920x1080 pixels and (b) normalized intensity at the focus of a beam converged by the SLM. The intensity temporal profile is sampled by using a Photron FastCam at 3000 fps.

Due to phase flickering characteristic of the Pluto SLM, the intensity of the reconstructed image will present fluctuating behaviour (Fig. 2(b)). It is therefore necessary to operate the camera at a time when the intensity is maximal, that is when the diffraction efficiency of the SLM is at its maximum. This is done by triggering the camera with an output voltage from the SLM. This limits the detection rate to 60 Hz. Phase flickering can be reduced by changing the addressing sequence of the SLM which would reduce the modulation phase levels, or by using an SLM with addressing frequencies higher than the desired operating frequency. These possibilities will be explored in the future.

In the following sections, we test the DHWS in different operation modes (single- or multi-channel) and study the dependence of the sensors output on: size of the detectors (or region-of-interest, $A_n$, on the CCD in our case), angle-of-arrival of the beam, misalignment and scintillation.

3. DEFOCUS OPERATION MODE

Initially, DHWS was realized for sensing defocus only. Apart from tip and tilt, which cannot be sensed by DHWS, defocus is the easiest aberration to generate and it has, together with astigmatism, the highest variance in propagation through turbulence. In the next section we consider the case of detecting defocus and astigmatism simultaneously. The setup, shown in Figs. 3(a) and 3(b), consists of a beam expander, a defocus generation system, CGH displayed on the SLM and the CCD. The two images on the camera corresponding to the defocus channel are separated by 16 mm and the plane of the camera is placed 800 mm from the SLM. In this experiment, $1.4 \lambda$ has been chosen as maximum positive and negative amplitude of defocus to be encoded in the CGH. Previous measurements of the atmospheric conditions in Ettlingen, Germany, showed that this amplitude range is sufficient to cover the defocus variability over our paths of interest. When recording higher amplitudes, the dynamic range of the used detector is a critical parameter.

We introduced known defocus to the beam and measured the sensor output value. Defocus was introduced by changing the position of the second lens of a telescope. Equation (7) gives the relation between the displacement $d$ of the second lens, see Fig. 3(b), and the strength of defocus $a$, of the resulting beam:

$$ a = \frac{D^2 d}{16f(f + d)}, $$

where $f$ is the combined focal length of the two lenses of the second telescope (Fig. 3(b)) and $D$ is the diameter of the beam.

This formula is deduced by comparing the optical path difference after the telescope with the optical path difference generated by a defocus term. The former is simplified to a parabola under paraxial approximation:

$$ W(x, y) = \frac{x^2 + y^2}{2F}, $$

where $F$ is the focal length of the telescope.
where $F$ is the back focal length of the telescope,

$$F = \frac{f(f + d)}{d};$$  

(9)

Optical path difference due to the defocus term is

$$W_3(\rho, \theta) = a(2\rho^2 - 1),$$  

(10)

where $a$ is the amplitude of the Zernike mode $Z_3$ and $\rho$ is the normalized radial coordinate,

$$\rho = \frac{x^2 + y^2}{(D/2)^2}.$$  

(11)

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**Figure 3:** (a) Setup used for DHWS calibration and defocus measurement and (b) diagram of the same setup.
Thus comparing Eq. (8) with Eq. (10), neglecting the constant term, we obtain the relation (7). By means of a conventional Shack-Hartmann sensor shown in Fig. 4(a), we are able to measure the defocus amplitude as a function of displacement $d$ to corroborate Eq. (7) experimentally. We also checked that the system does not introduce further aberrations (see Fig. 4(b)). The calibration/response curve obtained from the sensor output is shown in Fig. 5(a). The linear behaviour after correction for the non-zero offset can be seen in the $\pm 0.7 \lambda$ range.

![Shack-Hartmann sensor](image)

**Figure 4:** (a) Shack-Hartmann sensor was used to decompose the beam after the defocus generation system into Zernike polynomials. (b) Calibration of the defocus generation system; it was verified that, to first order, our setup generates pure defocus.

In DHWS light is integrated on the detector. This detector can have the form of either a specific region-of-interest on a CCD or an avalanche photodiode. To check the influence of the (relative) detector size, the curves of Fig. 5(a) are shown for different dimensions of the meta-detector. The radii are given in units of the diffraction-limited beam radius at the focus,

$$\omega_0 = \frac{2F\lambda}{\pi D}. \quad (12)$$

For our experiments $F = 800$ mm, $D = 7$ mm and $\lambda = 633$ nm which gives a diffracted-limited waist at the focus of $\omega_0 = 46 \ \mu m$. Fig. 5(b) shows the cross-section of the image formed by the SLM on the CCD.

The dynamic range of sensor output will decrease when increasing the size of the detector, Fig. 5(a). Naturally, the effect of noise will also be smaller for larger detectors. Sensitivity of the DHWS can be quantified using the slope of the response curve (cf. Fig. 5), the higher the magnitude of the slope the better. Some example values are listed in Table 2.

<table>
<thead>
<tr>
<th>Detector radius ($\omega_0$)</th>
<th>6.0</th>
<th>7.0</th>
<th>8.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope ($\lambda^{-1}$)</td>
<td>-0.963</td>
<td>-0.876</td>
<td>-0.781</td>
</tr>
</tbody>
</table>

One can observe that the response curves do not cross the origin. This offset $\Delta$, is due to static aberrations generated by the SLM and to misalignment between sensor and detector. The two images do not focus in the exact same plane perpendicular to the optical axis which causes one image to exhibit a higher intensity than
One can compensate for this offset by introducing the corresponding static modes with opposite sign into the SLM, by means of a CGH or when using a different detector for each image, by adjusting the measured intensities. Alternatively, one can remove the problem by shifting the measured response curves manually. The sensor can then yield the amplitude $A$ of defocus given the calibrated slope of the response curve, $m$:

$$A = \frac{1}{m} (S - \Delta),$$

where $S$ is the sensor output, and $\Delta$ is the offset.

In the second part of our work we sense more than one Zernike mode, making use of the SLM as both, aberrating medium and sensor. To test the efficacy of our SLM as a defocus generator we repeat the type of measurements outlined above but this time with the SLM as the defocusing element. This is shown in Fig. 6. The sensitivity of the DHWS for the same parameters as before (cf. Table 2) is shown now in Table 3. The last row shows the relative difference between the two approaches.

In Fig. 7(a) the sensor output curve from Fig. 6(a) for the integration area of radius $7\, \omega_0$, is presented with the mentioned offset corrected and together with the crosstalk from the Zernike modes $Z_4, Z_5, Z_6$ and $Z_7$. 

**Figure 5:** (a) Sensor’s response as a function of defocus introduced by the telescope, plotted as function of the meta-detector radius size and (b) cross-section of the image at the camera for various values of defocus.

**Figure 6:** (a) Sensors response as a function of defocus introduced by the SLM, plotted as function of the meta-detector radius size and (b) cross-section of the image at the camera for various values of defocus.
Table 3: Slope of the sensor output curve for different detector sizes when coding defocus in the SLM and relative difference with respect to the case when defocus is generated by the telescope.

<table>
<thead>
<tr>
<th>Detector radius ($\omega_0$)</th>
<th>6.0</th>
<th>7.0</th>
<th>8.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope ($\lambda^{-1}$)</td>
<td>-0.978</td>
<td>-0.814</td>
<td>-0.755</td>
</tr>
<tr>
<td>Relative difference with respect to the telescope system (%)</td>
<td>1.6</td>
<td>0.5</td>
<td>3.3</td>
</tr>
</tbody>
</table>

To determine the optimal detector size we consider two factors that characterize the performance of the DHWS. These are the (preferably high) slope $m$ and the (preferably low) cross-talk $\sigma_c$, caused by the presence of more than one wavefront aberrations. This performance optimization, given by

$$p = \frac{m}{\sigma_c},$$

is illustrated in Fig. 7(b).

As can be appreciated from Fig. 7(a), cross-talk between $Z_3$ and $Z_4$, $Z_5$, $Z_6$ or $Z_7$ is negligible. Next we consider other factors that directly affect the performance of the DHWS.

3.1 Effect of scintillation

In contrast to astronomy, wavefront sensing over long horizontal paths must deal with scintillation, i.e. amplitude variations across the pupil. To simulate this effect we place a mask, see Fig. 8(a), in front of the beam. This produces an intensity pattern in the SLM plane visually approximating scintillation effect, Fig. 8(b). The intensity is reduced by 90% due to the mask. Scintillation index defined by $\sigma_I^2 = (\langle I^2 \rangle / \langle I \rangle^2) - 1$, is 0.75. To study the effect of scintillation on the accuracy of DHWS, we introduce a known defocus aberration $b_n = 0.4 \lambda$ and we measure the sensor output while rotating the mask, creating different scintillation patterns. Using the linear approximation from Table 3 and taking the offset into account, we obtain the measured defocus amplitude. The resulting average defocus strength with scintillation is: $0.392 \pm 0.037 \lambda$.

Table 4 shows the standard deviations of defocus measurement for different detector sizes for ten rotations/measurements. We generally observe lower scattering for larger detector sizes. The variability due to scintillation is lowest for a detector radius of 6 $\omega_0$. This is very close to the radius previously found to be optimal, (Fig. 7(b)) considering cross-talk and linearity. Fig. 8(c) shows the sensor output under constant scintillation (no rotation of the mask) for different values of the introduced defocus and a detector size of 7 $\omega_0$. The black line shows the characteristic curve of the sensor for the corresponding detector size but without scintillation.
### Table 4: Standard deviation of ten experimental values under different scintillation patterns, for identical defocus magnitude of 0.4 \( \lambda \).

<table>
<thead>
<tr>
<th>Detector radius ( (\omega_0) )</th>
<th>2.4</th>
<th>4.0</th>
<th>6.0</th>
<th>7.0</th>
<th>8.0</th>
<th>9.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std.dev. ( (\lambda) )</td>
<td>0.062</td>
<td>0.054</td>
<td>0.036</td>
<td>0.037</td>
<td>0.038</td>
<td>0.039</td>
</tr>
</tbody>
</table>

**Figure 8:** (a) Scintillation mask use in the experiments. (b) Resulting intensity distribution in the conjugate pupil plane and (c) sensor output as a function of amplitude of defocus of the incoming beam for a constant scintillation.

### 3.2 Tip/tilt and misalignment

The effect of a tilted wavefront on the DHWS also has to be considered. When the direction of the incoming beam differs from the direction of the reference beam in Eq. (2), the positions of the reconstructed images will change. Unless an automatic system is implemented to always follow the center of the spot on the CCD, one has to make sure the sensor is robust against residual tip/tilt, especially when photodiodes are implemented as detectors. Optimization of detector size is of crucial importance here.

To study this effect we analyzed several scenarios of Gaussian beam propagation in turbulence. For beam radii between 5 and 15 cm, wavelengths in the range 500 to 1500 nm, propagation paths between 1 and 3 km, and \( C_n^2 \) values between \( 10^{-13} \) and \( 10^{-14} \) m\(^{-2/3}\) we found, using various formulae in the literature,\(^{21,22}\) that the rms beam wander or alternatively angle-of-arrival, is on the order of 10 \( \mu \)rad. The angular pixel size of the CCD in our setup also happens to be 10 \( \mu \)rad. Thus, in our case the consequence of wavefront tip/tilt would be a random change of spot position with a standard deviation of roughly one pixel in the detector plane, i.e. 9 \( \mu \)m (0.2 \( \omega_0 \)). It should be noted that both spots corresponding to one Zenike mode, will move by exactly the same amount. In the previous sections we have demonstrated that the meta-detector size optimized for sensitivity, minimization of cross-talk and scintillation effects is around 7 \( \omega_0 \) (300 \( \mu \)m for our detector). As this size is much bigger than the range of expected tip and tilt, we expect no effect on the DHWS performance. This is also demonstrated experimentally in Fig. 9(a). The blue line is the curve from Fig. 7(a). Fig. 9(b) shows the relative error as a function of the spots’ shift for a detector size of 7 \( \omega_0 \). Here, a constant defocus of value 0.4 \( \lambda \) was projected through the system. The values for \( I_n \) and \( I_{-n} \) (from Eq. (5)) were read out at off-axis positions separated from the center by values indicated on the x-axis. The sensor’s output in the presence of tip/tilt was then compared with the tip/tilt-free response curve (blue line from Fig. 9(a)). The absolute value of the relative error was then computed. The effect is small even for shifts ten times larger than the expected rms tip/tilt. The effect becomes significant for detector size around 3 \( \omega_0 \). Such a small detector would also be susceptible to scintillation (see Table 4) and is therefore of less interest. The effect of tip/tilt on the other modes, especially coma, is left for future work.

To study the effect of misalignment we consider two components: the first corresponds to a lateral beam shift that occurs with respect to the SLM display, and the second is the tilt with which the wavefront reaches the SLM. Fig. 10(a) shows the effect of a lateral shift amounting to 25% of the beam diameter. Fig. 10(b) shows the effect of a tilted wavefront with respect to the plane of the SLM display. The DWHS is very robust to these realistic misalignment values.
Figure 9: (a) Sensor’s response as a function of the defocus amplitude of the incoming beam, for different tilts of the beam with respect the optical axis of the sensor (b) relative difference, in percentage, of the sensor output for a defocus of 0.4 $\lambda$ when tilt is present, with respect to the sensor value in absence of tilt, the solid line corresponds to a detector size of 7 $\omega_0$ whereas the dashed line, for a detector size of 3.9 $\omega_0$.

Figure 10: (a) Sensor output when the beam experiences a lateral shift of 25% of the beam diameter. (b) Sensor output when the beam reaches the SLM with an angle of 5 mrad. The black curve in both graphics is the sensor response of Fig 7(a).

The previous sections were devoted to the defocus operation mode of the DHWS. In the same way the sensor can be built to detect any other Zernike mode. This was done for the Zernike modes $Z_4$ to $Z_{12}$. Fig. 11 shows the response curves for these modes with offsets removed. The size of the detector was optimized for each channel (cf. Fig. 7(b)), thus dynamic ranges and slopes vary from mode to mode (Table 5).

Table 5: Slopes of the characteristic curves for a mode amplitude interval of 1.4 $\lambda$, from -0.7 $\lambda$ to 0.7 $\lambda$.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
<th>$Z_6$</th>
<th>$Z_7$</th>
<th>$Z_8$</th>
<th>$Z_9$</th>
<th>$Z_{10}$</th>
<th>$Z_{11}$</th>
<th>$Z_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detector radius ($\omega_0$)</td>
<td>3.0</td>
<td>3.0</td>
<td>1.0</td>
<td>1.0</td>
<td>7.0</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Slope ($\lambda^{-1}$)</td>
<td>-1.05</td>
<td>-0.88</td>
<td>-0.61</td>
<td>-0.54</td>
<td>-2.08*</td>
<td>-0.50</td>
<td>-0.48</td>
<td>-2.30*</td>
<td>-2.33*</td>
</tr>
</tbody>
</table>

*Slope defined only for the interval -0.3 $\lambda$ to 0.3 $\lambda$
4. MULTICHANNEL OPERATION MODE

In this section we show the realization and test the multichannel version of the DHWS for measurement of more than one Zernike mode. We chose to implement the sensor for measurements of defocus and the two astigmatisms. We consider a circular distribution of the channels in the detector plane (Fig 12(a)), thus ensuring a uniform intensity distribution between the different spots because intensity of the diffraction pattern from the SLM decays as one moves from the optical axis.

To obtain the CGH a multiplex of six holograms, two for each channel, is necessary. Fig 12(a) shows an example of coded CGH used in the multichannel DHWS. It should be noted that increasing the number of holograms to multiplex directly affects the signal to noise ratio of the sensor. Fig. 12(b) shows the image observed in the detector plane (zero order from the SLM has been blocked); the colors have been inverted and central parts of the spots are deliberately saturated to bring out the faint structures.

![Figure 12](image1.png)

**Figure 12:** (a) Multiplex CGH for measuring simultaneously defocus and astigmatisms and (b) corresponding image of the detector plane, zero order from the SLM has been blocked, the colors have been inverted and central parts of the spots are deliberately saturated.

Just like in the single-channel mode, we are interested in knowing the sensor’s output for a given channel when applying different aberrations to the incoming beam. For the channel under investigation we optimize the meta-detector size following the same principle as before. Figs. 13(a)-13(c) show the sensor’s response curves with offset correction for the three channels individually, when other aberrations were present in the beam. In Table 6. the slopes of the characteristic curves are listed.

![Figure 11](image2.png)

**Figure 11:** Characteristic curves for the Zernike modes $Z_4$-$Z_{12}$ choosing the most convenient detector size separately for each channel.
Figure 13: Sensor response curves for the channels $Z_3$, $Z_4$ and $Z_5$ when a variable mode amplitude of (a) $Z_3$, (b) $Z_4$ and (c) $Z_5$, is introduced in the tested beam.

The cross talk is very small and does not change appreciably with the amplitude, this can be attributed to diffraction noise and dynamic range limitations of the sensor. Suitable pinholes would reduce the light coming from other diffraction orders and channels.

Table 6: Slopes of the characteristic curves from the three channels of the multimodal wavefront sensor, from $-0.7 \lambda$ to $0.7 \lambda$, for a recorded mode amplitude interval of $1.4 \lambda$.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope ($\lambda^{-1}$)</td>
<td>-0.871</td>
<td>-0.870</td>
<td>-0.982</td>
</tr>
<tr>
<td>Detector radius ($\omega_0$)</td>
<td>8.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Next we test the DHWS by introducing to the incoming beam a known aberration. We generate a random sample of Gaussian-distributed Zernike amplitudes with standard deviation of $0.5 \lambda$, see Table 7.

This approach can also be used to simultaneously measure higher number of Zernike modes. Fig. 14(a), as an example, corresponds to the CGH for 13 Zernike modes: $Z_3$ to $Z_{15}$, and Fig. 14(b) is the image displayed in the same way as Fig. 12(b), of the detector plane, when a defocus of $0.3 \lambda$ is present in the incoming beam.
Table 7: Amplitudes of the three Zernike modes implemented in the CGH, for some random but known aberrated wavefronts.

<table>
<thead>
<tr>
<th>Introduced amplitudes ((\lambda))</th>
<th>Sensor output amplitudes ((\lambda))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Z_3) (Z_4) (Z_5) (Z_3) (Z_4) (Z_5)</td>
</tr>
<tr>
<td>-0.19 0.26 0.44</td>
<td>-0.17 0.23 0.46</td>
</tr>
<tr>
<td>-0.25 -0.02 0.05</td>
<td>0.23 -0.03 0.09</td>
</tr>
<tr>
<td>-0.39 0.03 0.00</td>
<td>-0.36 0.02 0.01</td>
</tr>
<tr>
<td>0.04 0.18 -0.29</td>
<td>0.01 0.20 -0.29</td>
</tr>
<tr>
<td>-0.45 0.06 -0.39</td>
<td>-0.42 0.05 -0.38</td>
</tr>
<tr>
<td>-0.29 0.06 0.22</td>
<td>-0.28 0.04 0.22</td>
</tr>
<tr>
<td>0.45 -0.14 0.25</td>
<td>0.42 -0.12 0.26</td>
</tr>
<tr>
<td>-0.15 0.01 -0.18</td>
<td>-0.17 -0.02 -0.18</td>
</tr>
<tr>
<td>std.dev. ((\lambda))</td>
<td>0.03 0.02 0.02</td>
</tr>
</tbody>
</table>

Figure 14: (a)CGH for the performance of the sensor in detecting simultaneously the mode amplitude of the Zernike polynomials \(Z_3\)\(\ldots\)\(Z_{15}\) and (b) image of the detector plane.

5. CONCLUSIONS

We have implemented a digital holographic wavefront sensor for the case of single- and multi-mode operations. We have presented a strategy for the optimization of the sensor’s detector size, taking into account maximum sensitivity, minimum cross-talk and robustness to scintillation. Cross-talk can be virtually eliminated by the careful choice of detector size for each mode. The effects of three components of tip/tilt have also been investigated. Beam movement in the detector plane under realistic assumptions plays a small role at least for the measurement of defocus. Beam misalignment and tilted arrival on the SLM have equally small effect. We have shown that DHWS when operated in conditions characterized by the scintillation index of 0.75 yields errors as small as 0.05 \(\lambda\). To our knowledge this is the only sensor that is robust against scintillation.

REFERENCES


